Quantum Uncertainty of Light Fields and Energy Quantization

- how comes light is best described by operators and what does it mean?

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pre view

- field quantization

- coherent state
- real laser
- generation of high power light beams
- relevance of quantum uncertainty
- modification of the quantum uncertainty by light matter interaction
- quantum dynamics & Wigner flow
- effect of losses
- designing material with high nonlinear coefficient
- quantum metrology with high power laser
- nonlinear optics with squeezed vacuum
- the quantum vacuum

Fig. 1: optical resonator

$$\left(\vec{\nabla}^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{E}(\vec{r},t) = 0$$

$$\vec{E}(\vec{r},t) = \vec{u}(\vec{r})\,q(t) \quad \Rightarrow \quad \int_V \varepsilon_0 \vec{u}^2(\vec{r})\,dV = 1$$

→ q^2 has dimension **energy**

$$\begin{cases} (a) \quad \vec{\nabla}^2\vec{u}(\vec{r}) - R\vec{u}(\vec{r}) = 0 \\ (b) \quad \dfrac{1}{c^2}\dfrac{\partial^2}{\partial t^2}q(t) - Rq(t) = 0 \end{cases}$$

Maxwell's equations

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\vec{\nabla}\cdot\vec{B} = 0$$

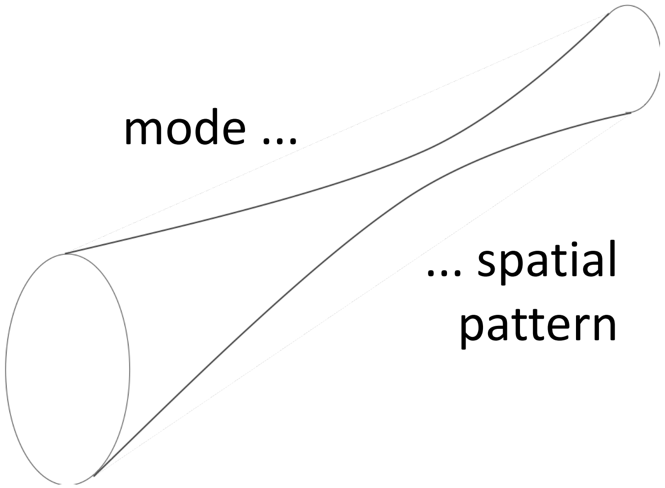
$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

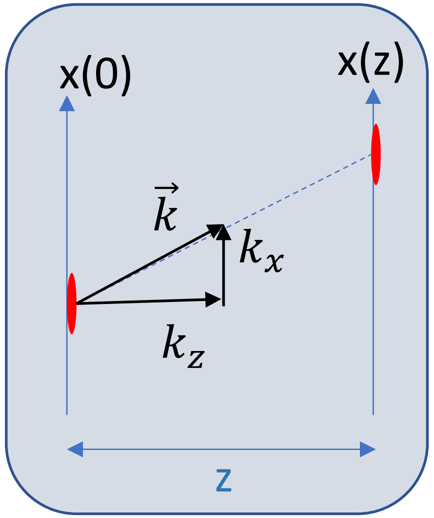
$$\vec{\nabla}\times\vec{B} = \mu_0\vec{j} + \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$$

diffraction

Helmholtz - Equation

$$\begin{cases} (a) \quad \vec{\nabla}^2 \vec{u}(\vec{r}) - R\vec{u}(\vec{r}) = 0 \\ (b) \quad \frac{1}{c^2}\frac{\partial^2}{\partial t^2} q(t) - Rq(t) = 0 \end{cases}$$





$x(0)\,,\; k_x(0)$

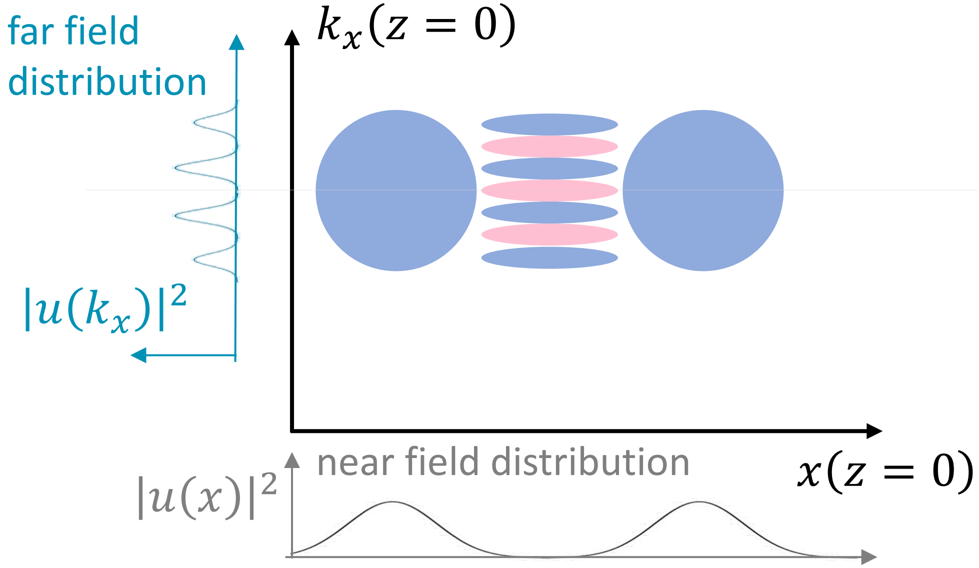
$x\,, \qquad \frac{\partial x}{\partial z} = \frac{k_x}{k_z} \approx \frac{k_x}{k}$

$I(x)\,, \qquad I(k_x)$

$u(x)\,, \qquad u(k_x)$

(fractional) Fourier transform

“lab“ phase space of classical optics



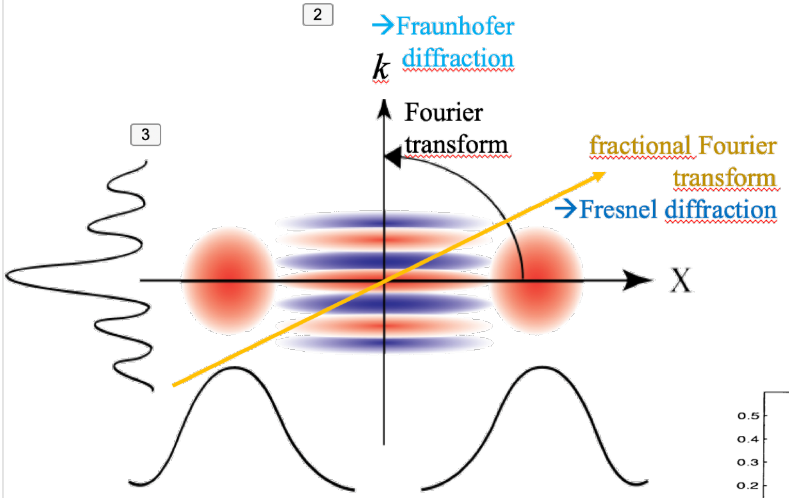
Phase space distribution function:

Wigner function

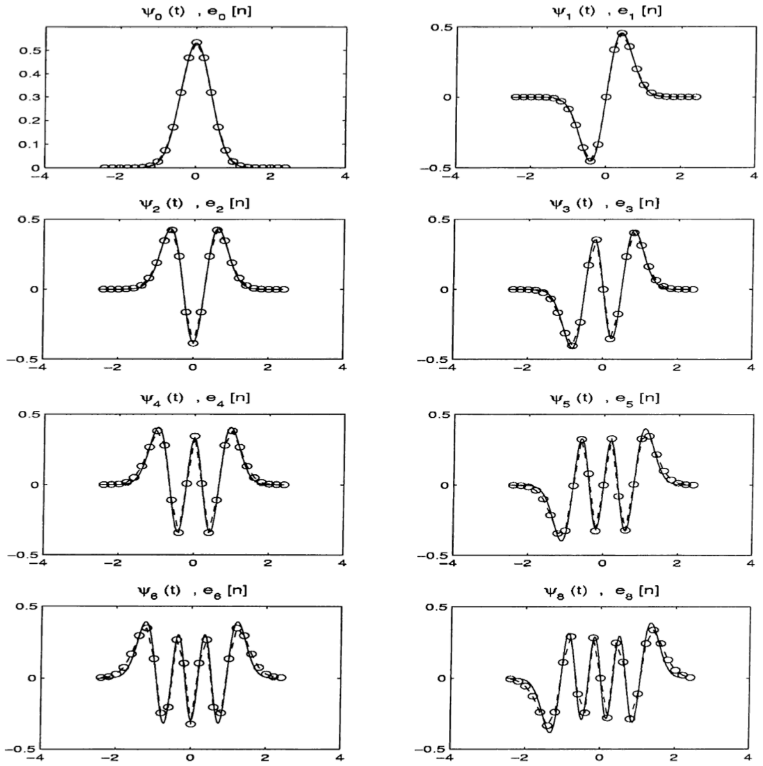
Transverse mode pattern in lasers

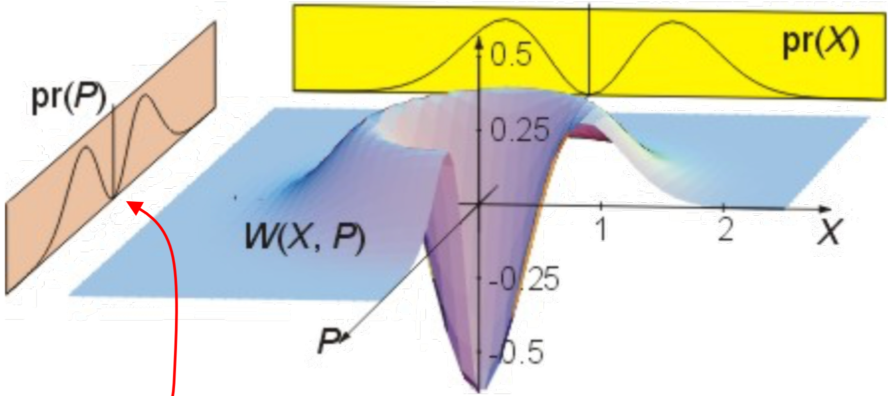
→ invariant under proagation
→ invariant under Fourier transformation
→ rotationally symmetric Wigner function

W(x,k) → Wigner function



Hermite Gaussian functions





zero requires negative values of pr(*P*)

$$q(t)=q(0)\cos(\omega t)+\frac{\dot{q}(0)}{\omega}\sin(\omega t)$$

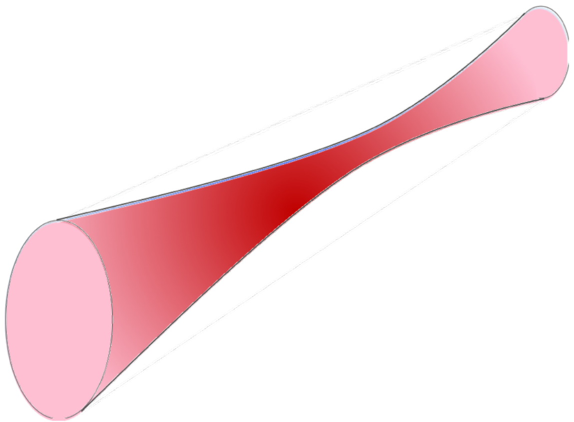
(b) → $$\frac{1}{\omega^2}\left(\dot{q}(0)\right)^2+\left(q(0)\right)^2=S$$

Equation describing temporal evolution

$$\begin{cases} (a) \quad \vec{\nabla}^2\vec{u}(\vec{r})-R\vec{u}(\vec{r})=0 \\ (b) \quad \dfrac{1}{c^2}\dfrac{\partial^2}{\partial t^2}q(t)-Rq(t)=0 \end{cases}$$

→ Harmonic oscillator

excitation of mode



phase space diagram

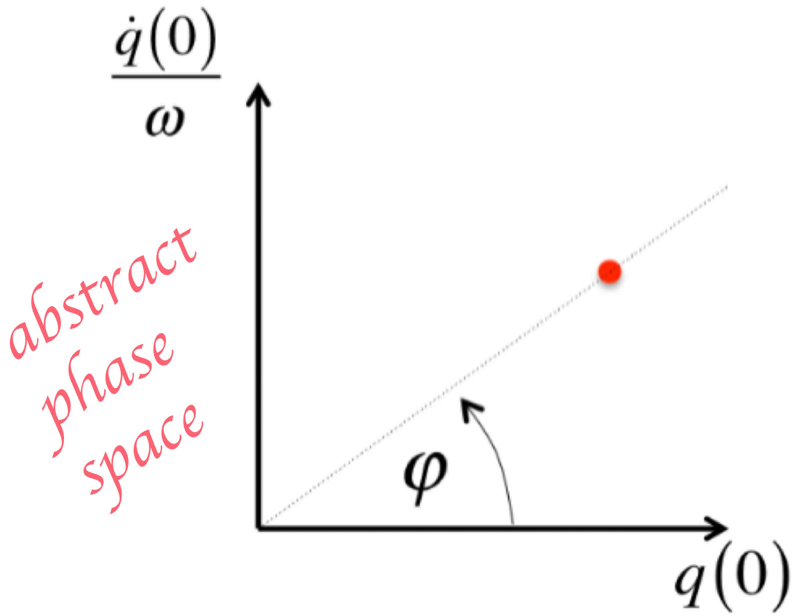
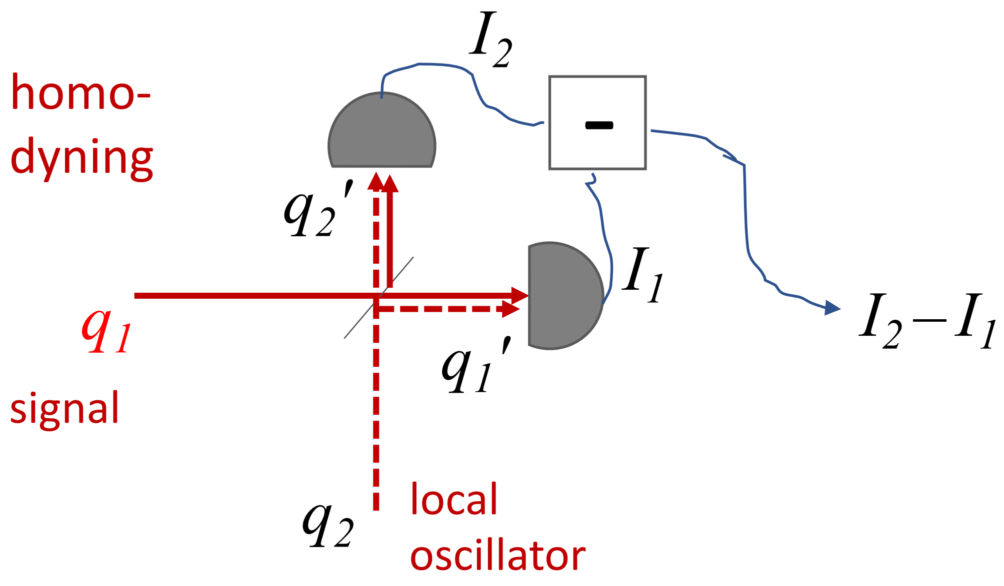


Fig. 2: phase space representation of a classical field: a single point.

→ q^2 has dimension energy

Measurement of q and $\dot{q}$ with homodyning

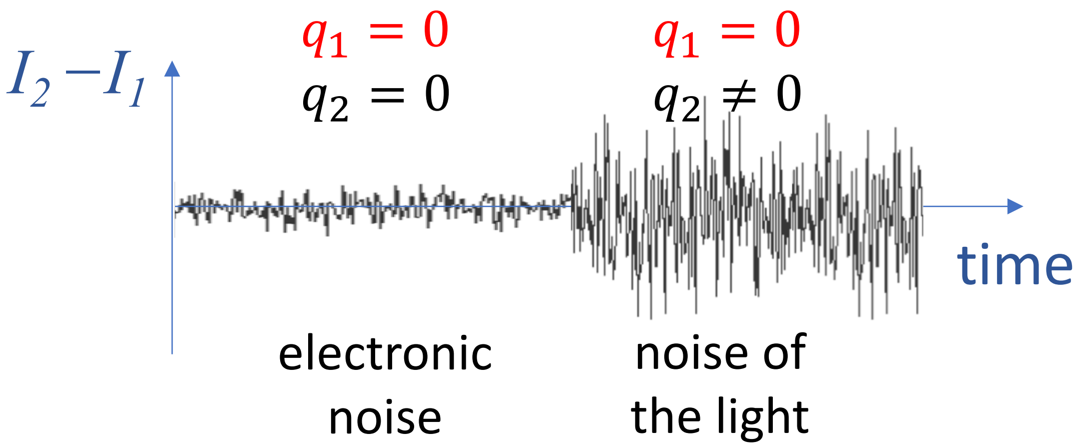
amplitude & phase measurement



$$I_2 - I_1 \propto 2 \cdot \Re\{q_1 q_2^*\}$$

If $q_1 = 0$ → $I_2 - I_1 = 0$

But experiment shows noise:



$$q_1' = (q_1 - q_2)/\sqrt{2}$$
$$q_2' = (q_1 + q_2)/\sqrt{2}$$

$$I_1 \propto |q_1'|^2 = (|q_1|^2 + |q_2|^2 - q_1 q_2^* - q_2 q_1^*)/2$$

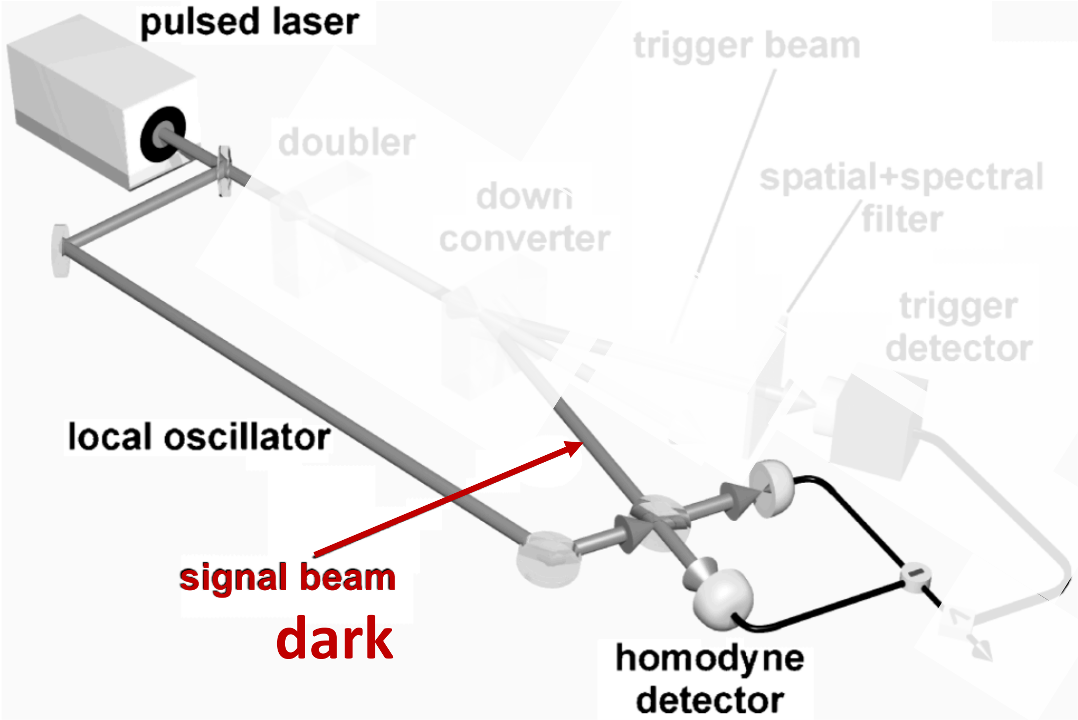
$$I_2 \propto |q_2'|^2 = (|q_1|^2 + |q_2|^2 + q_1 q_2^* + q_2 q_1^*)/2$$

→obviously:
$\langle q_1 \rangle = 0$ and $\langle q_1^2 \rangle \neq 0$

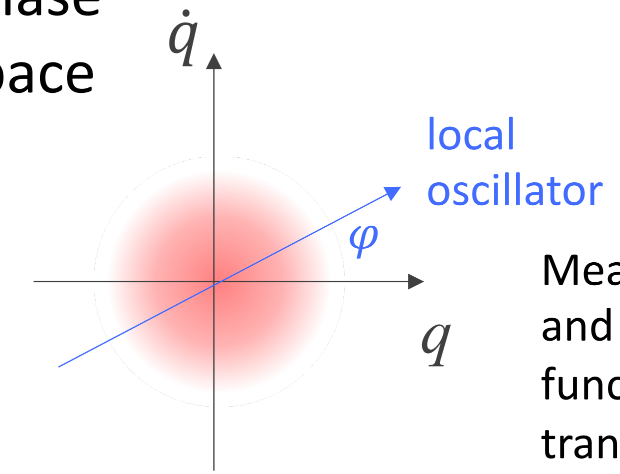
quantum state reconstruction of the vacuum state (zero-photon Fock state)

A I Lvovsky et al., Phys. Rev. Lett. 87, 050402 (2001)

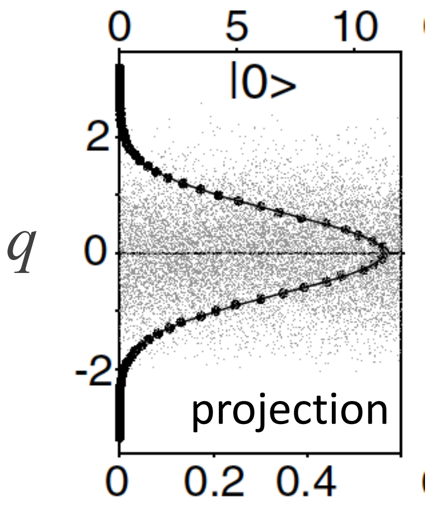
Homodyning → projection of phase space distribution onto “φ“

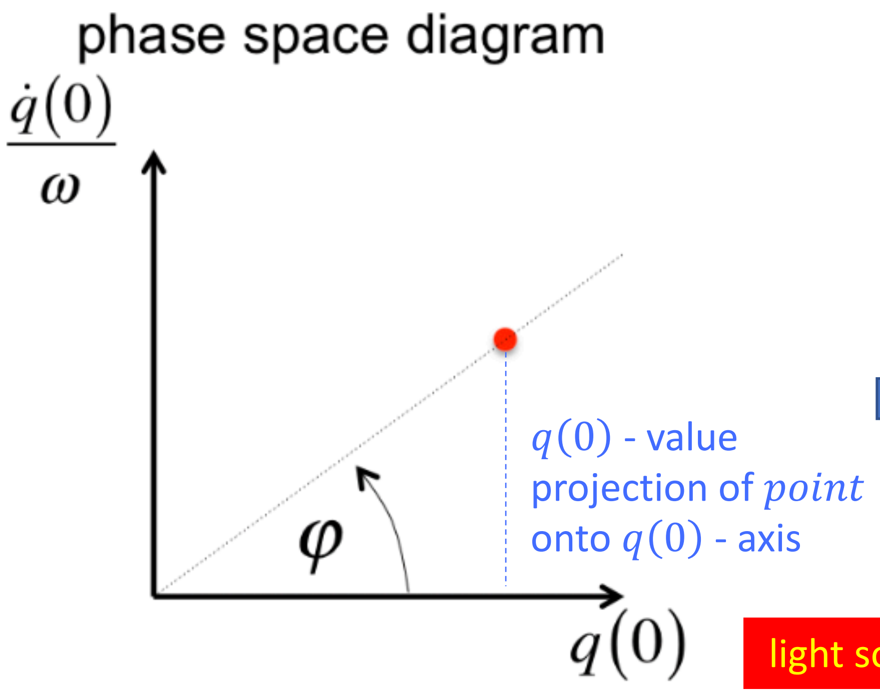


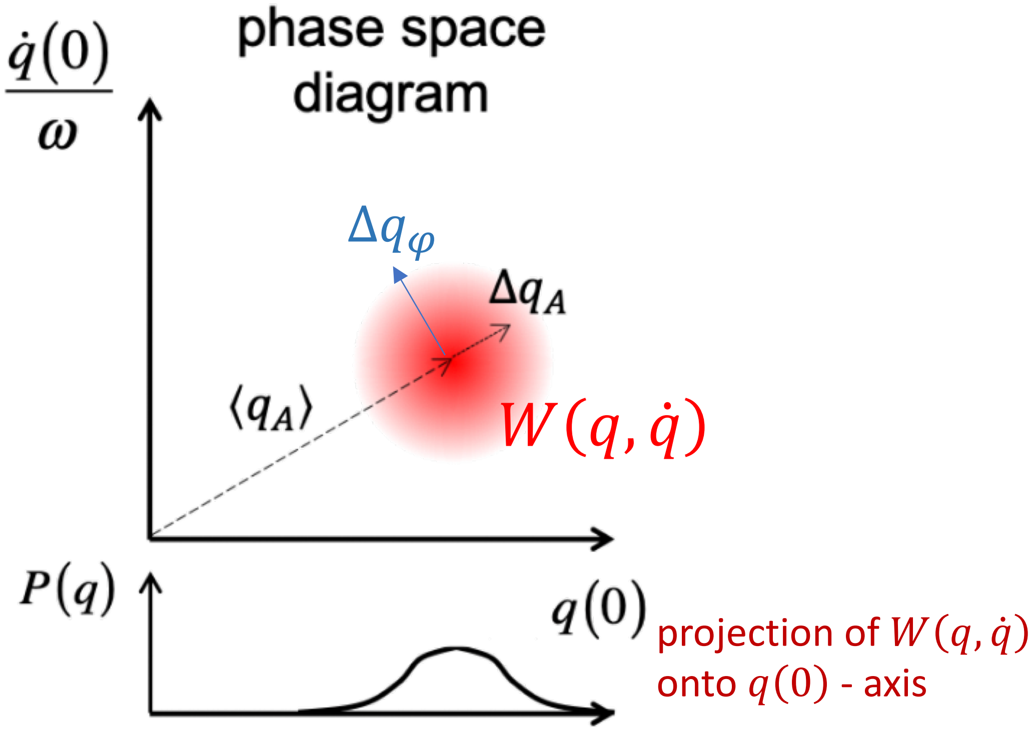
phase space

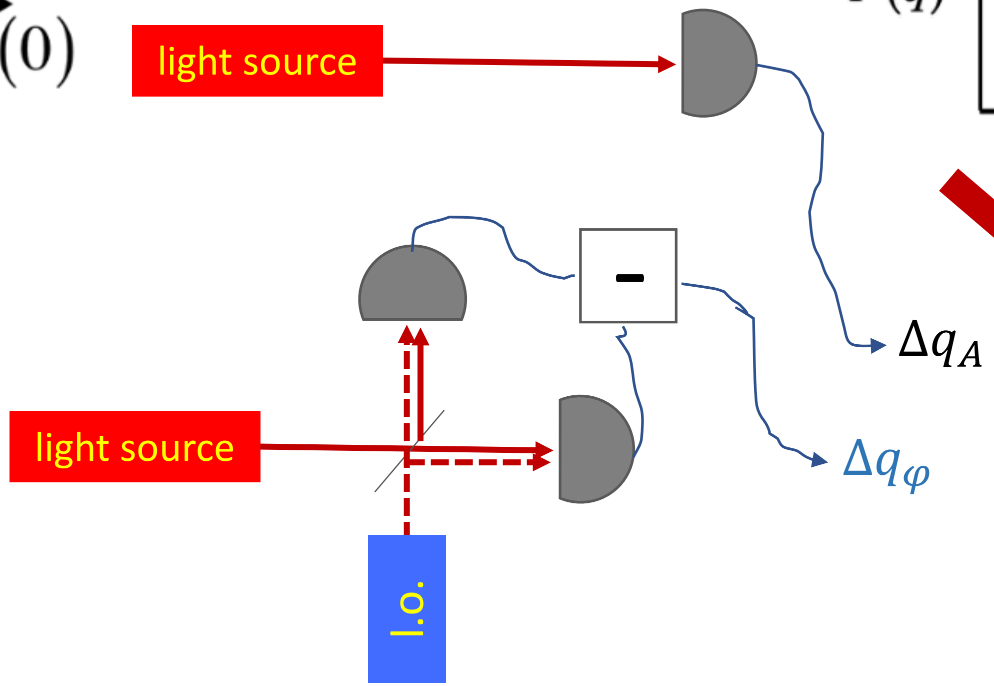


Measure for different φ and reconstruct Wigner function, e.g. by Radon transformation







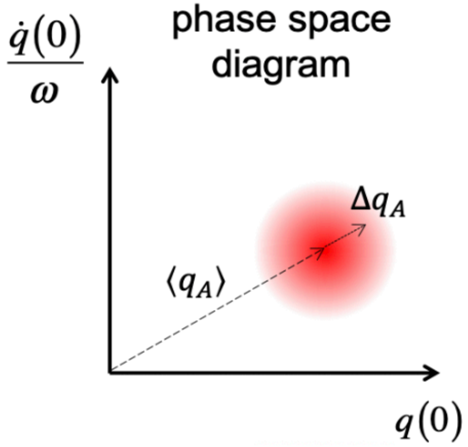


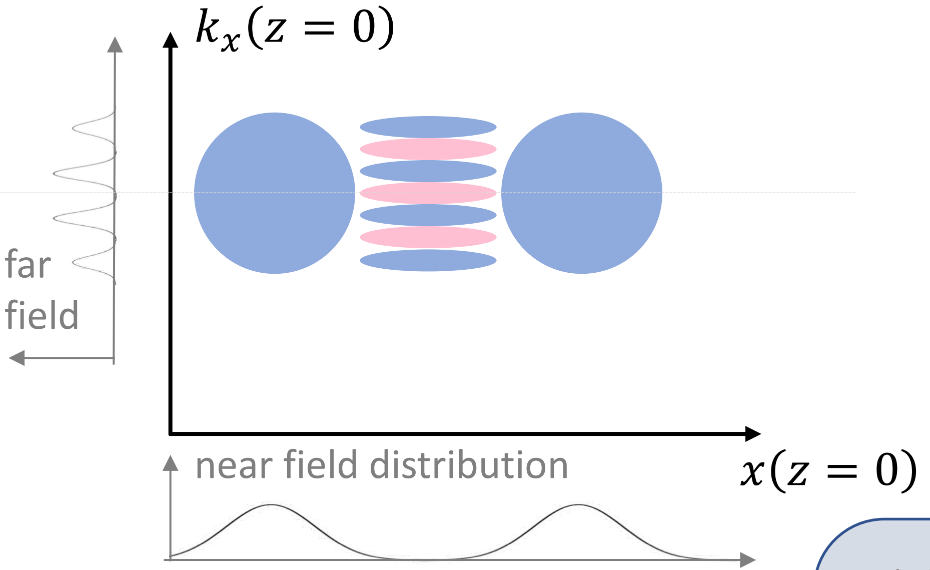
- distributions $P(q), P(\dot{q})$
- minimum area in phase space
- for laser: symmetric

➢ $P(q)$ dependent on $P(\dot{q})$

abstract phase space where excitation "lives"

"lab" phase space of classical optics





$q(t)\,,\ \dot{q}(t)$

$q = q(0), \quad \dot{q} = \dot{q}(0)$

$P(q)\,, \qquad P(\dot{q})$

$\psi(q), \qquad \psi(\dot{q})$

!

(fractional) Fourier transform

$t \Leftrightarrow z$

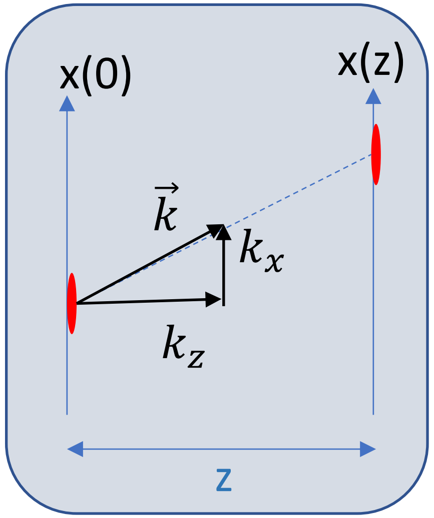
$x(0)\,,\ k_x(0)$

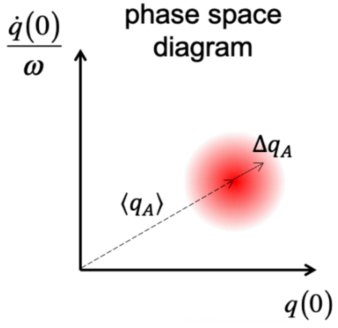
$x\,, \qquad \frac{\partial x}{\partial z} = \frac{k_x}{k_z} \approx \frac{k_x}{k}$

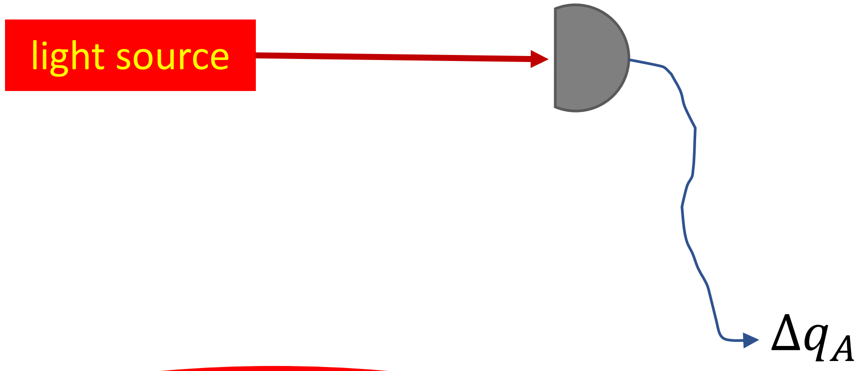
$I(x)\,, \qquad I(k_x)$

$u(x)\,, \qquad u(k_x)$

(fractional) Fourier transform







p is conjugate to variable q

→ $p \sim \dot q$

$$P(q) = \Psi^*(q)\Psi(q)$$

$$\Psi(q) = \int dp\,\tilde\Psi(p)e^{iqp}$$

$$\int dq\,\Psi^*(q)(-i)\frac{\partial}{\partial q}\Psi(q) = \int dp\,\tilde\Psi^*(p)\,p\,\tilde\Psi(p)$$

$$p \equiv -i\frac{\partial}{\partial q}$$

$$\frac{1}{\omega^2}(\dot q(0))^2 + (q(0))^2 = S$$

$$-\left(\frac{\hbar\omega}{2}\frac{\partial}{\partial q}\right)^2\Psi(q) + q^2\Psi(q) = S\Psi(q)$$

quantitative result:

He Ne laser of 1mW

root mean square power fluctuations in radio frequency band of 1 MHz:

$2.5 \cdot 10^{-8}$ Watt

same for phase .....

$$\pi\sqrt{\langle(\Delta q)^2\rangle}\sqrt{\left\langle\left(\Delta\left(\frac{\dot q}{\omega}\right)\right)^2\right\rangle} = \frac{\pi}{2}1.56\cdot 10^{-19}J$$

$$\frac{\dot q}{\omega} \approx 0.5\cdot 10^{-34}[Js]\,\omega p$$

$\hbar$

$$\frac{1}{\omega^2}(\dot{q}(0))^2 + (q(0))^2 = S$$

$$-\left(\frac{\hbar\omega}{2}\frac{\partial}{\partial q}\right)^2 \Psi(q) + q^2\Psi(q) = S\Psi(q)$$

interestingly enough:
eigen functions the same as for
laser modes !!!

eigen functions

$$\hat{a} = \frac{1}{\sqrt{\hbar\omega}} q + \frac{\sqrt{\hbar\omega}}{2}\frac{\partial}{\partial q} = \frac{1}{\sqrt{\hbar\omega}} q + i\frac{\sqrt{\hbar\omega}}{2} p$$

$$\hat{a}^\dagger = \frac{1}{\sqrt{\hbar\omega}} q - \frac{\sqrt{\hbar\omega}}{2}\frac{\partial}{\partial q} = \frac{1}{\sqrt{\hbar\omega}} q - i\frac{\sqrt{\hbar\omega}}{2} p$$

$$\left.\begin{aligned} \Psi_0(q) &= N_0 e^{-\frac{q^2}{\hbar\omega}} \\ \Psi_1(q) &= N_1 q e^{-\frac{q^2}{\hbar\omega}} \end{aligned}\right\} |n\rangle$$

$$\vdots$$

field operators

$$\int dq \Psi^*(q)\left(\hbar\omega\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right)\right)\Psi(q) = S$$

$$\hat{S} = \hbar\omega\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right)$$

$$\hat{a}|n\rangle = \sqrt{n}\,|n-1\rangle$$

$$\hat{a}^\dagger|n\rangle = \sqrt{n+1}\,|n+1\rangle$$

eigen values

$$S_n = \left(n + \frac{1}{2}\right)\hbar\omega$$

field operators

$\hat{a}|n\rangle = \sqrt{n}\,|n-1\rangle$

$\hat{a}^\dagger|n\rangle = \sqrt{n+1}\,|n+1\rangle$

$\hat{a}^\dagger\hat{a}|n\rangle = n\,|n\rangle$

$\hat{a}\hat{a}^\dagger|n\rangle = (n+1)\,|n\rangle$

$[\hat{a}, \hat{a}^\dagger] = \hat{a}\hat{a}^\dagger - \hat{a}^\dagger\hat{a} = \hat{1}$

$\{|n\rangle, n = 0, 1, 2, \ldots\}$
ortho-normal
basis of Hilbert space

eigen functions of $\hat{a}$?

$$\hat{a}\sum_{n=0}^{\infty} c_n|n\rangle \overset{?}{=} \alpha \sum_{n=0}^{\infty} c_n|n\rangle \equiv \alpha|\alpha\rangle$$

$$|\alpha\rangle = e^{-|\alpha|^2/2}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle$$ ✓

coherent state

eigen functions of $\hat{a}^\dagger$?

$$\hat{a}^\dagger\sum_{n=0}^{\infty} c_n|n\rangle \overset{?}{=} \beta \sum_{n=0}^{\infty} c_n|n\rangle \equiv \beta|\beta\rangle$$

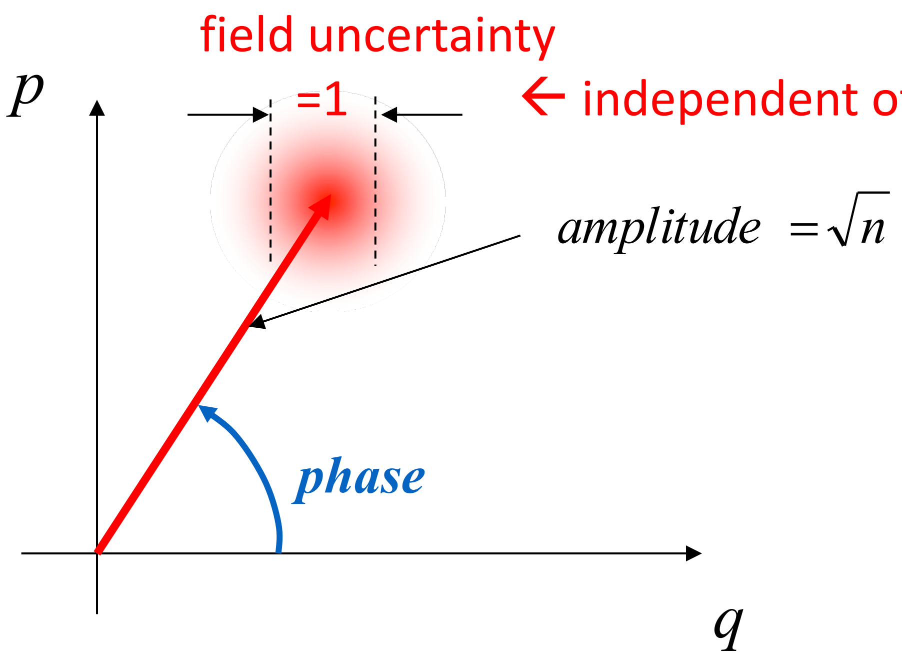
No!

end of introductory part

coherent state

phase space diagram for light field

equal and minimum uncertainty of X and P: coherent state
→ **laser**



← independent of *amplitude* !!!

q, p - mean has continuous spectrum
- variances obey uncertainty relation

→ q, p are non-commuting operators,

$$\hat{q} = \left(\hat{a} + \hat{a}^\dagger\right)/2$$
$$\hat{p} = \left(\hat{a} - \hat{a}^\dagger\right)/(2i)$$

$$\langle(\Delta q)^2\rangle = \frac{1}{4}$$

$$\begin{aligned} n &= \langle n\rangle + \Delta n \\ &= (\langle q\rangle + \Delta q)^2 \\ &= \langle q\rangle^2 + 2\langle q\rangle\Delta q + \langle \Delta q\rangle^2 \\ &= \langle n\rangle + \sqrt{\langle n\rangle}\cdot 2\Delta q + 1/4 \end{aligned}$$

$$\langle(\Delta n)^2\rangle = \langle n\rangle$$

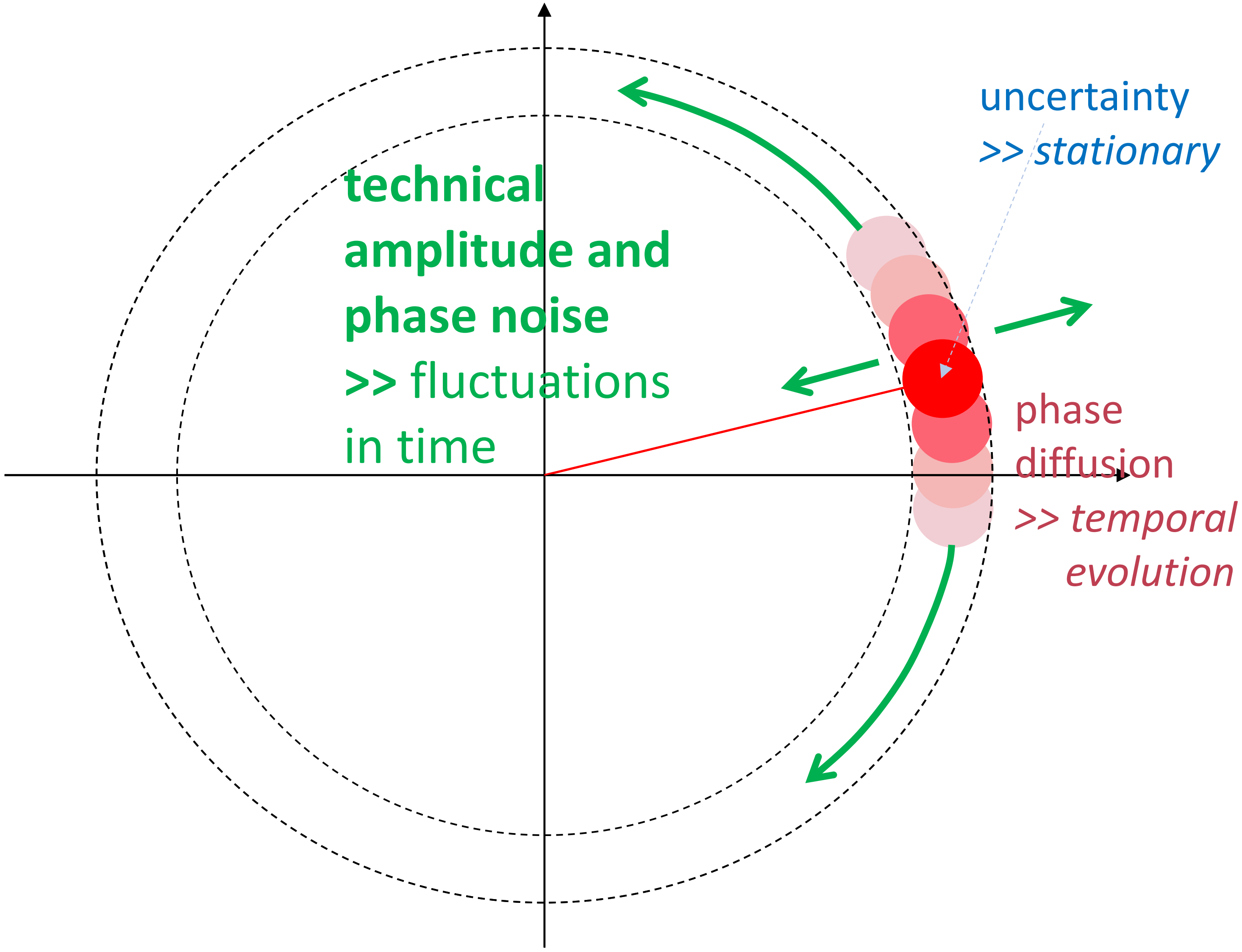
Poisson statistics

real laser

if measurement time interval short enough
>>> coherent state

short enough:

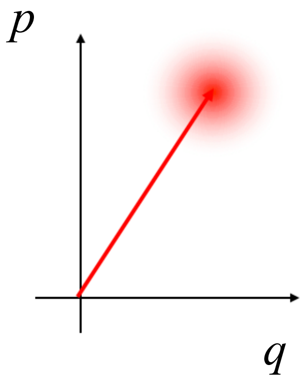
→ shorter than time constant of fluctuation and phase diffusion



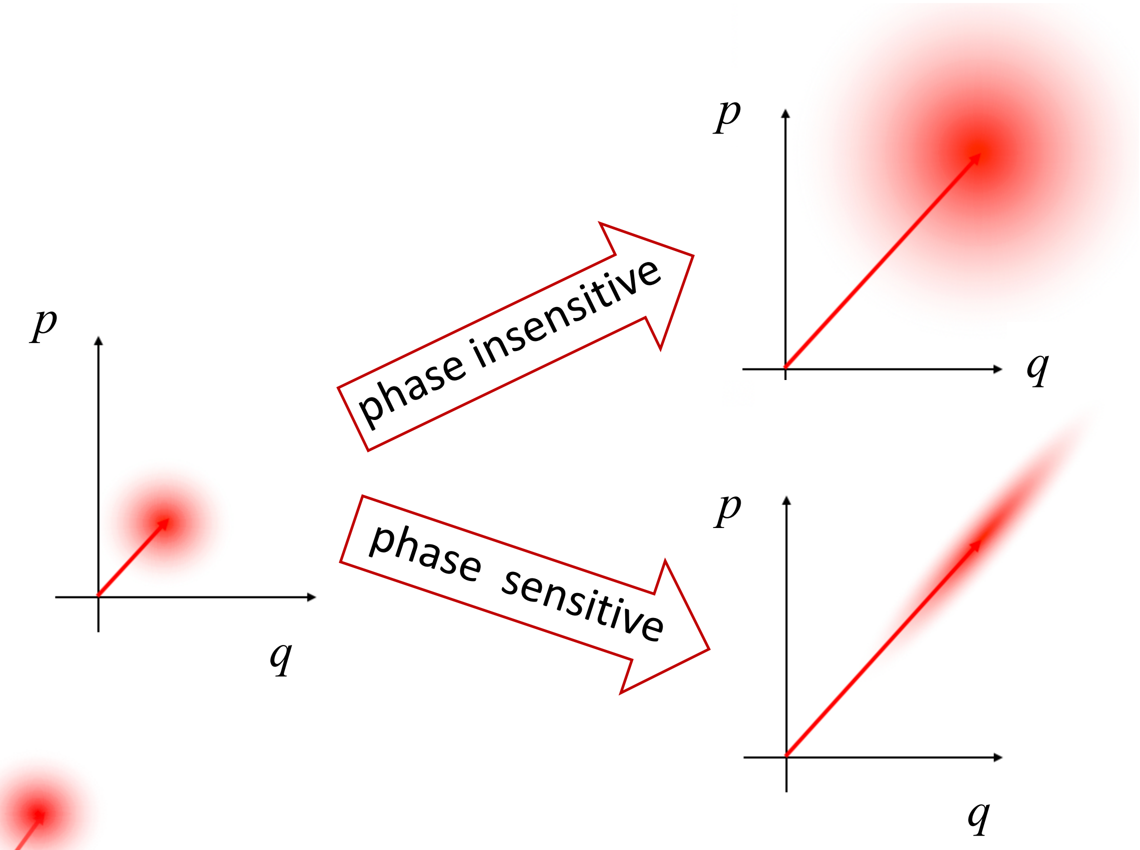
generation of high power light beams

laser resonator emitting high power

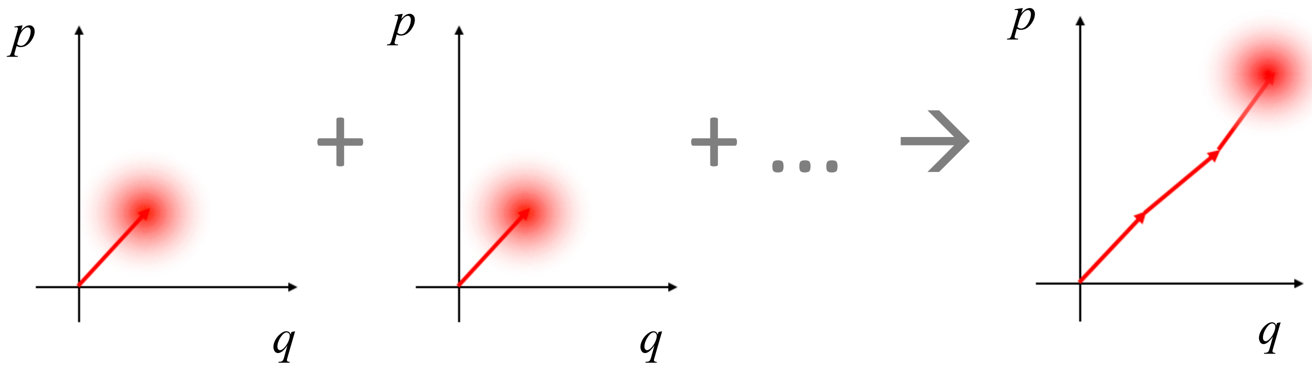
(& injection locking with seed laser)



amplification of laser radiation



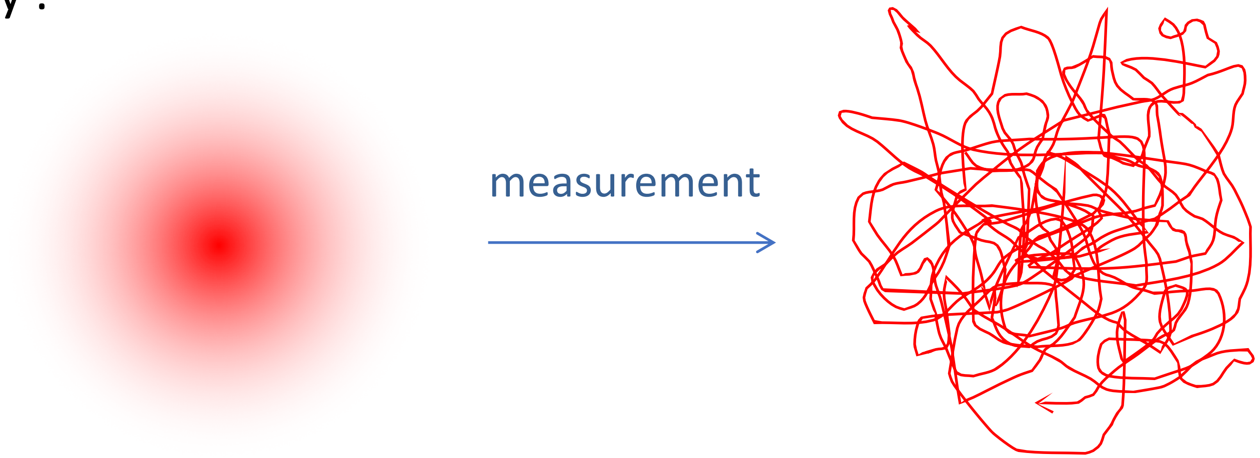
coherent combination



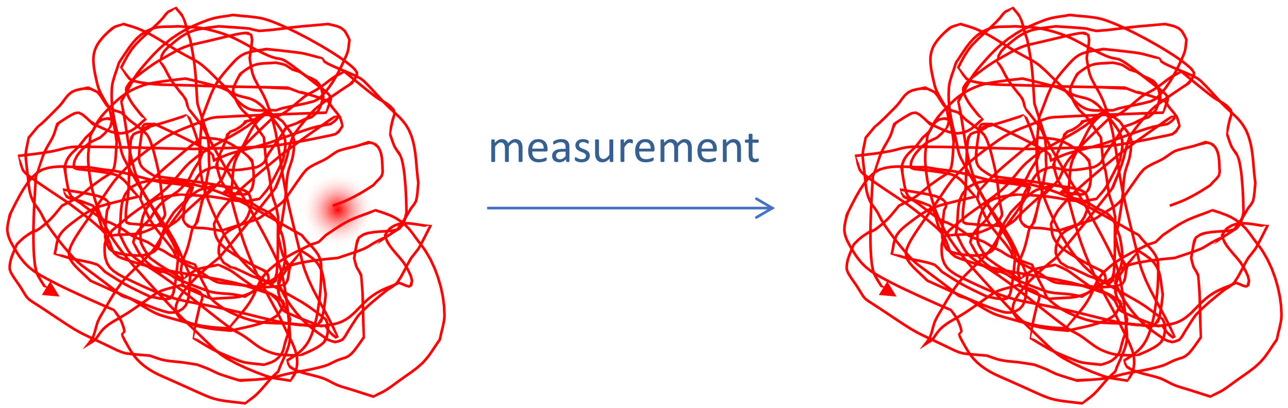
The standard quantum limit of coherent beam combining C.R. Müller, F. Sedlmeir, V.O. Martynov et al. New J. Phys. 21, 093047 (2019)

pure state versus mixed state

→ quantum uncertainty :



→ thermal noise :

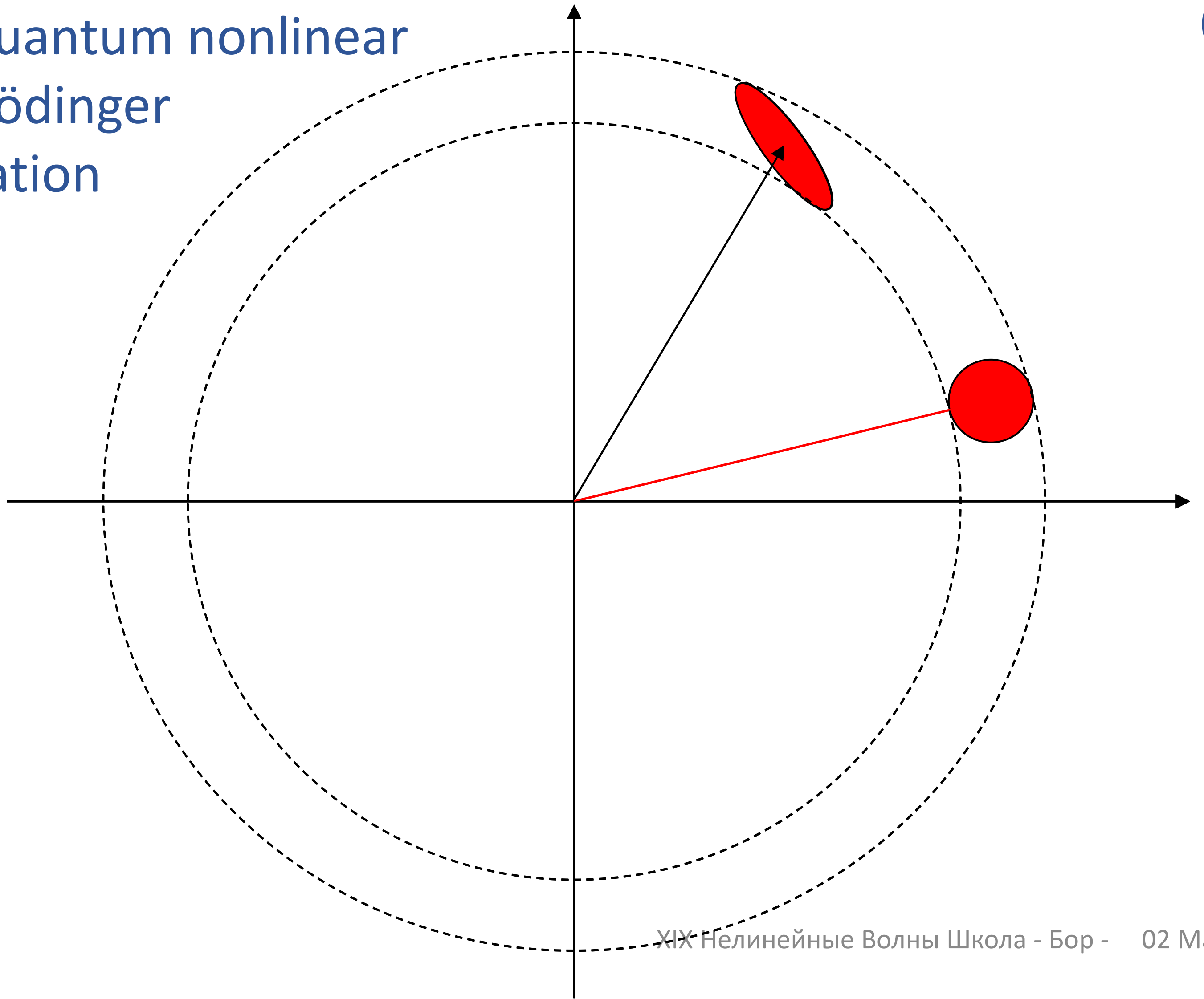


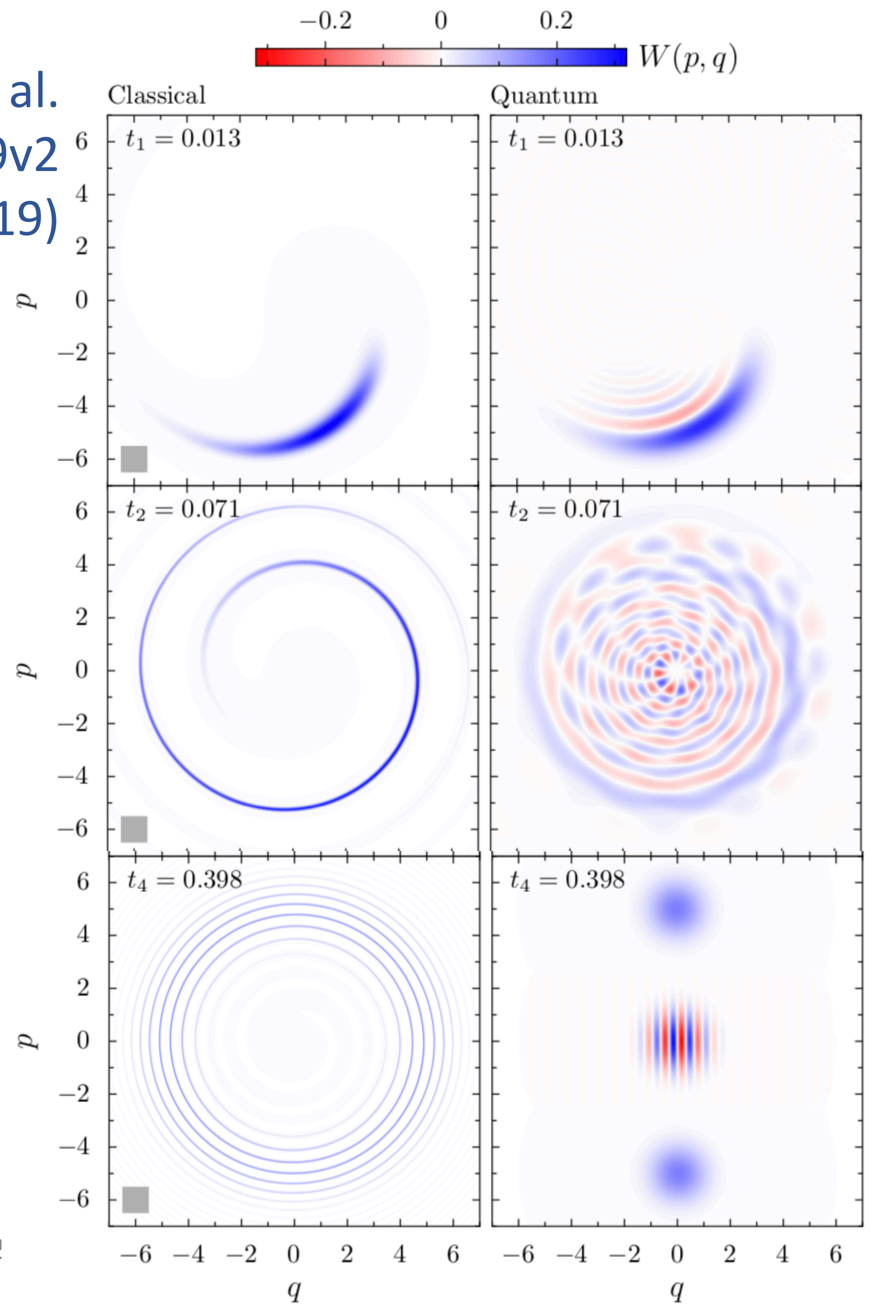
modification of the quantum uncertainty by light matter interaction

example Kerr effect

→ quantum nonlinear Schrödinger equation

G.M. Lando et al.
arXiv:1809.04139v2
(2019)

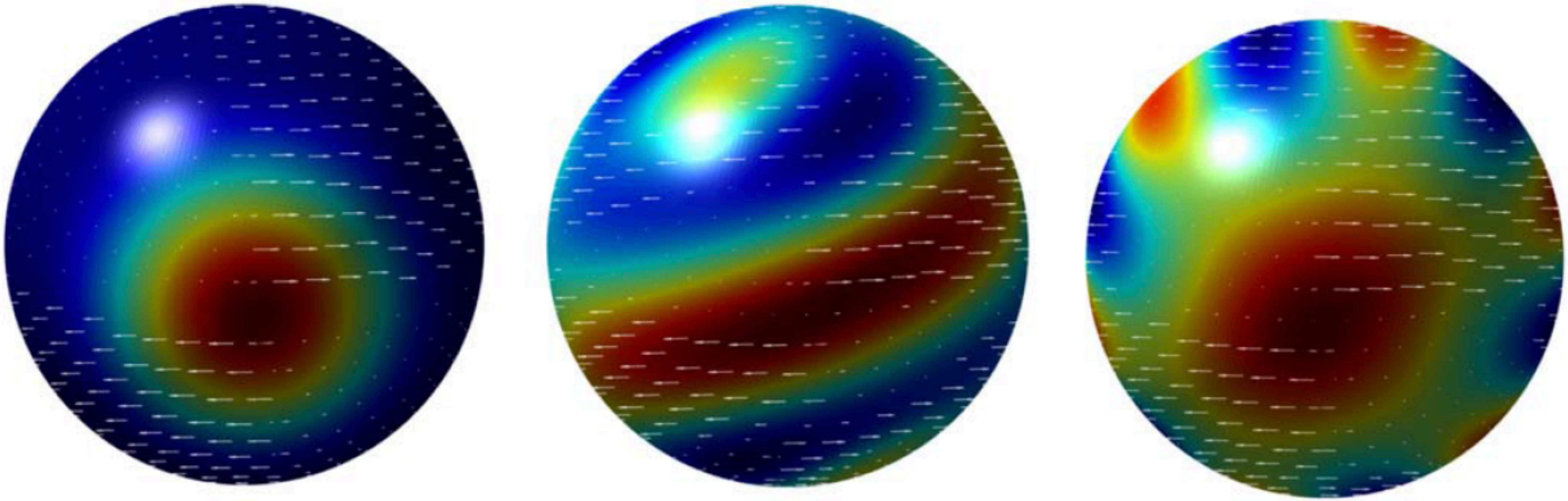




Quantum dynamics & Wigner flow

Wigner flow on the sphere
... the SU(2) Wigner function under nonlinear Kerr evolution

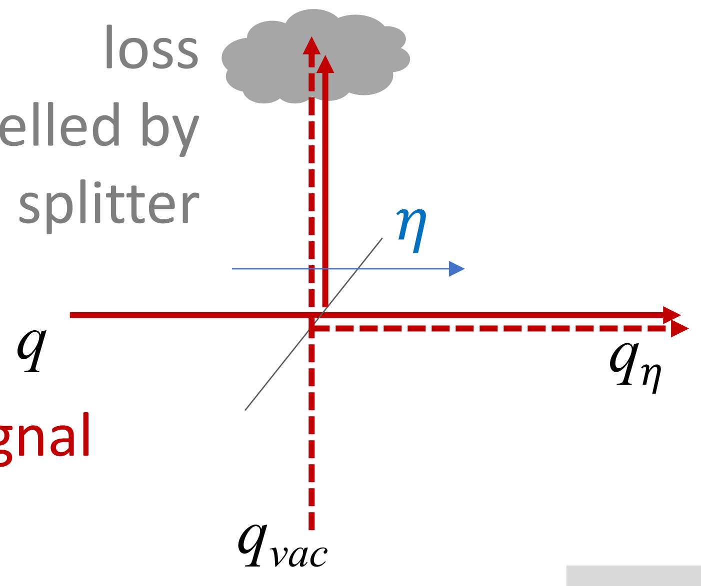
P. Yang, I.F. Valtierra, A.B. Klimov, S.-T. Wu, R.-K. Lee, L.L. Sánchez-Soto and G. Leuchs, Phys. Scr. 94, 044001 (2019)



$$\frac{\partial f(q,p|t)}{\partial t} = -\nabla \cdot \mathrm{J}(q,p|t)$$

$$\partial_t W_\rho(\Omega) = \varepsilon\{W_\rho(\Omega), W_{\mathrm{H}}(\Omega)\} + O(\varepsilon^3)$$

effect of losses



$$\langle \Delta q_\eta^2 \rangle = \eta \langle \Delta q^2 \rangle + \frac{(1-\eta)}{4}$$

See: Bogolyubov transformation

$$\hat{a} \;\rightarrow\; \hat{a}_\eta = \sqrt{\eta}\,\hat{a} + \sqrt{(1-\eta)}\,\hat{a}_{vac}$$

A.O. Caldeira and A.J. Leggett,
Phys. Rev. A 31, 1057 (1985)

if energy decays as $e^{-\gamma}$,

then quantum states containing n photons decay as $e^{-n\gamma}$

← formula compatible

G. Leuchs, U.L. Andersen
Laser Physics 15, 129 (2005)

designing material with high nonlinear coefficient

1 Tellurite TeO_2-WO_3-La_2O_3 (TWL) glasses have a non-linear refractive index $n_2 \sim 20$ times higher than fused silica

E.A. Anashkina , M.Y. Koptev, A.V. Andrianov et al., J. Lightwave Techn. 37, 4375 (2019)

goal → high nonlinearity & low loss

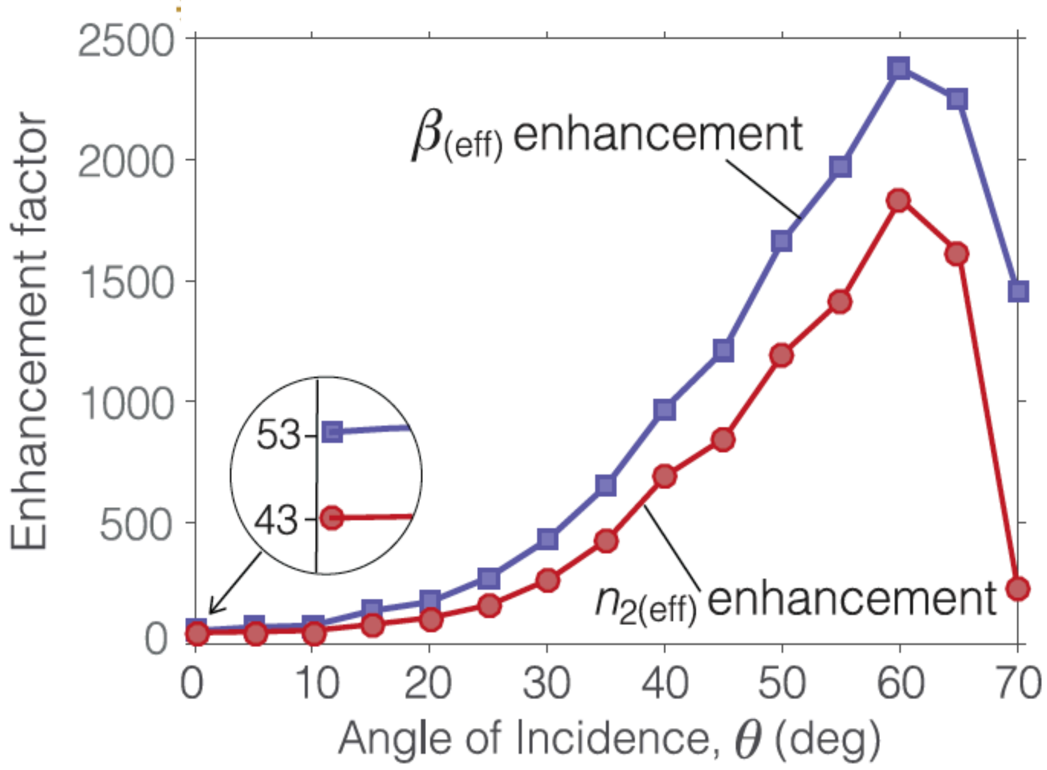
2 special case: permittivity close to zero: "epsilon near zero (ENZ) material"

$$n_2 = \frac{3\chi^{(3)}}{4\epsilon_0 c\, n_0\, \mathrm{Re}(n_0)}$$

Non-perturbative regime

$$n = \sqrt{n_0^2 + 2 n_0 n_2}$$

!

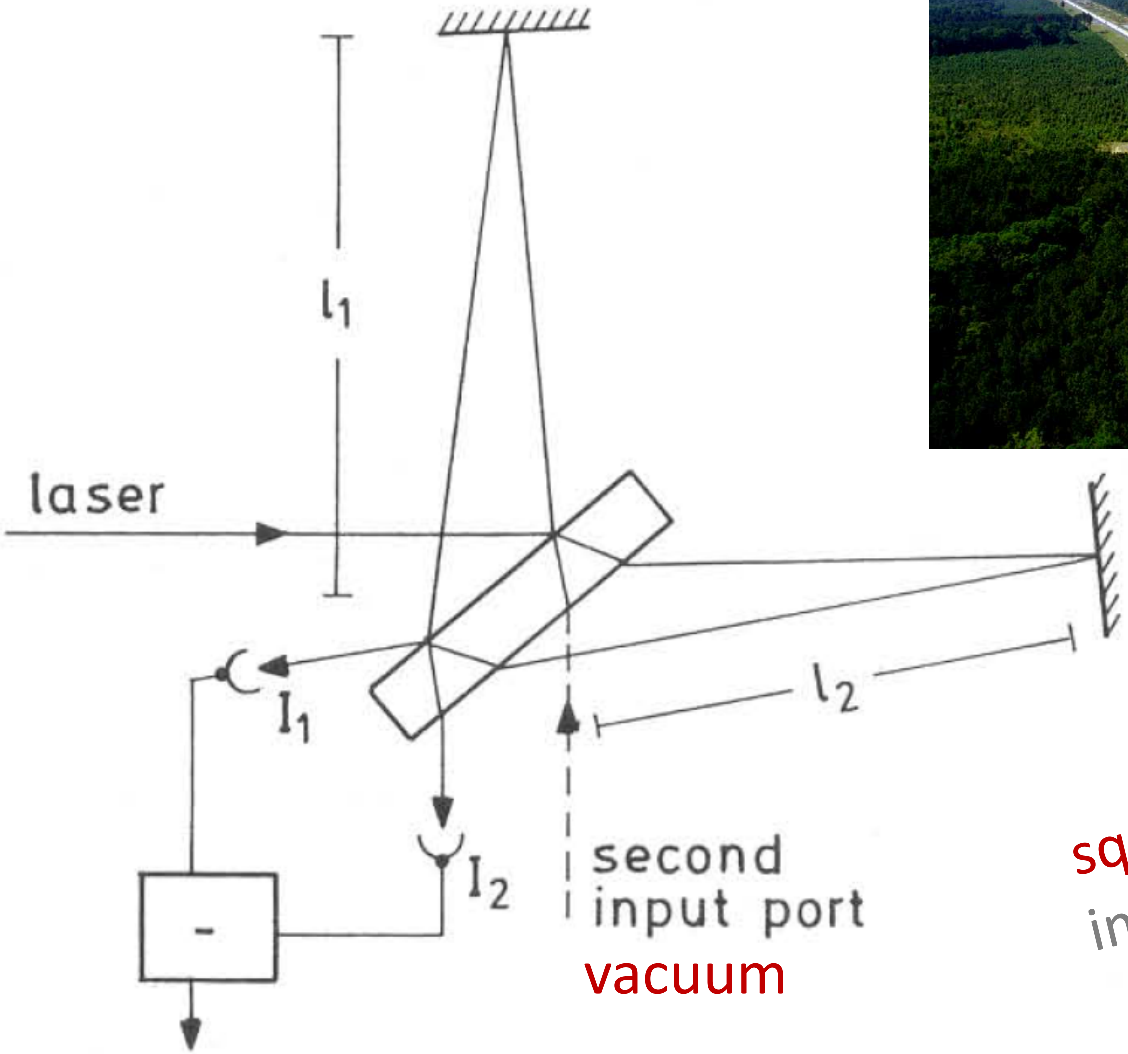


Large optical nonlinearity of indium tin oxide in its epsilon-near-zero region, M. Zahirul Alam, I. De Leon, R. W. Boyd, Science 352, 795 (2016).

quantum metrology with high power laser

gravitational waves

(interference)

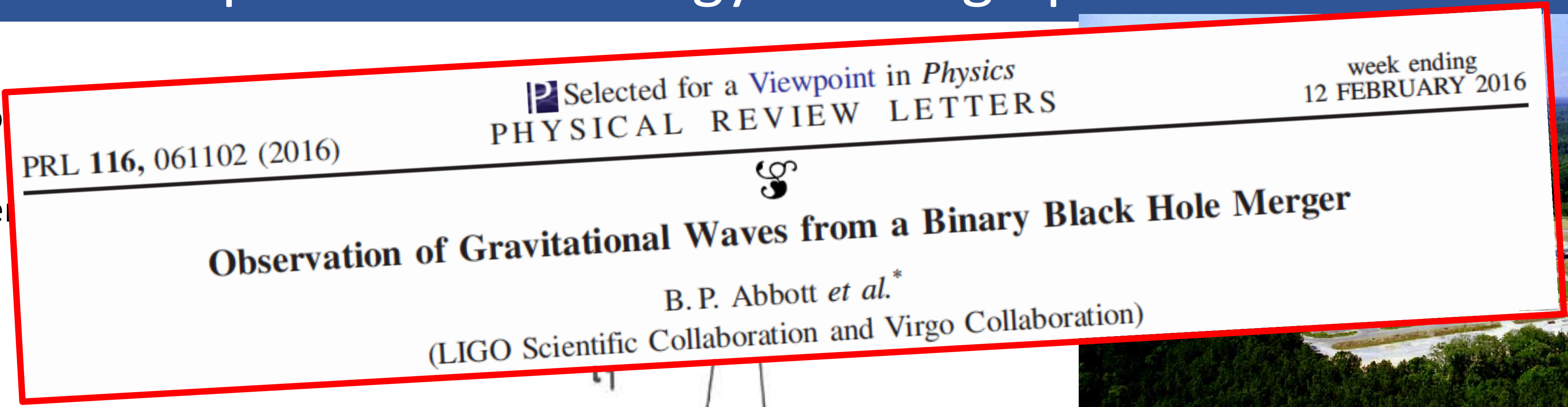


squeezed vacuum
improves sensitivity !

quantum metrology with high power laser

gravitatio

(interfere



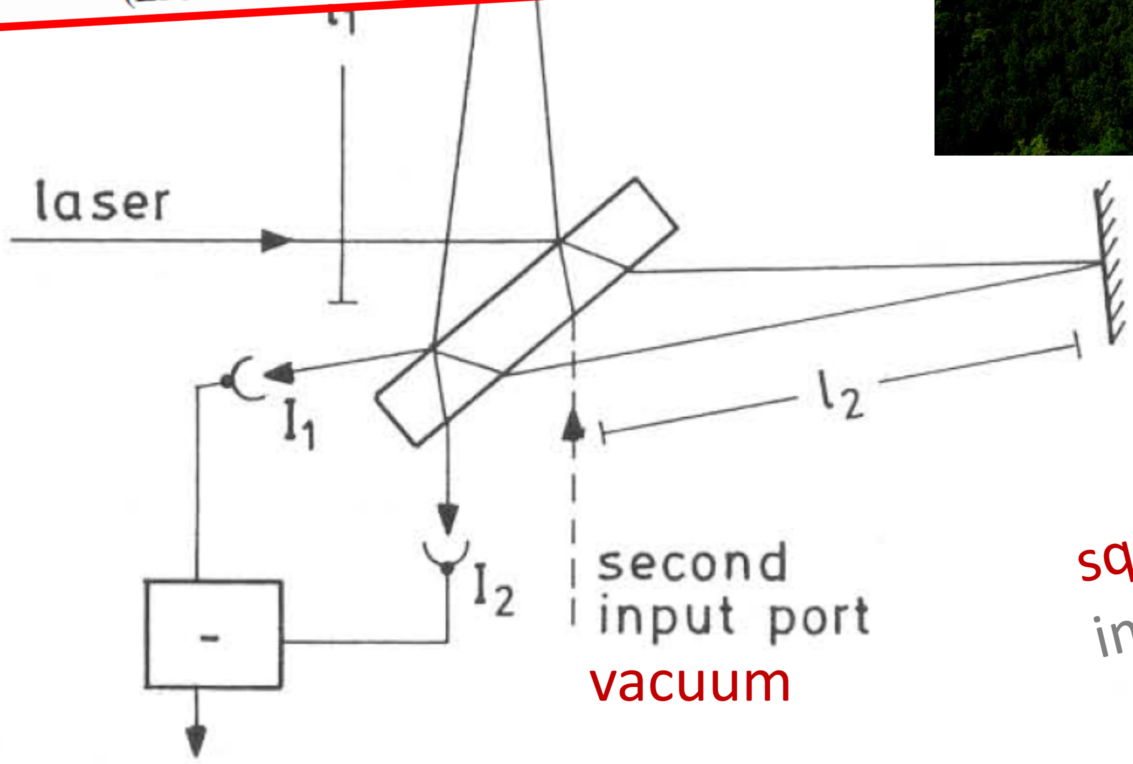
Selected for a Viewpoint in *Physics*

PRL **116,** 061102 (2016) PHYSICAL REVIEW LETTERS week ending 12 FEBRUARY 2016

Observation of Gravitational Waves from a Binary Black Hole Merger

B. P. Abbott *et al.**

(LIGO Scientific Collaboration and Virgo Collaboration)

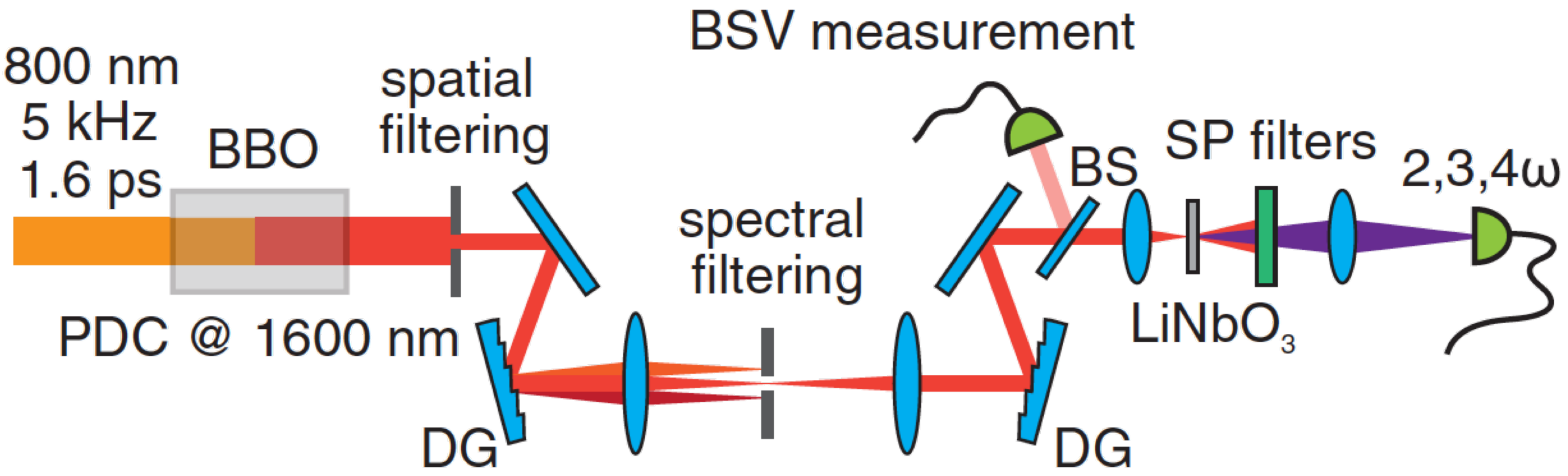


squeezed vacuum
improves sensitivity !

nonlinear optics with squeezed vacuum

Multiphoton Effects Enhanced due to large Photon-Number Uncertainty

K.Yu. Spasibko, D.A. Kopylov, V.L. Krutyanskiy, T.V. Murzina, G. Leuchs, M.V. Chekhova, Phys. Rev. Lett. 119, 223603 (2017)

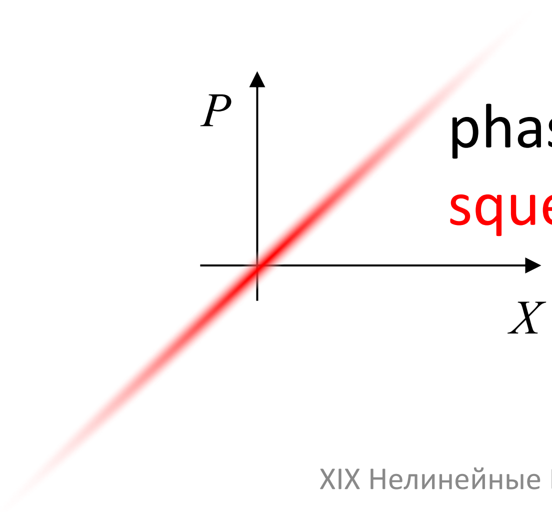


thermal statistics

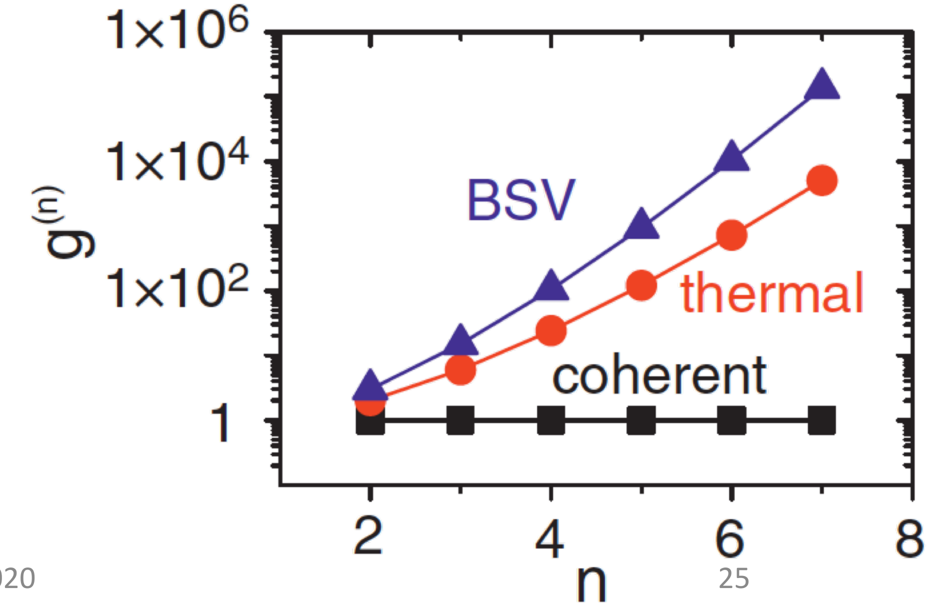
$$g^{(n)} = n!$$

squeezed light statistics

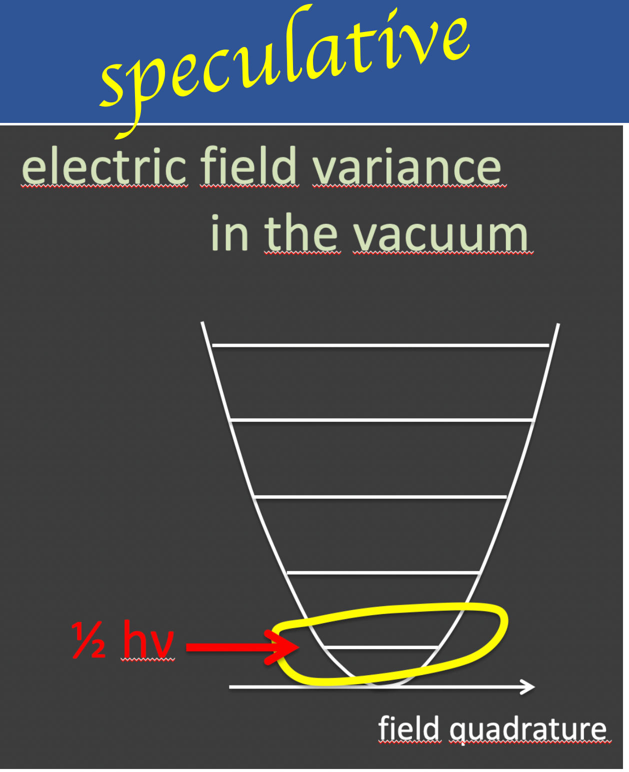
$$g^{(n)} = (2n-1)!!$$



phase space squeezing

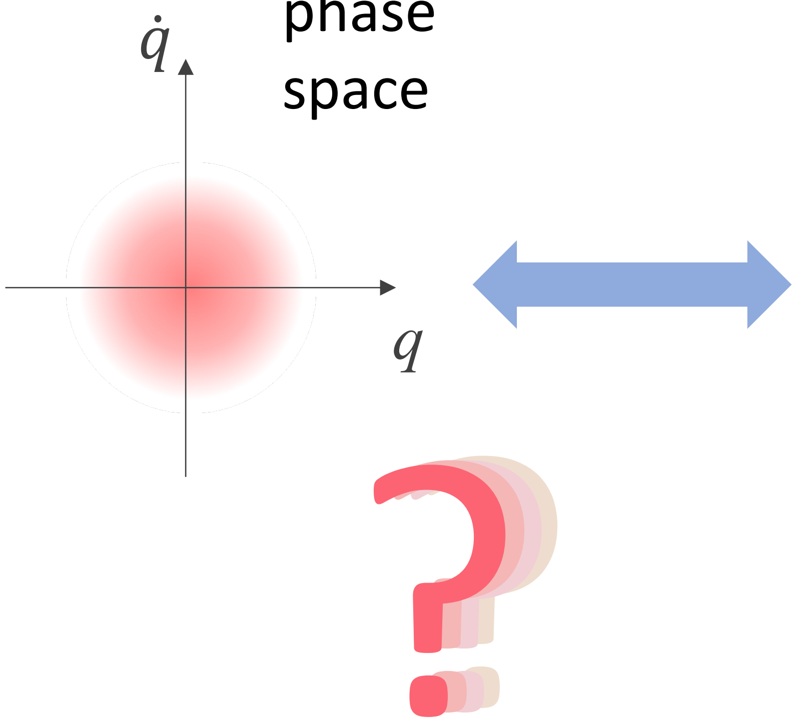


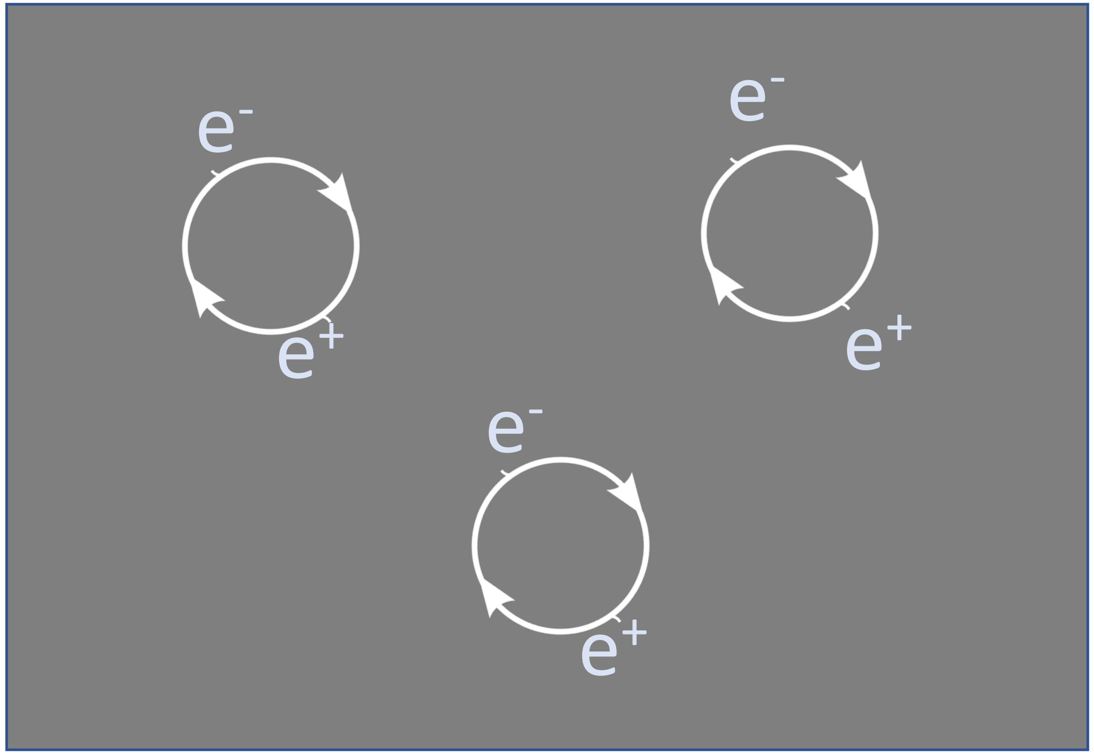
the quantum vacuum



the quantum vacuum as a dielectric-diamagnetic medium

G. Leuchs, M. Hawton, L.L. Sánchez-Soto,
Physics 2, 14 (2020) *(mdpi)*





post view

- field quantization

- coherent state
- real laser
- generation of high power light beams
- relevance of quantum uncertainty
- modification of the quantum uncertainty by light matter interaction
- quantum dynamics & Wigner flow
- effect of losses
- designing material with high nonlinear coefficient
- quantum metrology with high power laser
- nonlinear optics with squeezed vacuum
- the quantum vacuum

спасибо