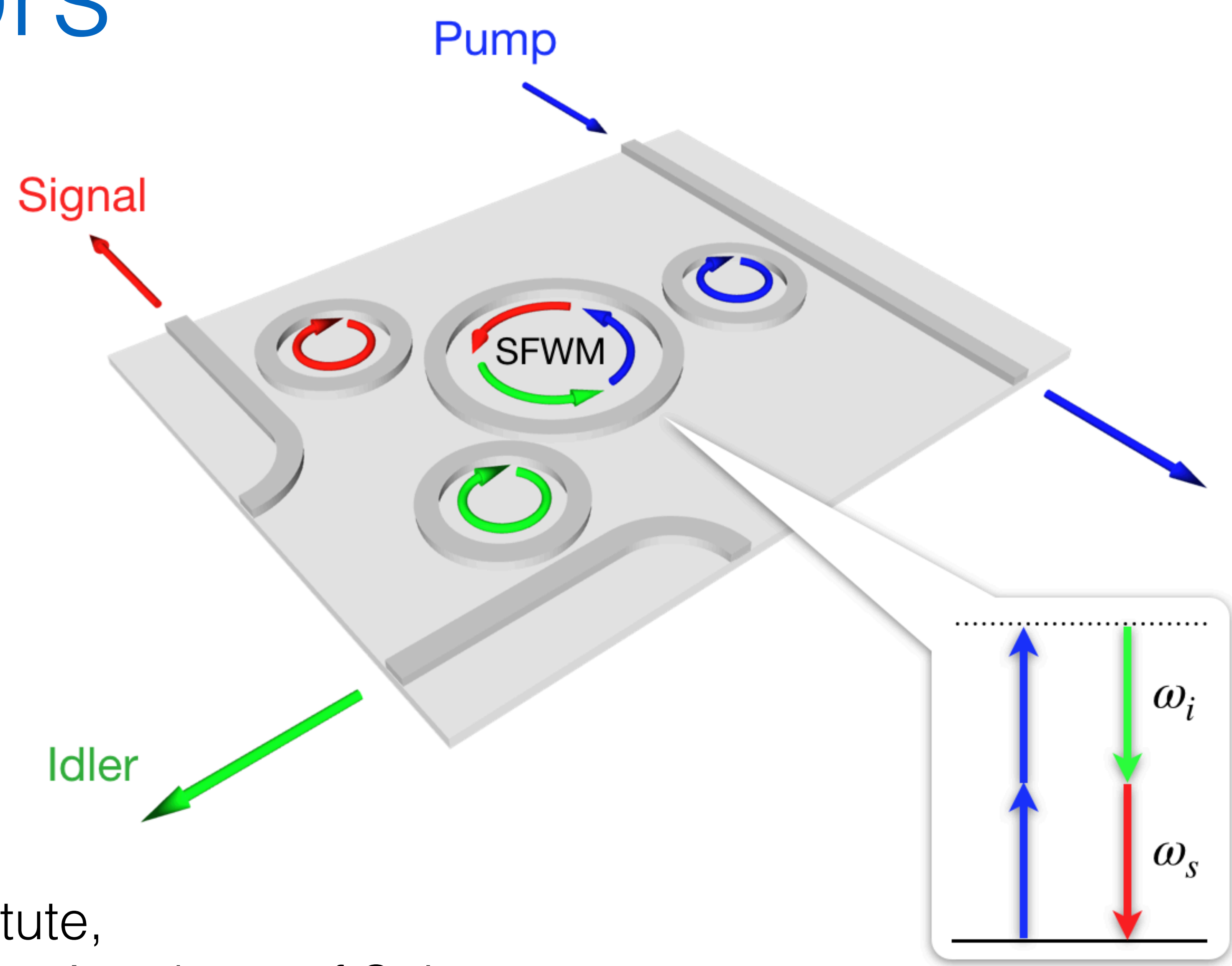
Nonclassical light sources based on nonlinear effects in systems of microresonators



Alexey Kalachev

Zavoisky Physical-Technical Institute,
Kazan Scientific Center of Russian Academy of Sciences

Nonlinear Waves 2020, Nizhny Novgorod, March 2, 2020

- **Introduction**

  Basic approaches to developing single-photon sources

- **Motivation**

  Why systems of coupled microresonators?

- **Examples of promising schemes of heralded sources**

  The model

  Pure single-photon states

  Frequency-bin qubits

  Heralded two-photon states

- **Conclusion**

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Nonclassical light sources

Single-photons Two-photon entangled states NOON states Cluster sates Squeezed states …	Quantum communications Quantum computing Quantum imaging Quantum metrology …

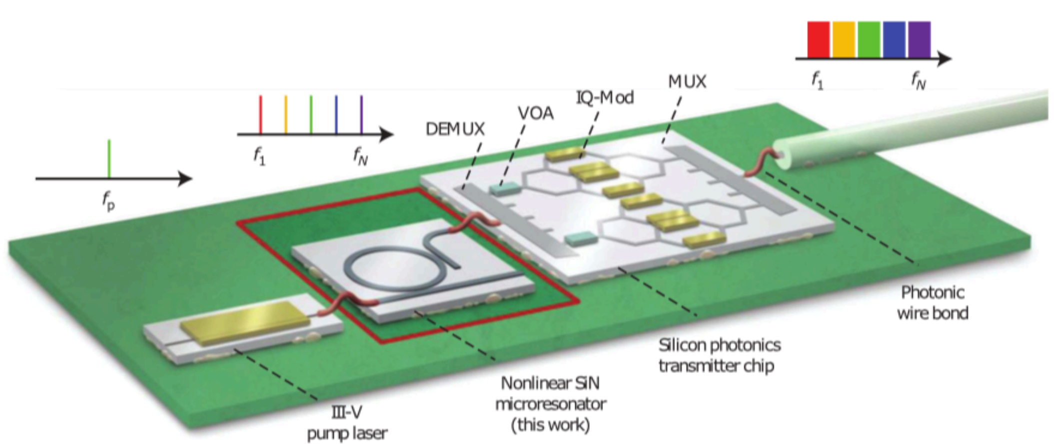


Image: M. Borghi, et al. J. Opt., 2017

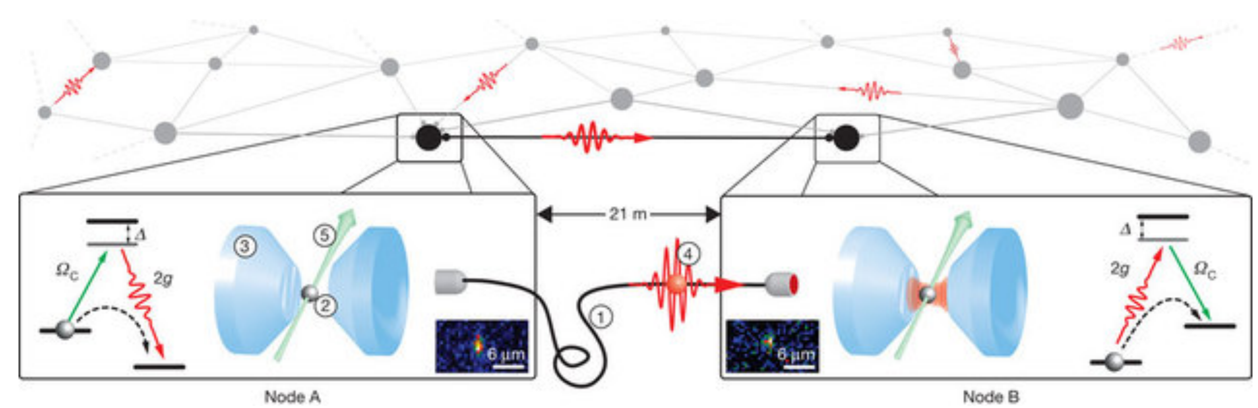


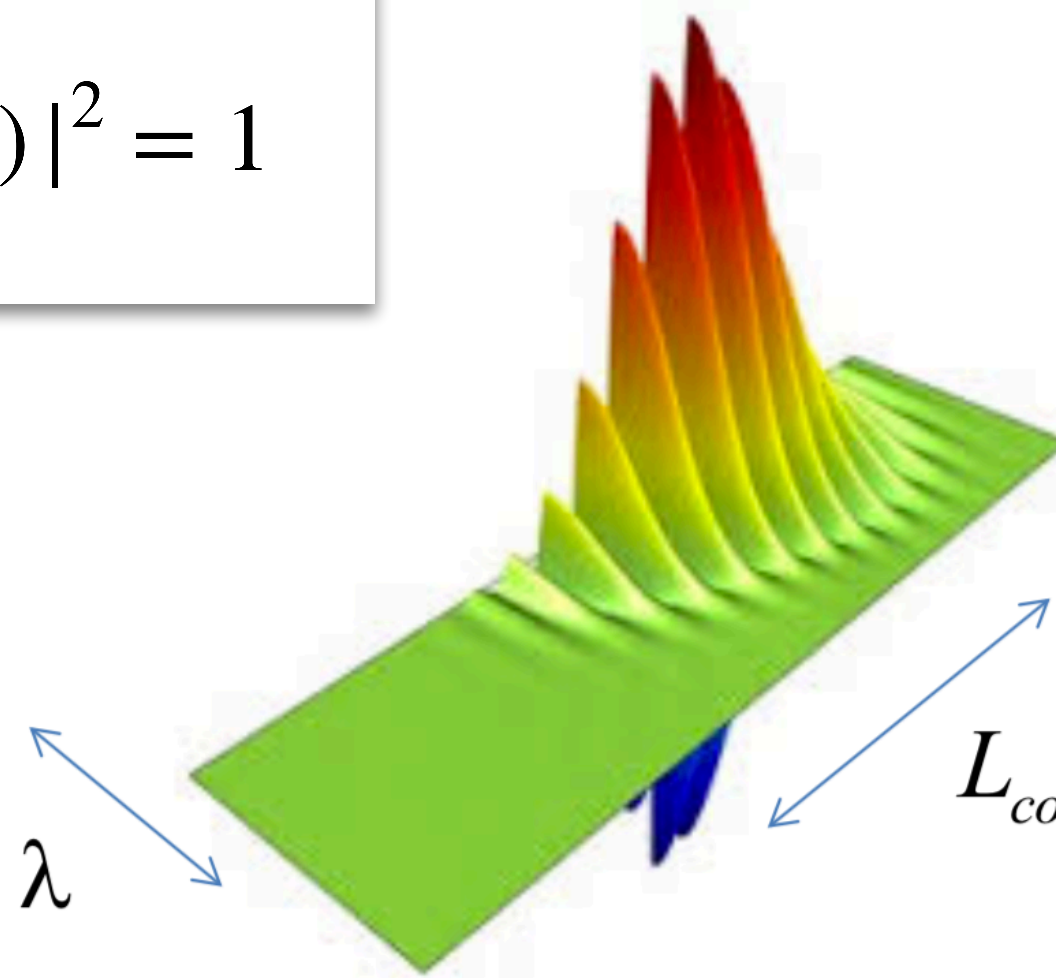
Image: S. Ritter at al., Nature, 2012

From small-scale photonic chips … to large-scale quantum networks

Single-photon wave packets

$$|\psi\rangle = \int d\nu F(\nu)\,|\nu\rangle$$

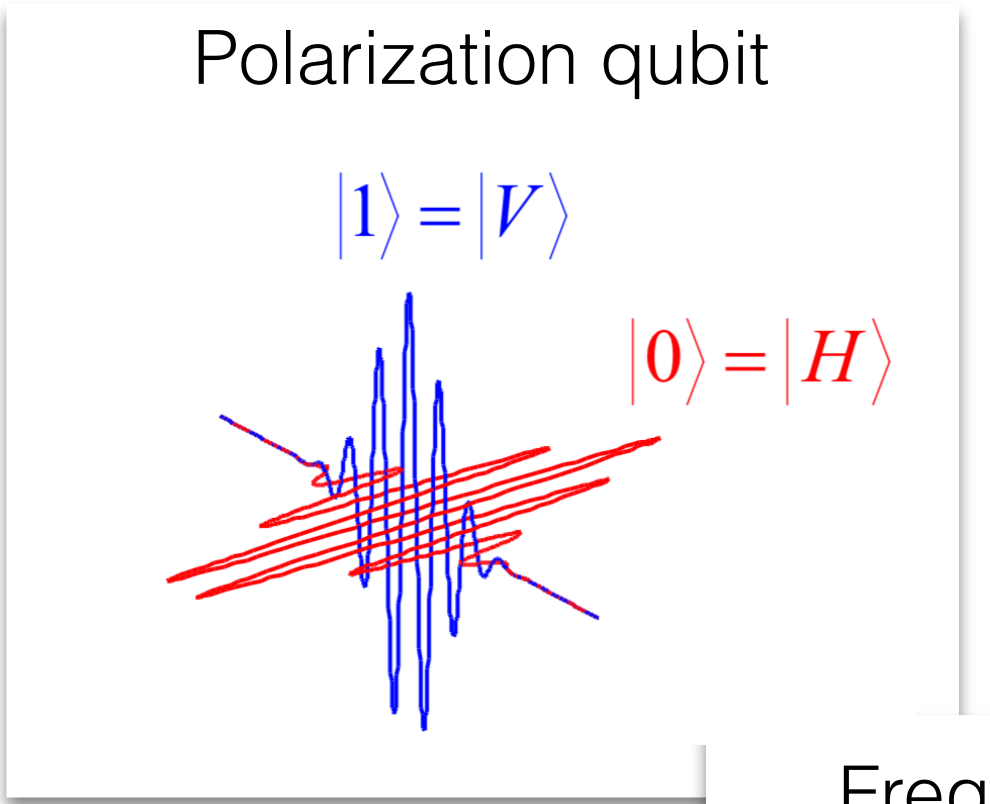
$$|\nu\rangle = a^{\dagger}(\nu)\,|0\rangle, \qquad \int d\nu\,|F(\nu)|^2 = 1$$

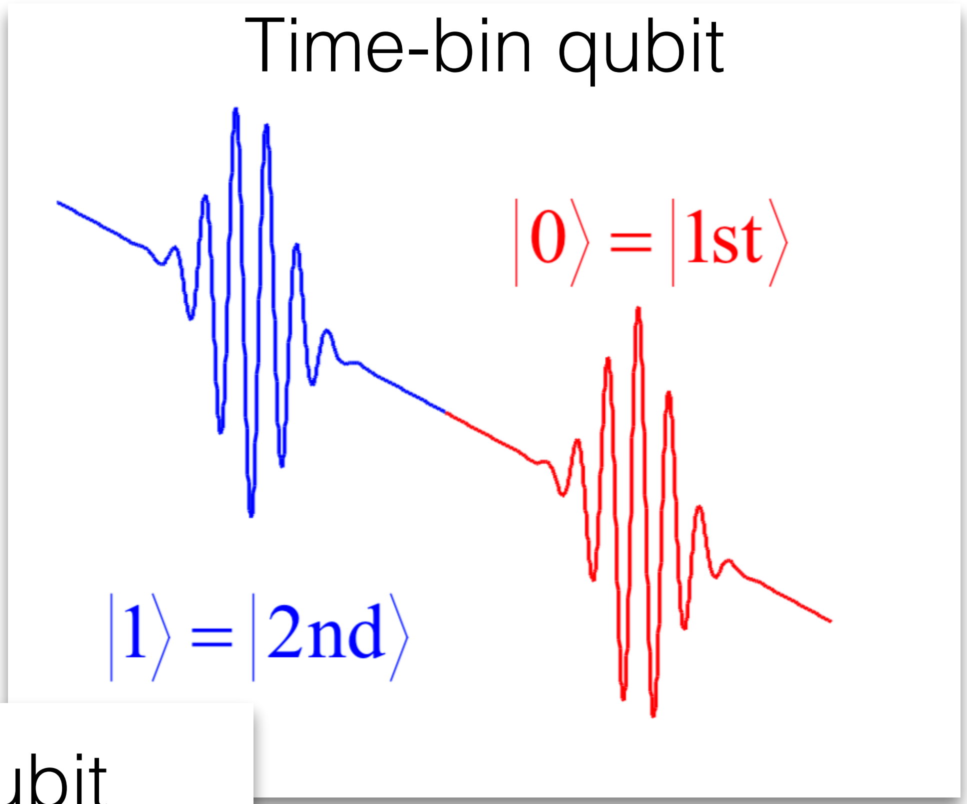


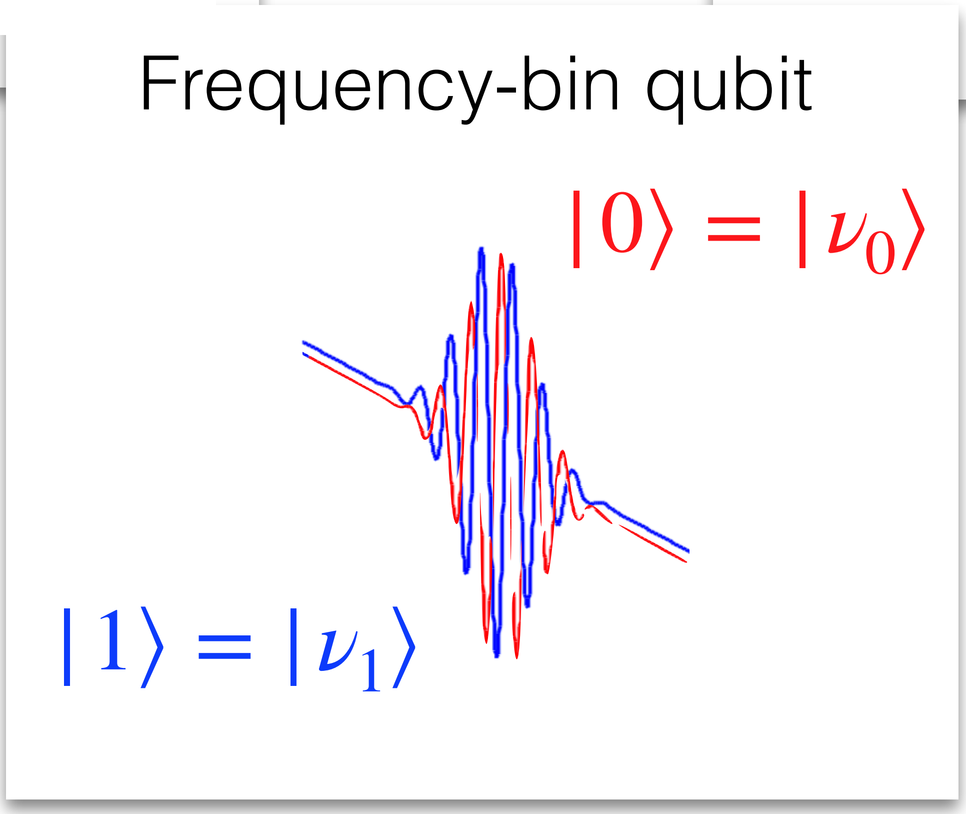
Pure single-photon state ≡ Transform-limited pulse

Flying qubits

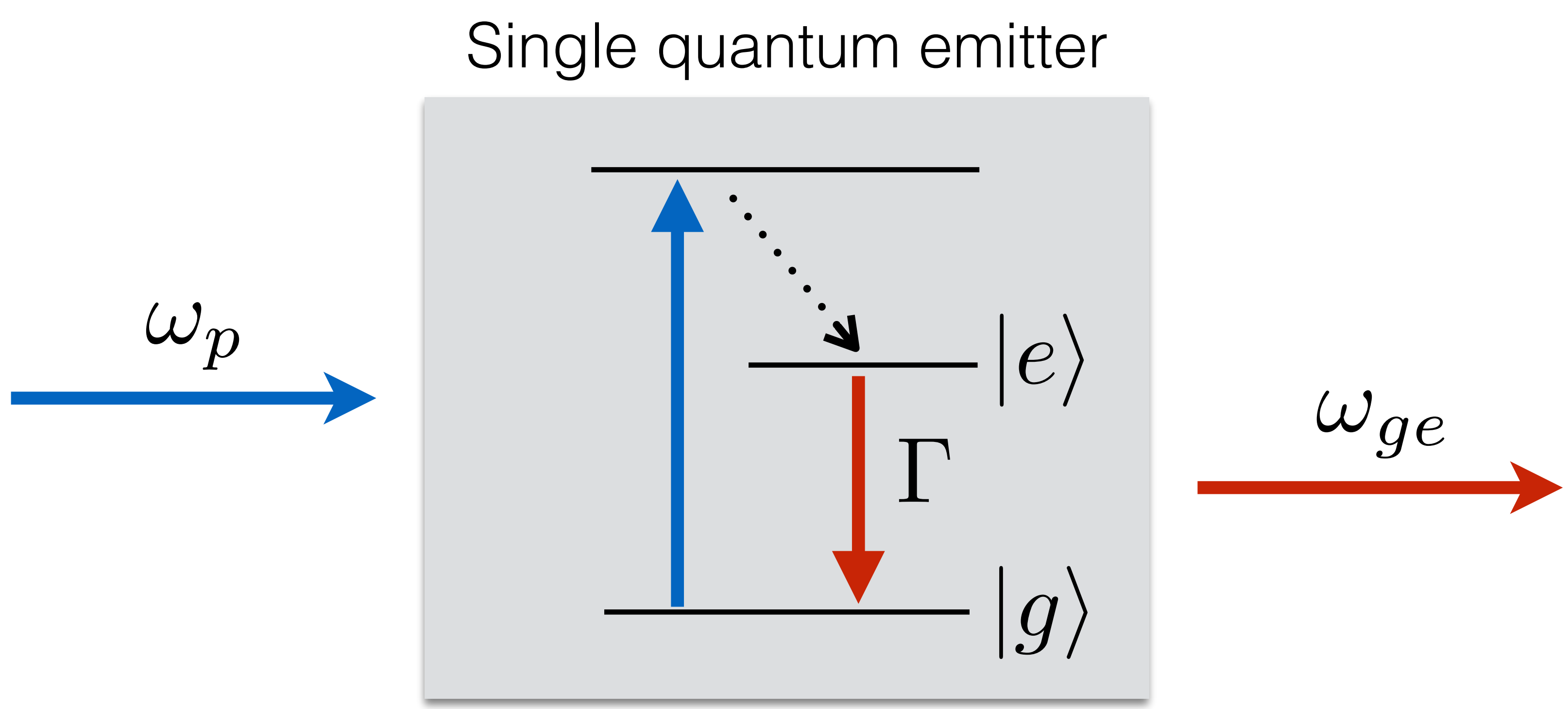
$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$







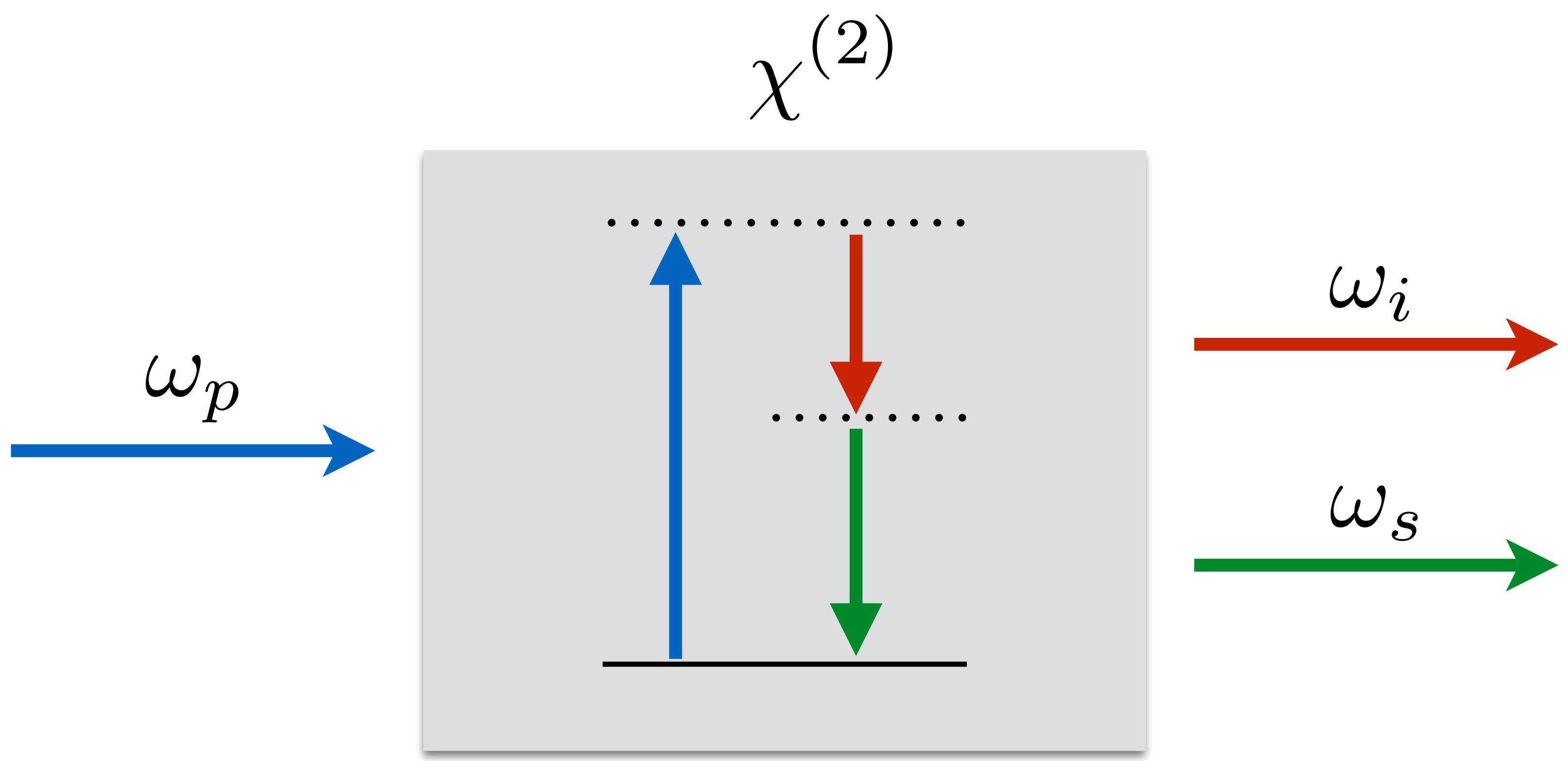
Spontaneous emission



$$|\psi\rangle = \int d\omega \frac{g(\omega)}{(\omega - \omega_{ge}) + i\Gamma/2} |\omega\rangle$$

Single photons on demand

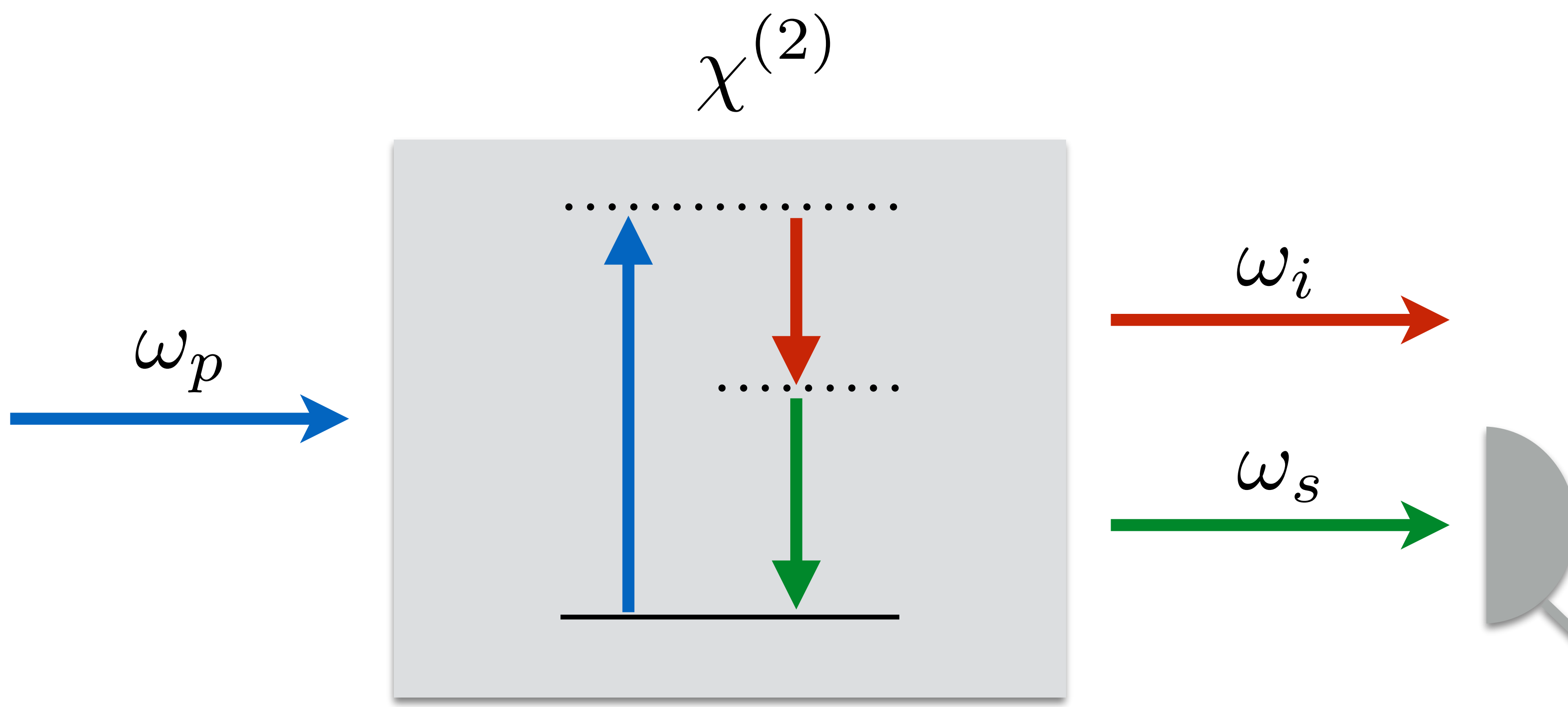
Spontaneous parametric down-conversion



$$|\psi\rangle = |0\rangle + \iint d\omega_s d\omega_i \, F(\omega_s, \omega_i)|\omega_s\rangle\omega_i\rangle$$

Entangled photon pairs

Spontaneous parametric down-conversion



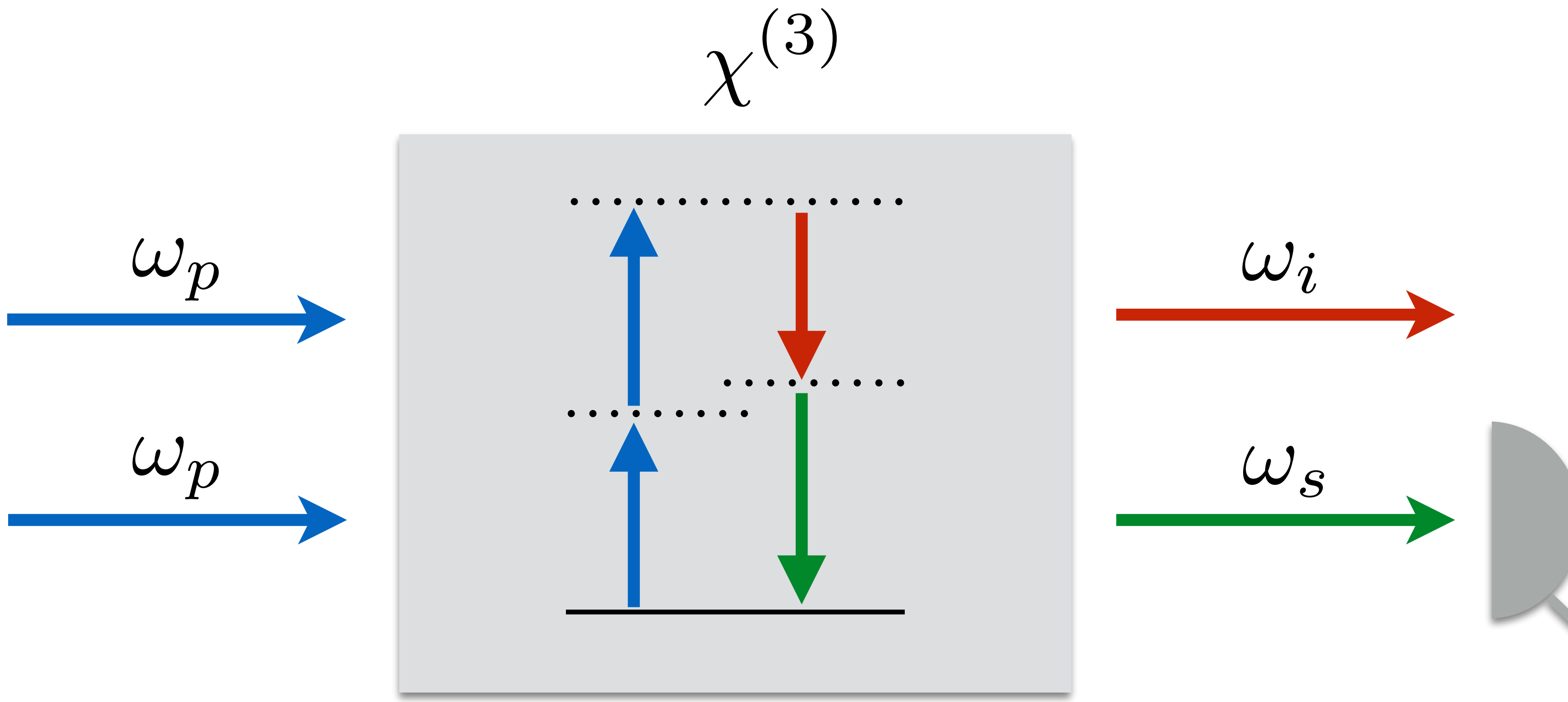
$$|\psi\rangle = |0\rangle + \iint d\omega_s d\omega_i \, F(\omega_s, \omega_i)|\omega_s\rangle\omega_i\rangle$$

$$\rho(\omega_i) = \int d\omega_s \langle\omega_s|\psi\rangle\langle\psi|\omega_s\rangle$$

Entangled photon pairs

Heralded single photons

Spontaneous four-wave mixing

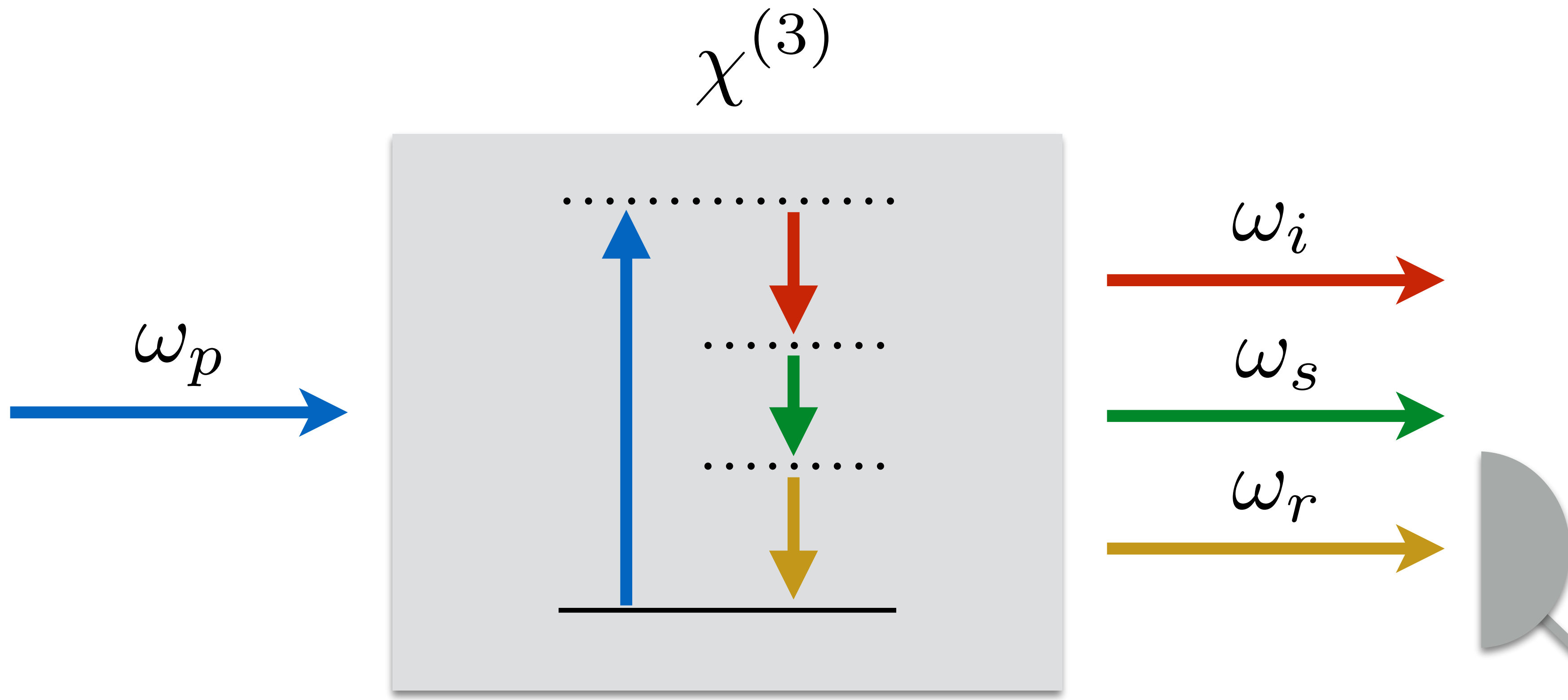


$$|\psi\rangle = |0\rangle + \iint d\omega_s d\omega_i \, F(\omega_s, \omega_i)|\omega_s\rangle\omega_i\rangle$$

$$\rho(\omega_i) = \int d\omega_s \langle\omega_s|\psi\rangle\langle\psi|\omega_s\rangle$$

Entangled photon pairs
Heralded single photons

Third-order spontaneous parametric down-conversion



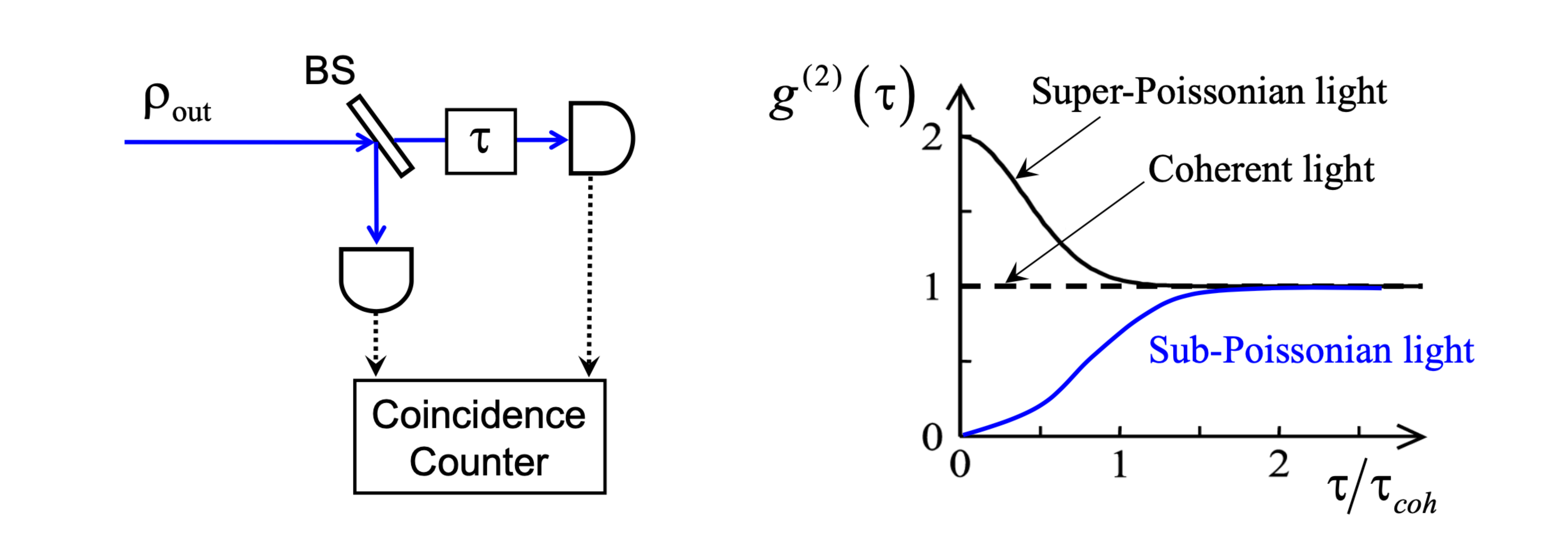
$$|\psi\rangle = |0\rangle + \iint d\omega_s d\omega_i d\omega_r \, F(\omega_s, \omega_i, \omega_r)|\omega_s\rangle\omega_i\rangle\omega_r\rangle$$

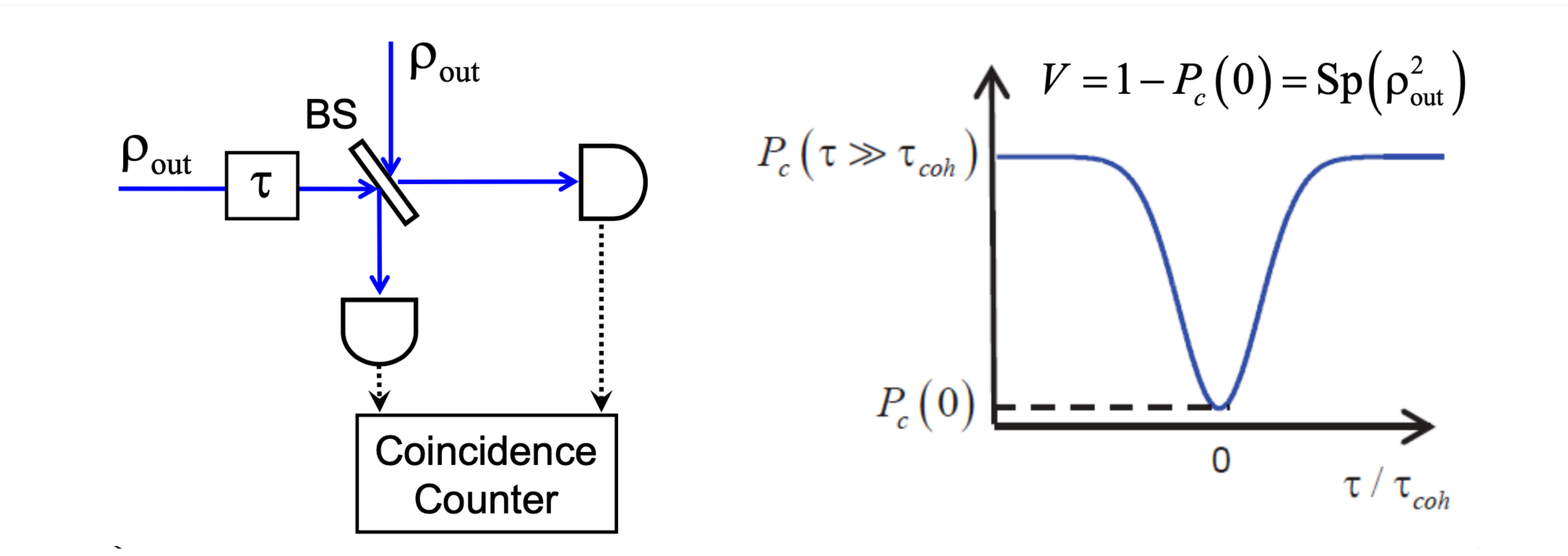
$$\rho(\omega_s, \omega_i) = \int d\omega_r \langle\omega_r|\psi\rangle\langle\psi|\omega_r\rangle$$

Entangled photon triples

Heralded photon pairs

Basic figures of merit





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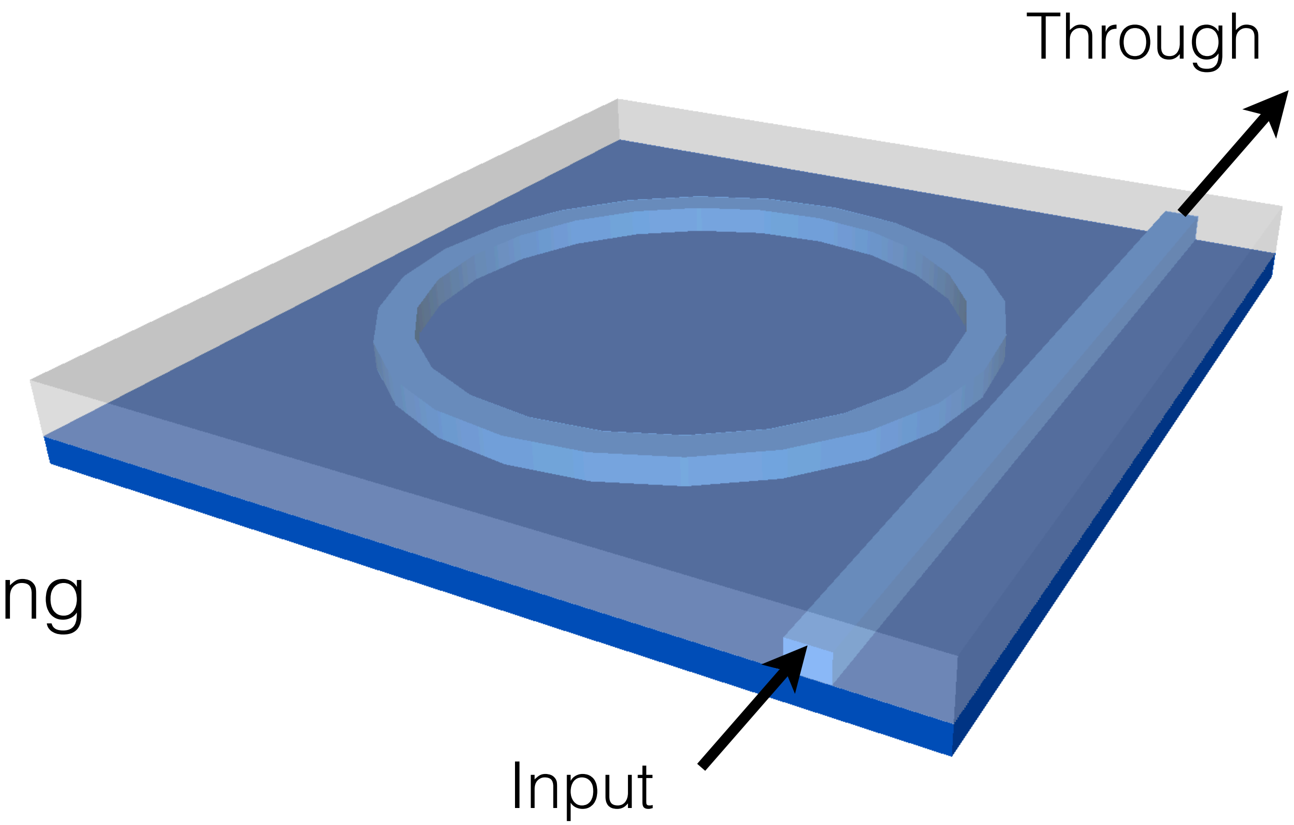
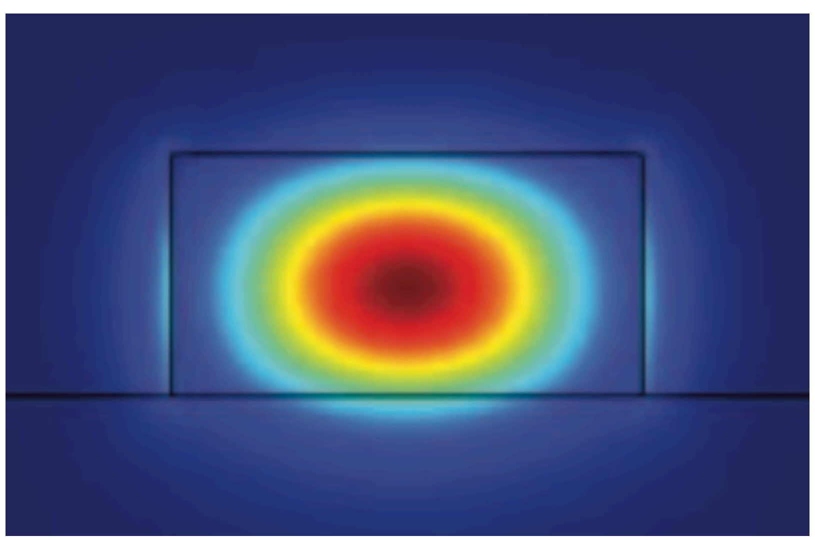
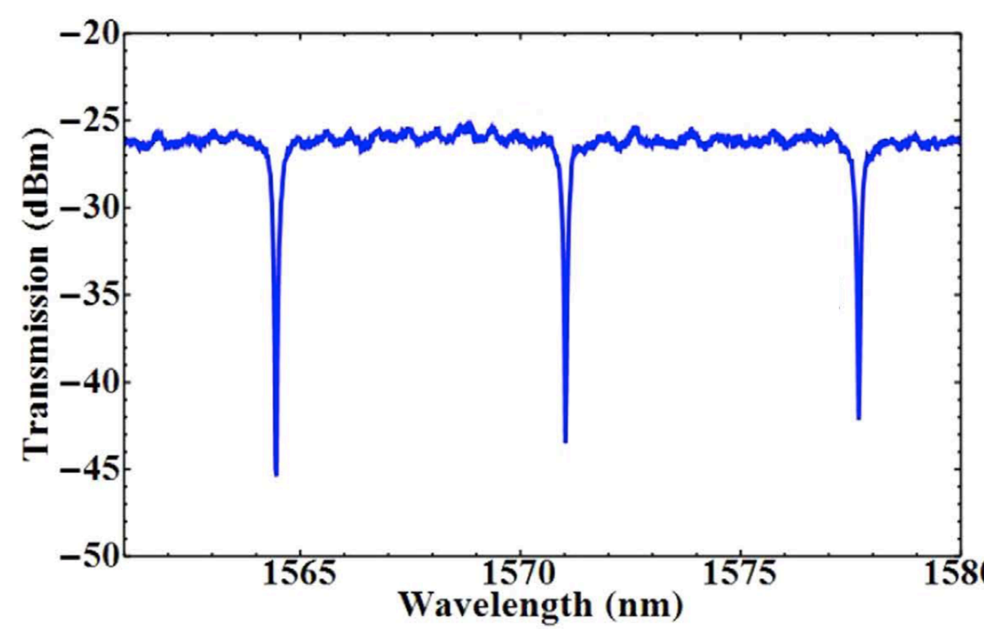
Microring resonators

- High efficiency due to small mode volume and high Q
- High FSR due to small size
- Small bandwidth
- CMOS-compatible manufacturing

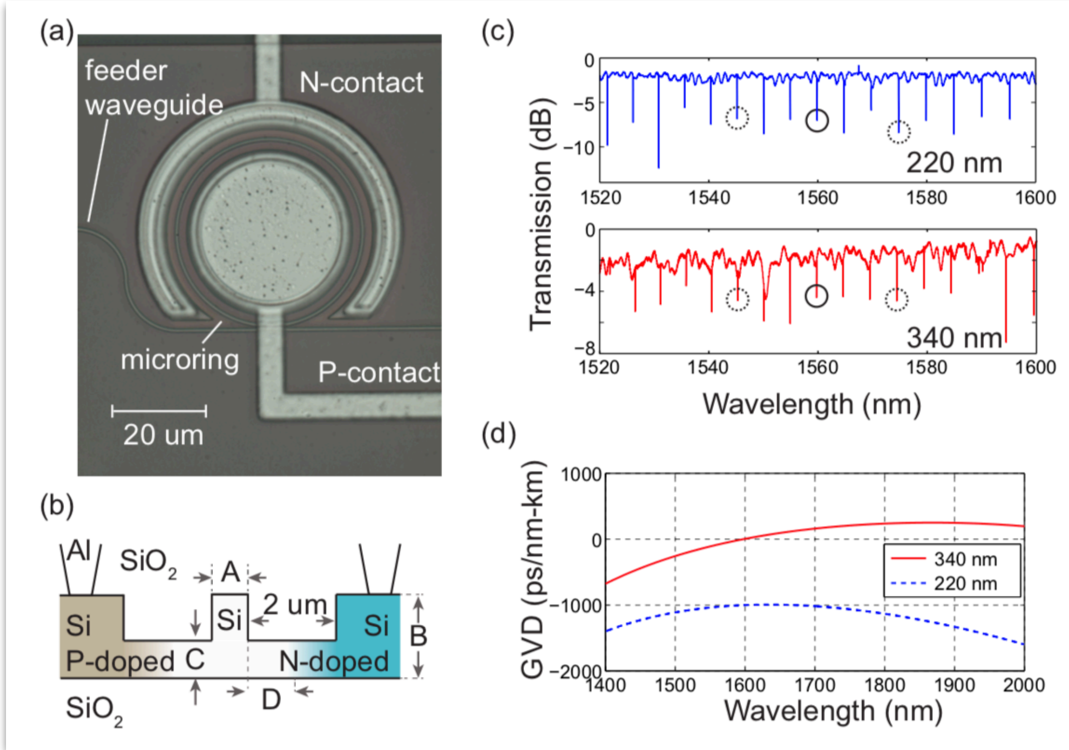




$$N \sim 1/A_{eff}^2$$

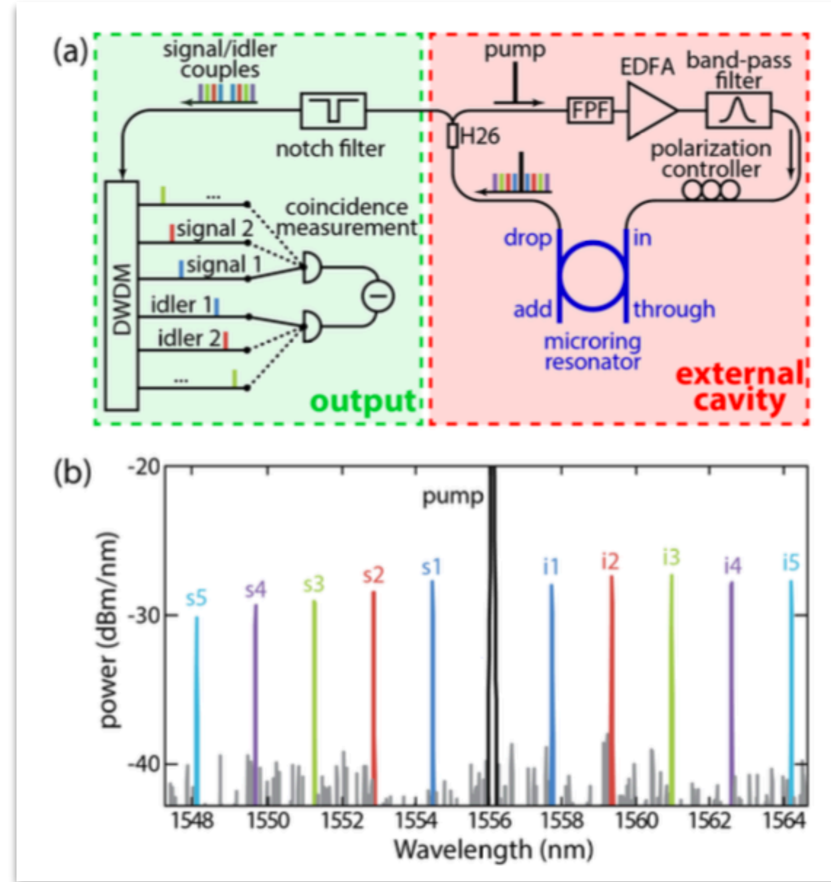


$$N \sim \sqrt{F_p F_p F_s F_i}$$



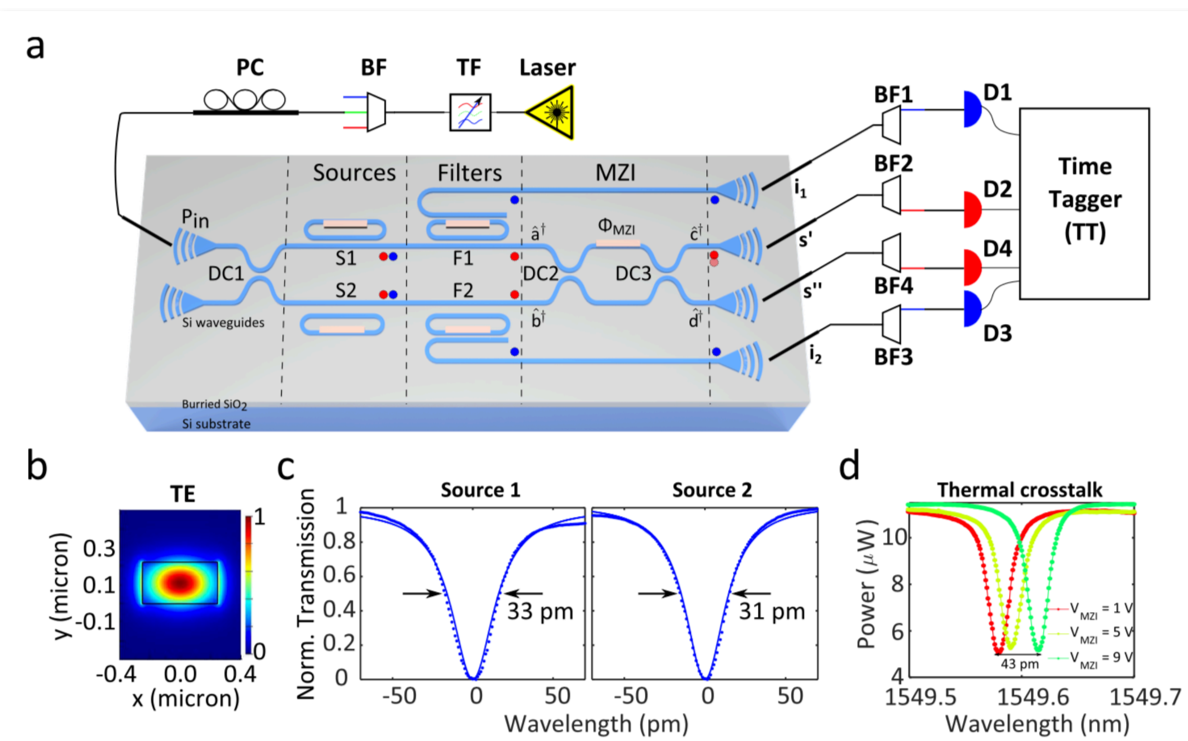
~ 100 kHz at ~10 μW pump

Savanier M, et al. Opt. Express 24 3313 (2016)



110 MHz bandwidth

Reimer C, et al. Opt. Express 22 6535 (2014)

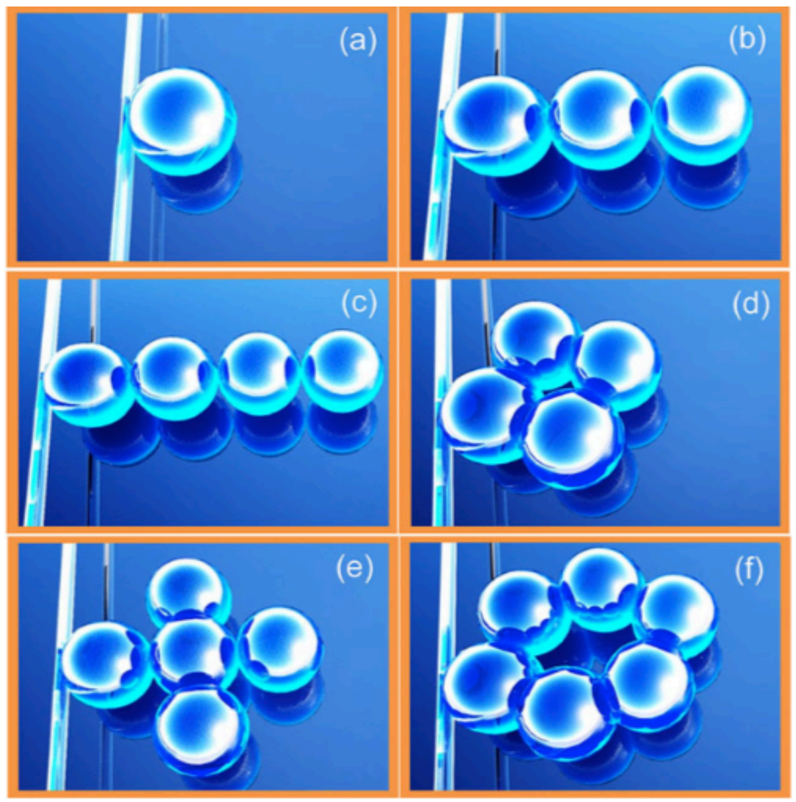


$$P = Sp(\rho^2)$$

Heralded photons
from independent sources
with the purity of 0.92

Faruque I.I., et al. Opt. Express 26 20379 (2018)

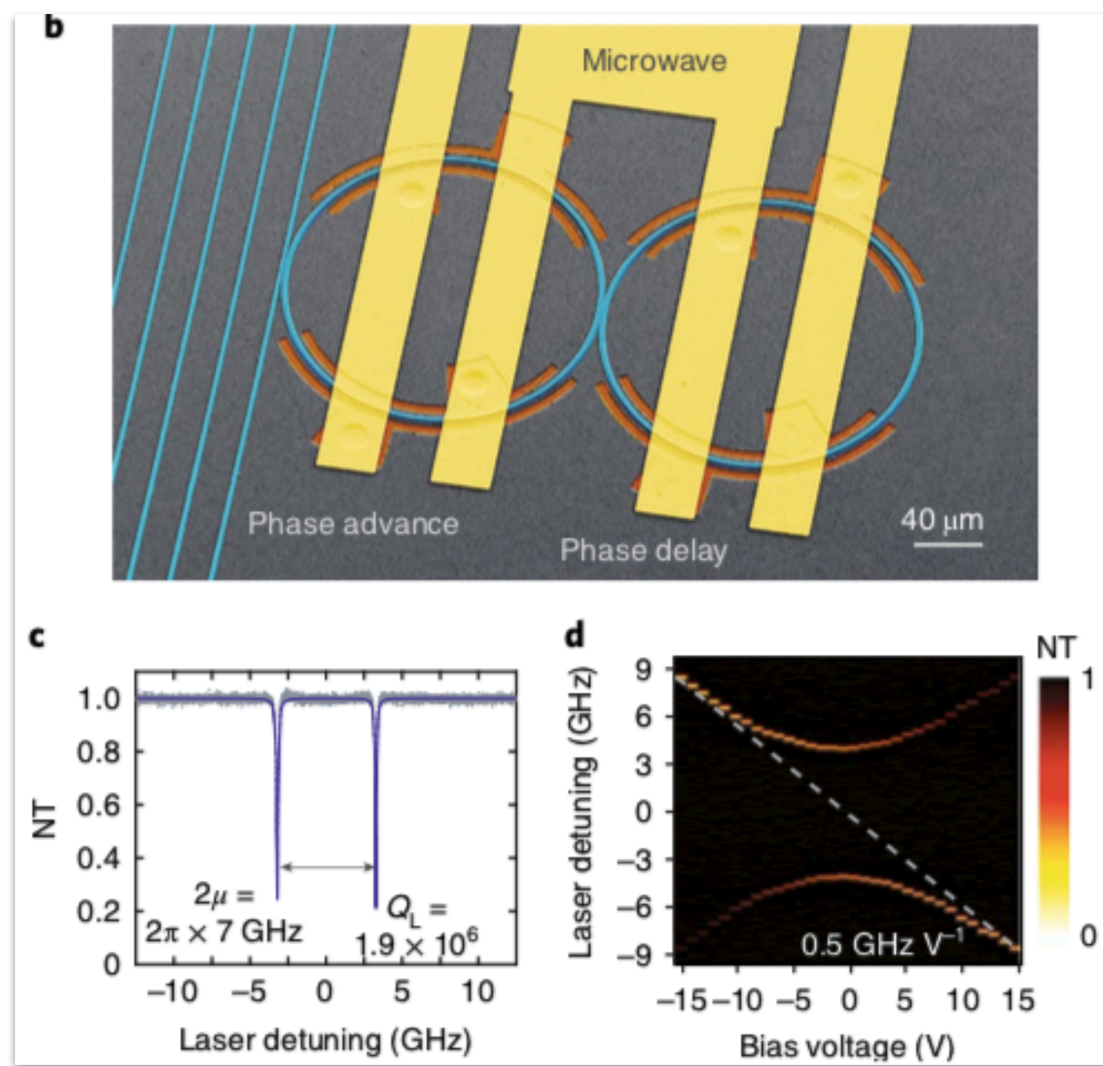
Coupled microresonators (photonic molecules)



Laser Photonics Rev, 11(2), 1600278 (2017)

- Engineering the absorption and dispersion properties in the transmission spectrum
- Correlation between spectral features and spatial configuration
- E-field enhancement

- Coupled resonator optical waveguides with controllable pulse delay
- High-order optical filters
- Enhanced optical sensors
- Low-threshold lasers
- Optical/microwave interface
- Optical storage



Nature Photonics, 13(1), 36 (2019)

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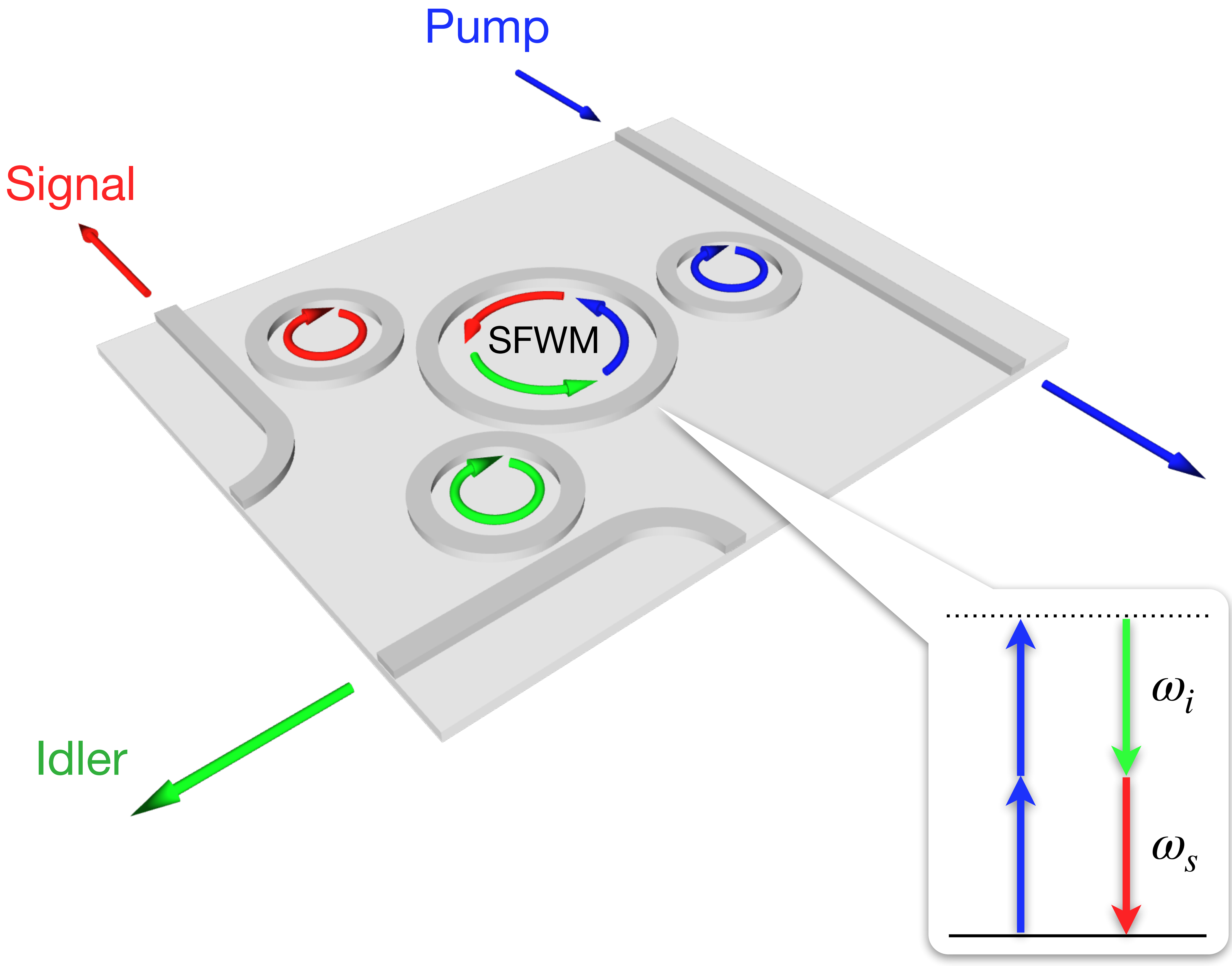
The model



The model

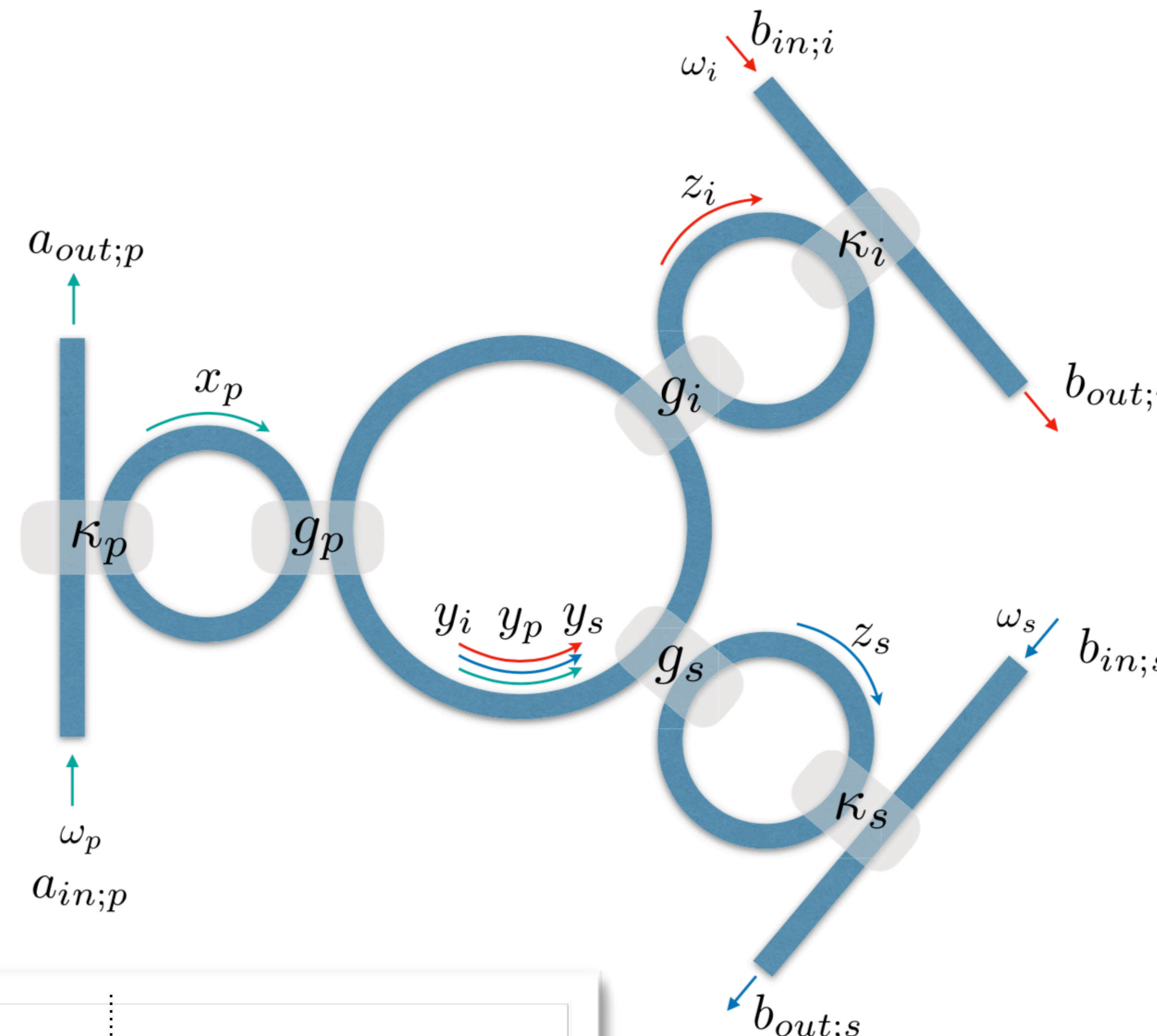
$$\mathcal{H}_{sys} = \hbar\omega_{0x,p}\, x_p^\dagger x_p + \sum_{n=p,i,s} \hbar\omega_{0y,n}\, y_n^\dagger y_n + \sum_{m=i,s} \hbar\omega_{0z,m}\, z_m^\dagger z_m$$

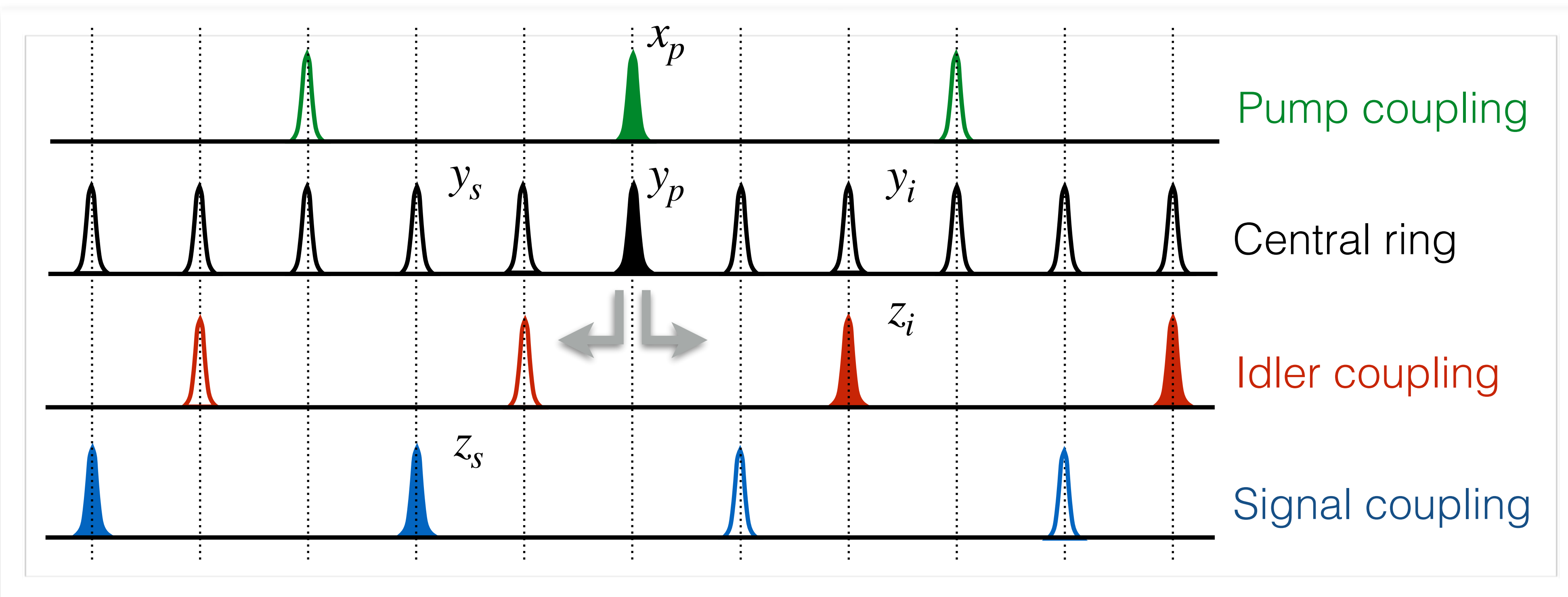
$$\mathcal{H}_{bath} = \int d\omega\, \hbar\omega \left[a_p^\dagger(\omega) a_p(\omega) + \sum_{m=i,s} b_m^\dagger(\omega) b_m(\omega) \right]$$

$$\mathcal{H}_{int}^{internal} = i\hbar g_p x_p^\dagger y_p + i\hbar g_i z_i^\dagger y_i + i\hbar g_s z_s^\dagger y_s + h.c.$$

$$\mathcal{H}_{int}^{bath} = \frac{i\hbar}{\sqrt{2\pi}} \int d\omega \, [\sqrt{\kappa_p}\, x_p^\dagger a_p(\omega) + \sqrt{\kappa_i}\, z_i^\dagger b_i(\omega) + \sqrt{\kappa_s}\, z_s^\dagger b_s(\omega) + h.c.]$$

$$\mathcal{H}_{SFWM}(t) = \zeta y_p(t) y_p(t) y_s^\dagger(t) y_i^\dagger(t),$$





Input-output relations

$$\left[\partial_t + i\omega_{0x,p} + \frac{\kappa_p}{2}\right] x_p - g_p y_p = \sqrt{\kappa_p}\, a_{in;p},$$

$$[\partial_t + i\omega_{0y,i}]y_i + g_i z_i = 0,$$

$$[\partial_t + i\omega_{0y,p}]y_p + g_p x_p = 0,$$

$$[\partial_t + i\omega_{0y,s}]y_s + g_s z_s = 0,$$

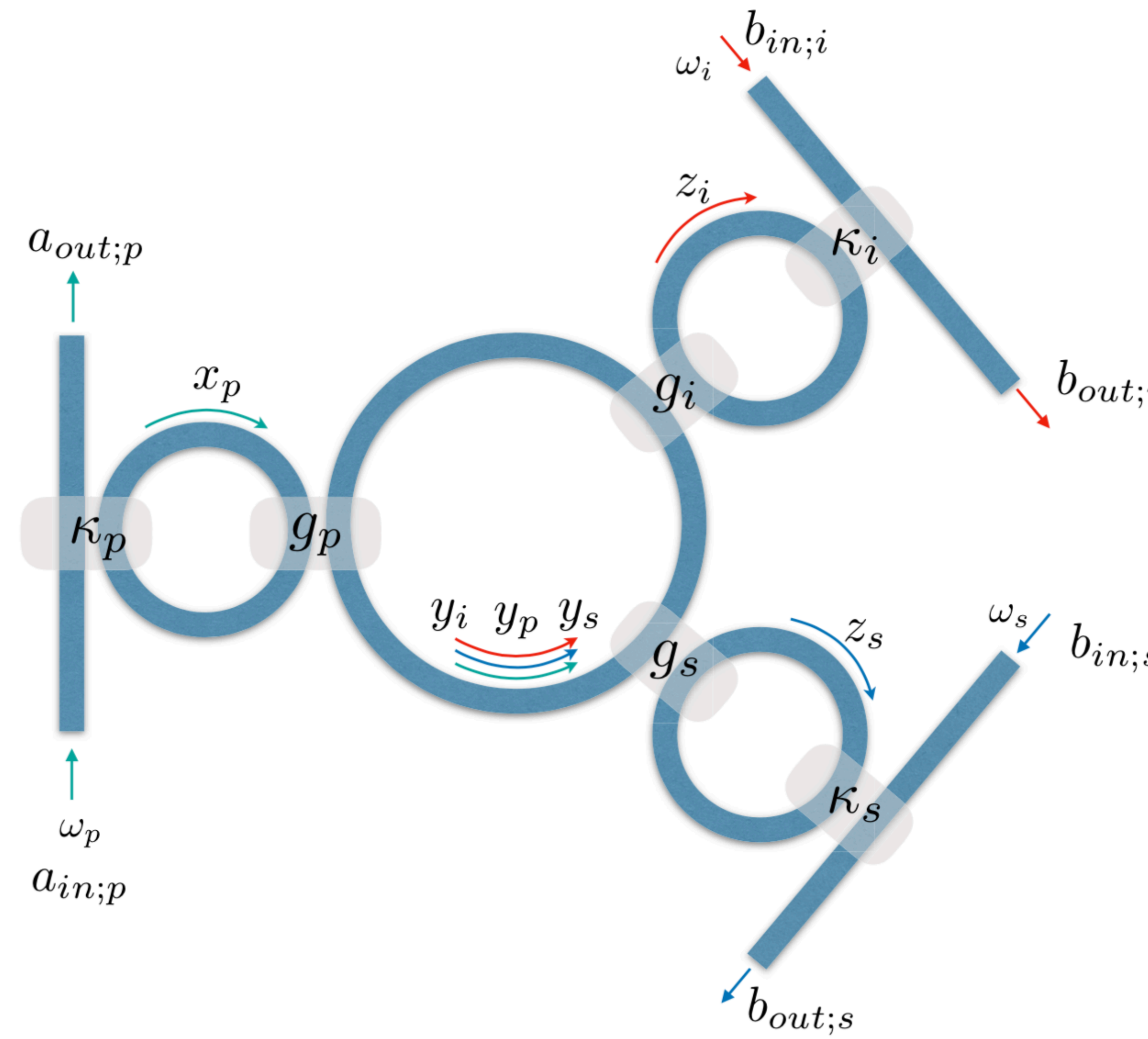
$$[\partial_t + i\omega_{0z,i} + \frac{\kappa_i}{2}]z_i - g_i y_i = \sqrt{\kappa_i}\, b_{in;i},$$

$$[\partial_t + i\omega_{0z,s} + \frac{\kappa_s}{2}]z_s - g_s y_s = \sqrt{\kappa_s}\, b_{in;s},$$

$$a_{in;p} - a_{out;p} = \sqrt{\kappa_p}\, x_p,$$

$$b_{in;i} - b_{out;i} = \sqrt{\kappa_i}\, z_i,$$

$$b_{in;s} - b_{out;s} = \sqrt{\kappa_s}\, z_s,$$

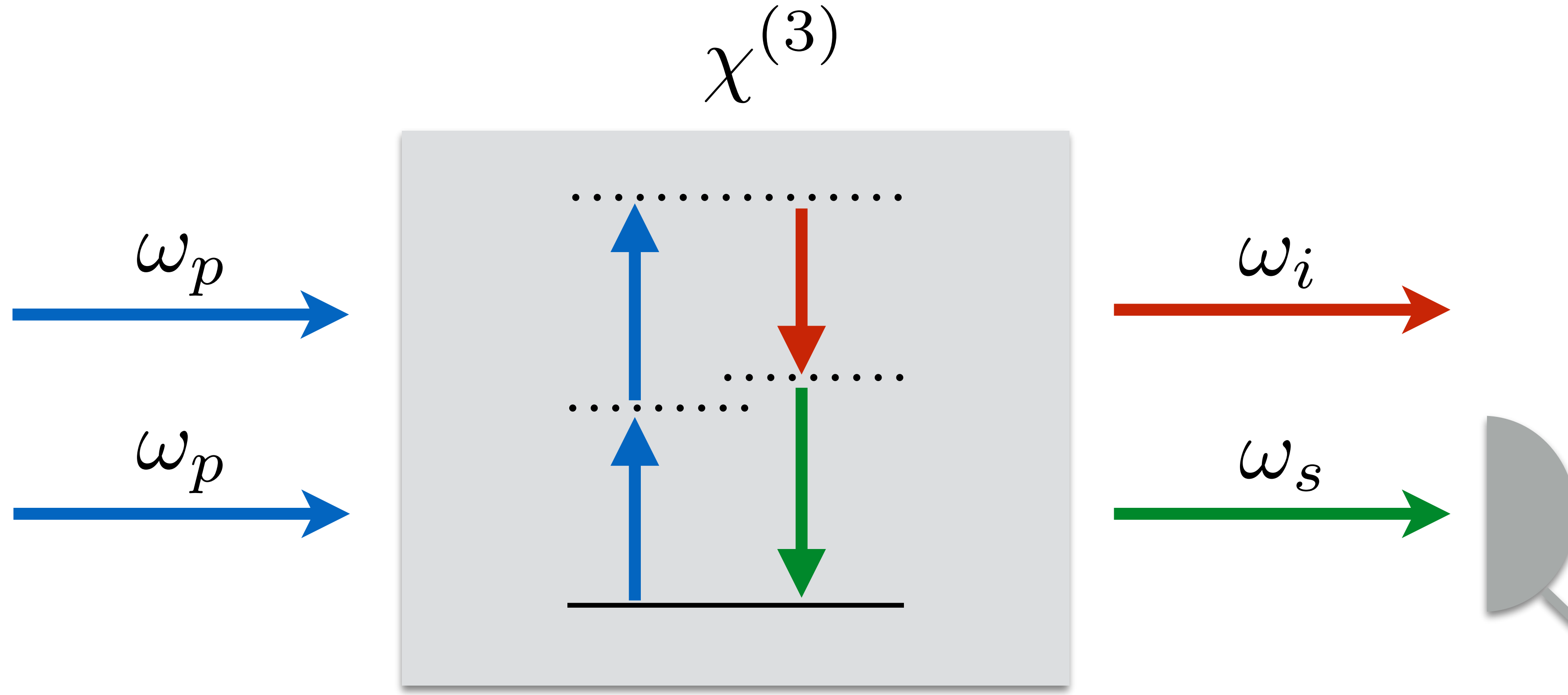


$$\Delta = \omega_0 - \omega$$

$$m = i, s$$

$$y_p(\omega) = M_p a_{in;p}(\omega) = \frac{2g_p\sqrt{\kappa_p}\, a_{in;p}(\omega)}{-2\Delta_p^2 + 2g_p^2 + i\Delta_p\kappa_p}$$

$$y_m(\omega) = M_m b_{in;m}(\omega) = \frac{2g_{is}\sqrt{\kappa_{is}}\, b_{in;m}(\omega)}{-2\Delta_m^2 + 2g_{is}^2 + i\Delta_m\kappa_{is}}$$



$$|\psi\rangle = |0\rangle + \iint d\omega_s d\omega_i \, F(\omega_s, \omega_i)|\omega_s\rangle\omega_i\rangle$$

$$\rho(\omega_i) = \int d\omega_s \langle\omega_s|\psi\rangle\langle\psi|\omega_s\rangle$$

Factorable JSA

$$F(\omega_i, \omega_s) = F_i(\omega_i)F_s(\omega_s)$$

Pure heralded states

$$\rho(\omega_i) = |\psi_i\rangle\langle\psi_i|$$

A.B.U'Ren, K. Banaszek, I.A. Walmsley // Quantum Inf. Comp. 3, 480 (2003)
A.B.U'Ren, C. Silberhorn, et al. // Laser Physics, 15, 146 (2005)

Outline of calculation

$$\mathcal{H}_{SFWM}(t) = \zeta y_p(t) y_p(t) y_s^\dagger(t) y_i^\dagger(t)$$

$$u(t) = \frac{1}{\sqrt{2\pi}} \int d\omega\, e^{-i\omega t} u(\omega)$$

$$y_p(\omega) = M_p\, a_{in;p}(\omega)$$

$$y_i(\omega) = M_i^*\, a_{out;i}(\omega) \qquad y_s(\omega) = M_s^*\, a_{out;s}(\omega)$$

$$|\psi\rangle = [1 - i/\hbar \textstyle\int dt\, \mathcal{H}_{SFWM}(t)] |0\rangle |\alpha\rangle$$

$$|\psi\rangle = |0\rangle|\alpha\rangle - \frac{i\zeta}{\hbar\sqrt{2\pi}^3} \int d\omega_i d\omega_s \mathcal{F}(\omega_i, \omega_s)\, y^\dagger_{out;i}(\omega_i) y^\dagger_{out;s}(\omega_s) |0\rangle|\alpha\rangle$$

Purity of the state

Joint Spectral Amplitude in the present model:

$$\mathcal{F}(\omega_i, \omega_s) = \mathcal{I}_p(\omega_i, \omega_s) M_i(\omega_i) M_s(\omega_s)$$

$$\mathcal{I}_p(\omega_i, \omega_s) = \int d\omega_p M_p(\omega_s + \omega_i - \omega_p) M_p(\omega_p) \alpha(\omega_s + \omega_i - \omega_p) \alpha(\omega_p)$$

Schmidt decomposition:

$$\mathcal{F}(\omega_i, \omega_s) = \sum_n \sqrt{\lambda_n}\, \psi_n(\omega_i) \phi_n(\omega_s), \quad \sum_n \lambda_n = 1$$

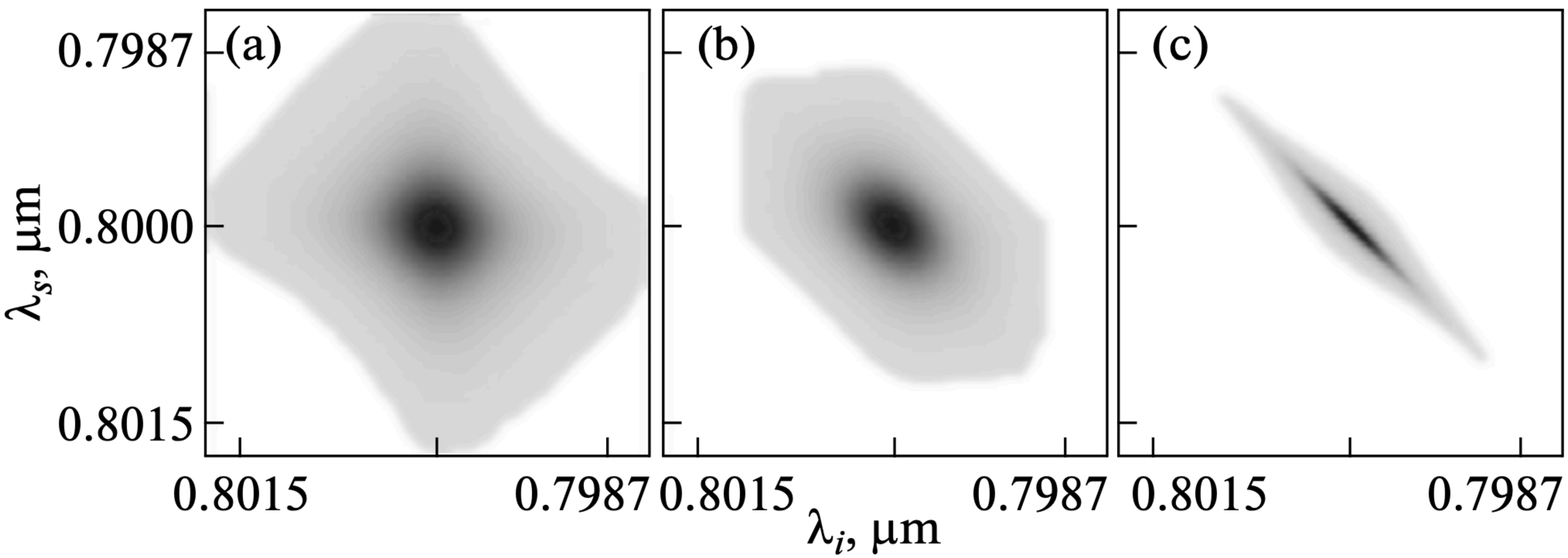
Schmidt number: $K = 1/\sum_n \lambda_n^2 \quad (K \geq 1)$

Purity: $\gamma = 1/K \quad (0 \leq \gamma \leq 1)$

$K = 1$ → Factorable JSA → Pure state of heralded photons

Condition I: Broadband pump field

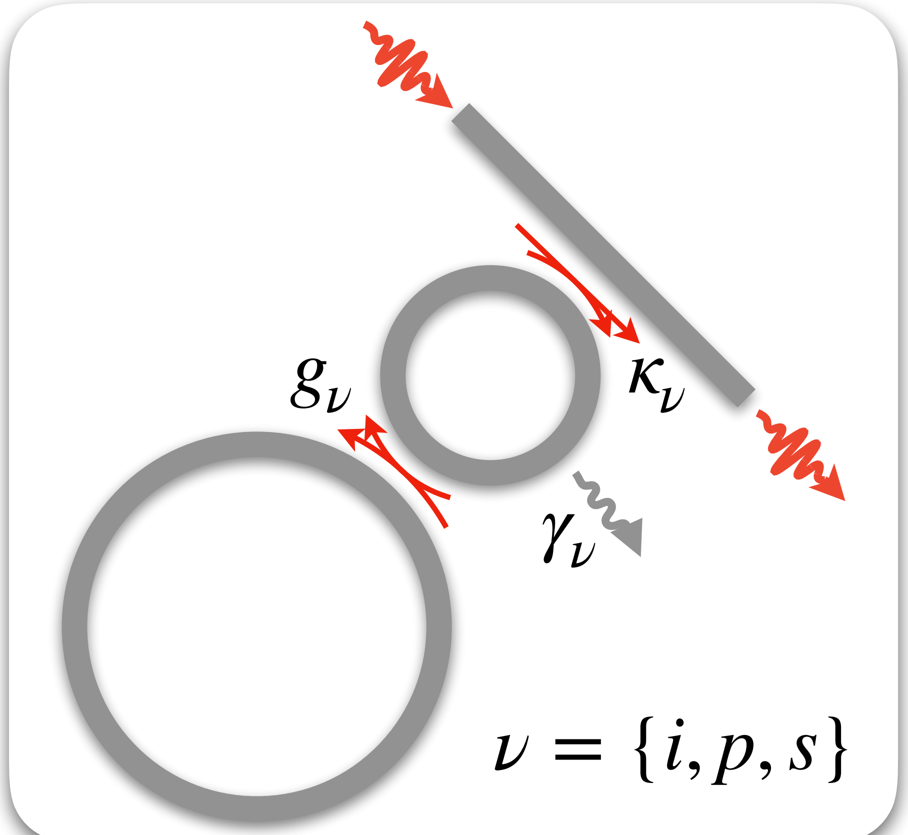
$R_p = 0.30$ $\qquad$ $R_p = 0.65$ $\qquad$ $R_p = 0.95$



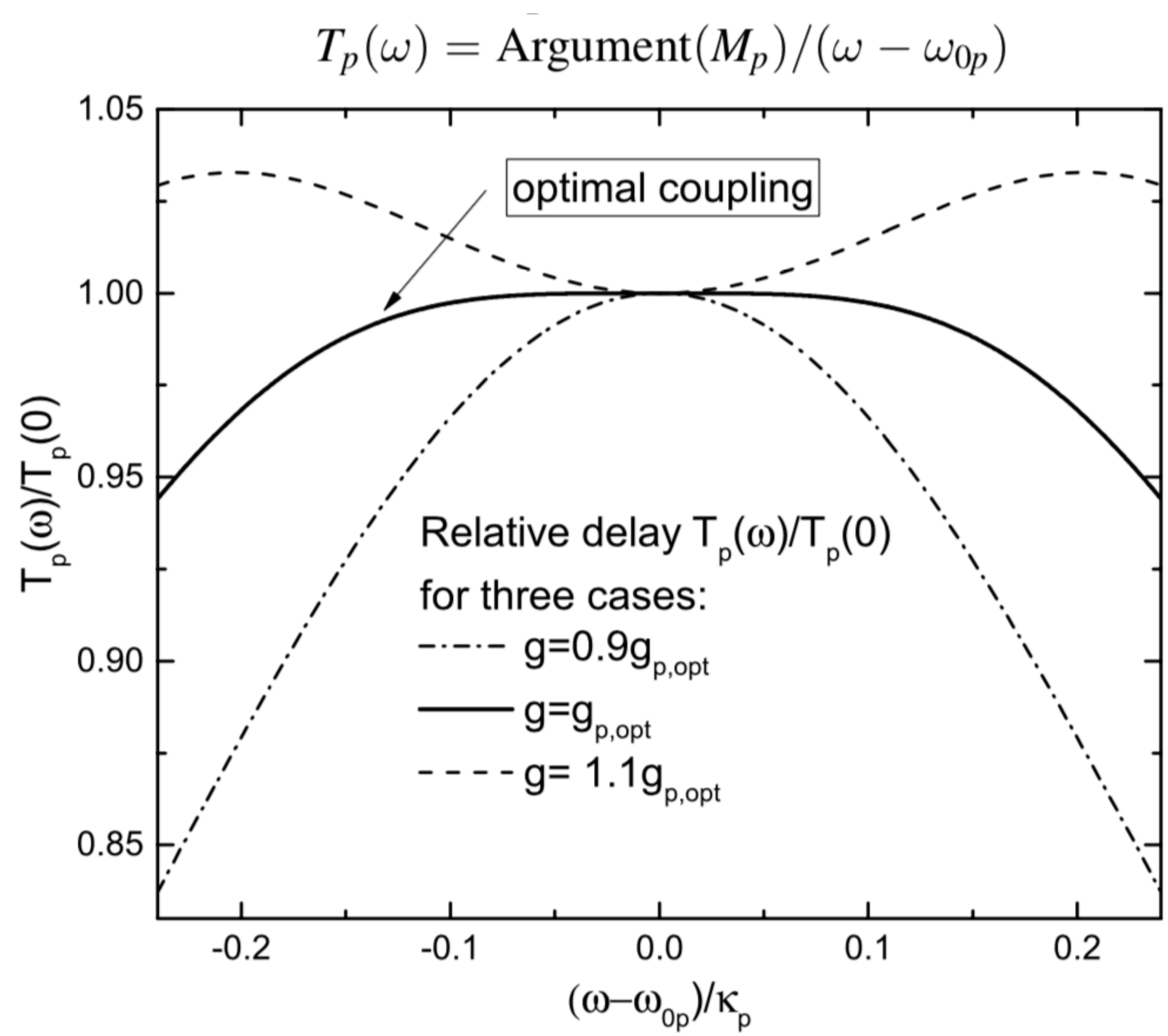
$$R_i = R_s = \text{const}$$

Y. Jeronimo-Moreno et al. // Laser Physics, 20, 1221 (2010)

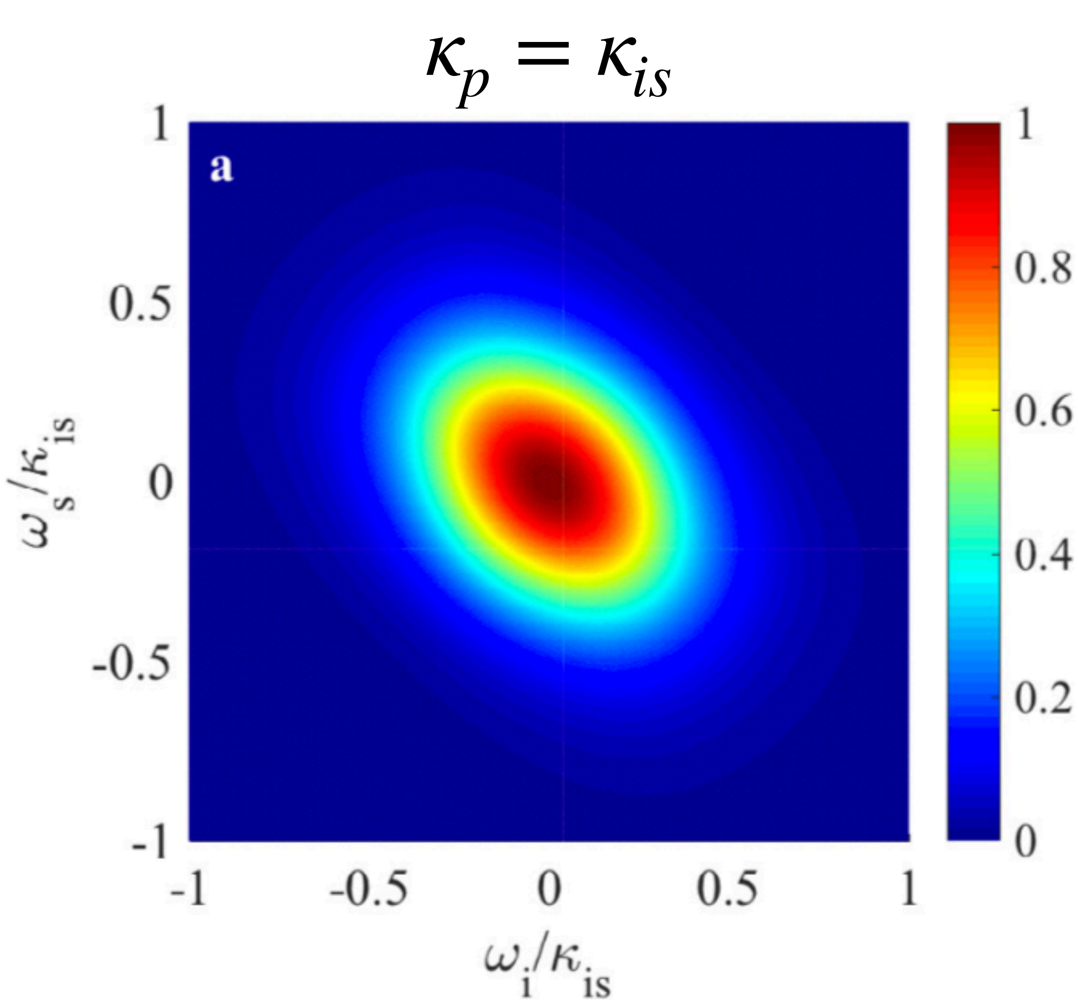
Condition II: Optimal coupling



$$g_{p,\mathrm{opt}} = \kappa_p/\sqrt{12},\ g_{is,\mathrm{opt}} = \kappa_{is}/\sqrt{12}.$$

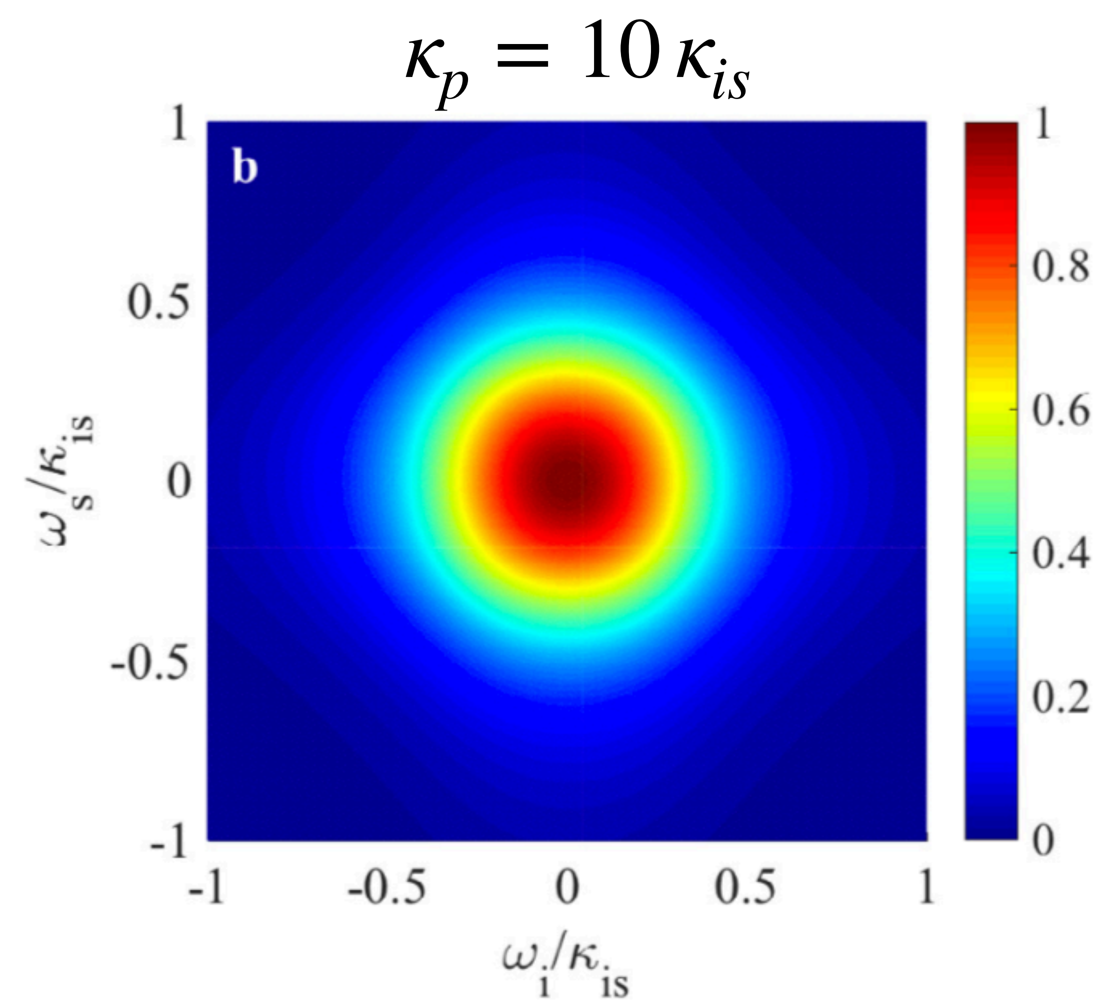


Basic result



$K = 1.07$

$\gamma = 0.94$



$K = 1.00006$

$\gamma = 0.9999$

- Optimal ratios between coupling constants
- Gaussian pump pulses with optimal spectral width $\Delta\omega_{1/2} \approx \kappa_p/2$

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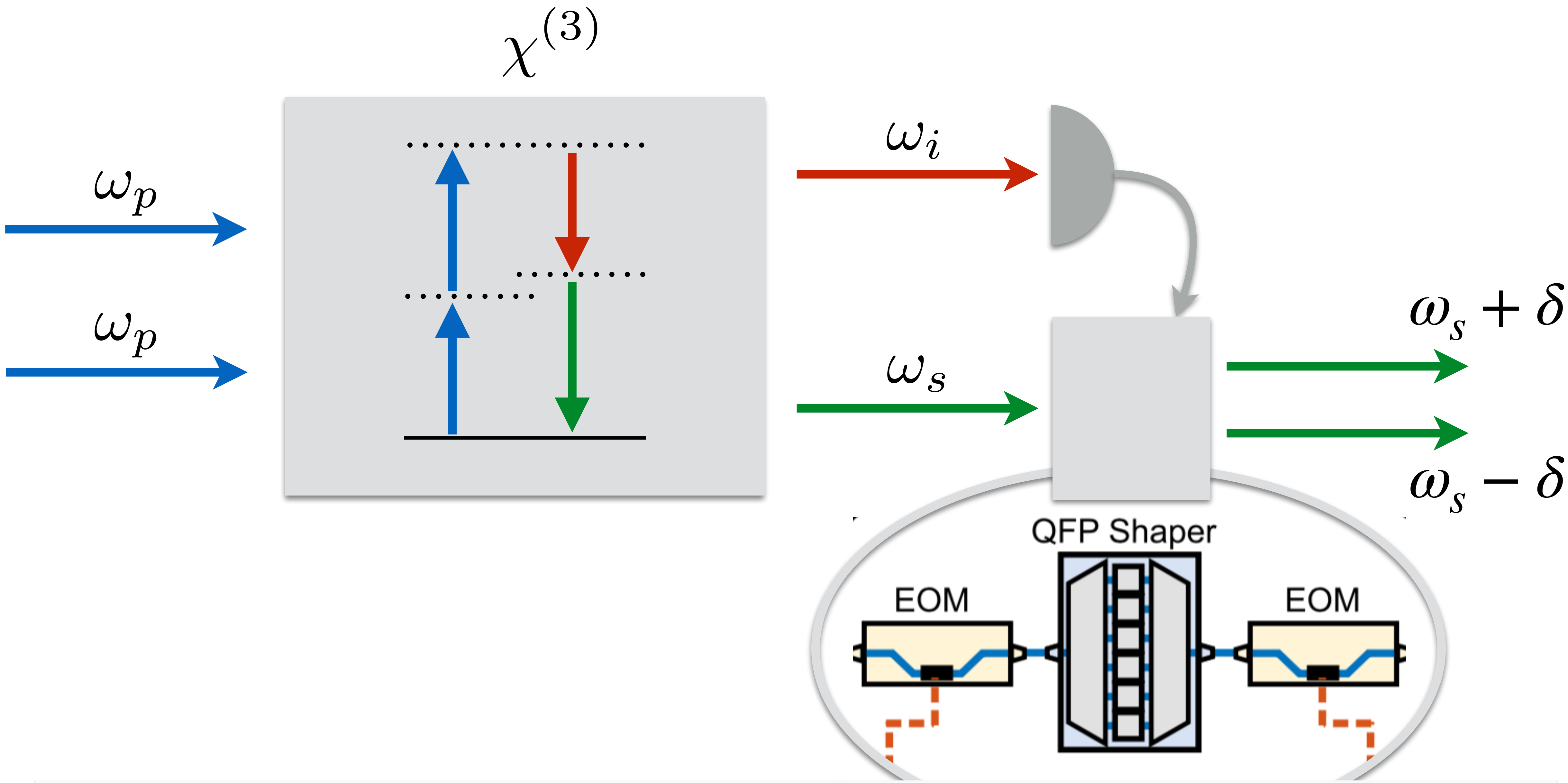
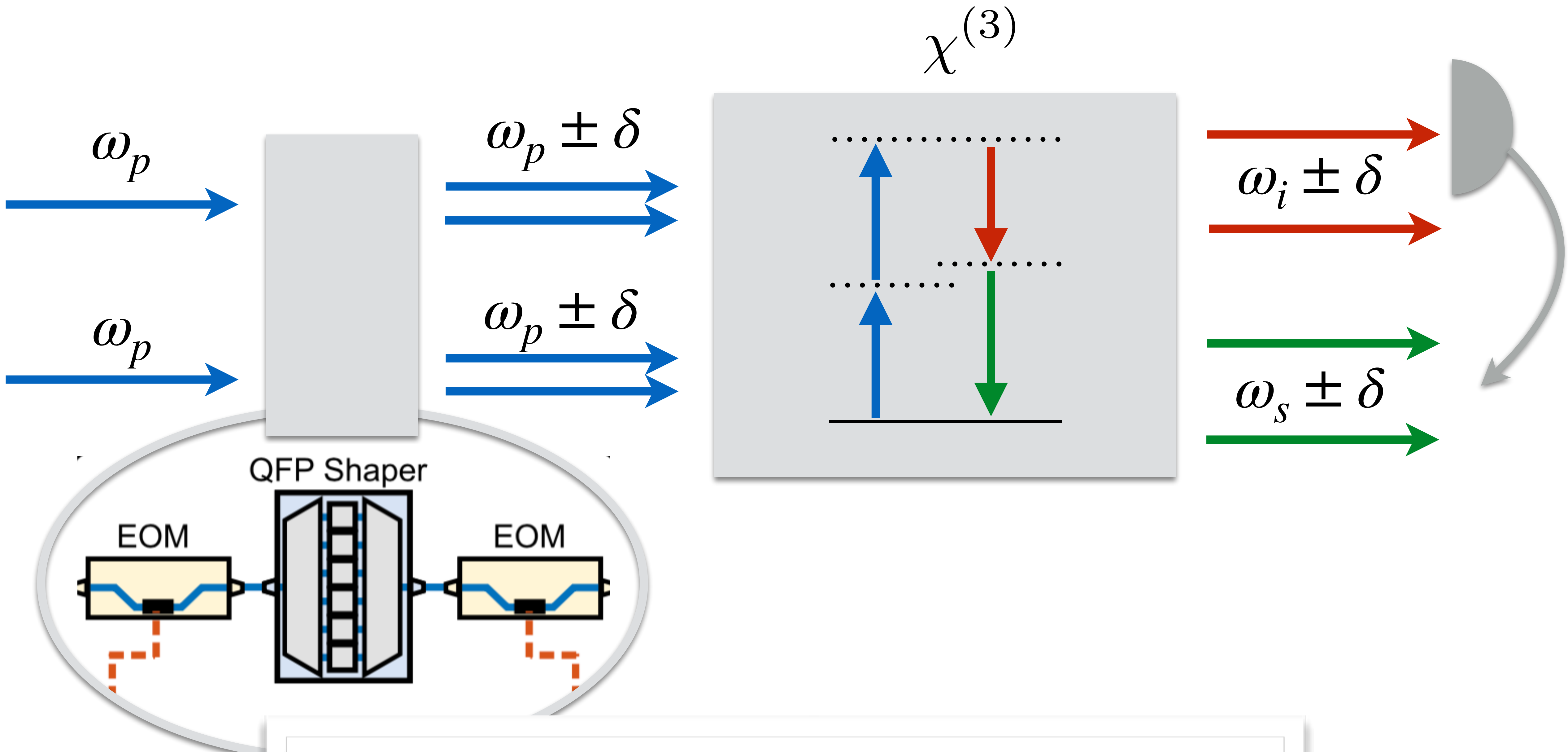
The simplest approach: modulation at the output



The heralding efficiency is reduced by insertion losses and by less than unit success probability of the gate

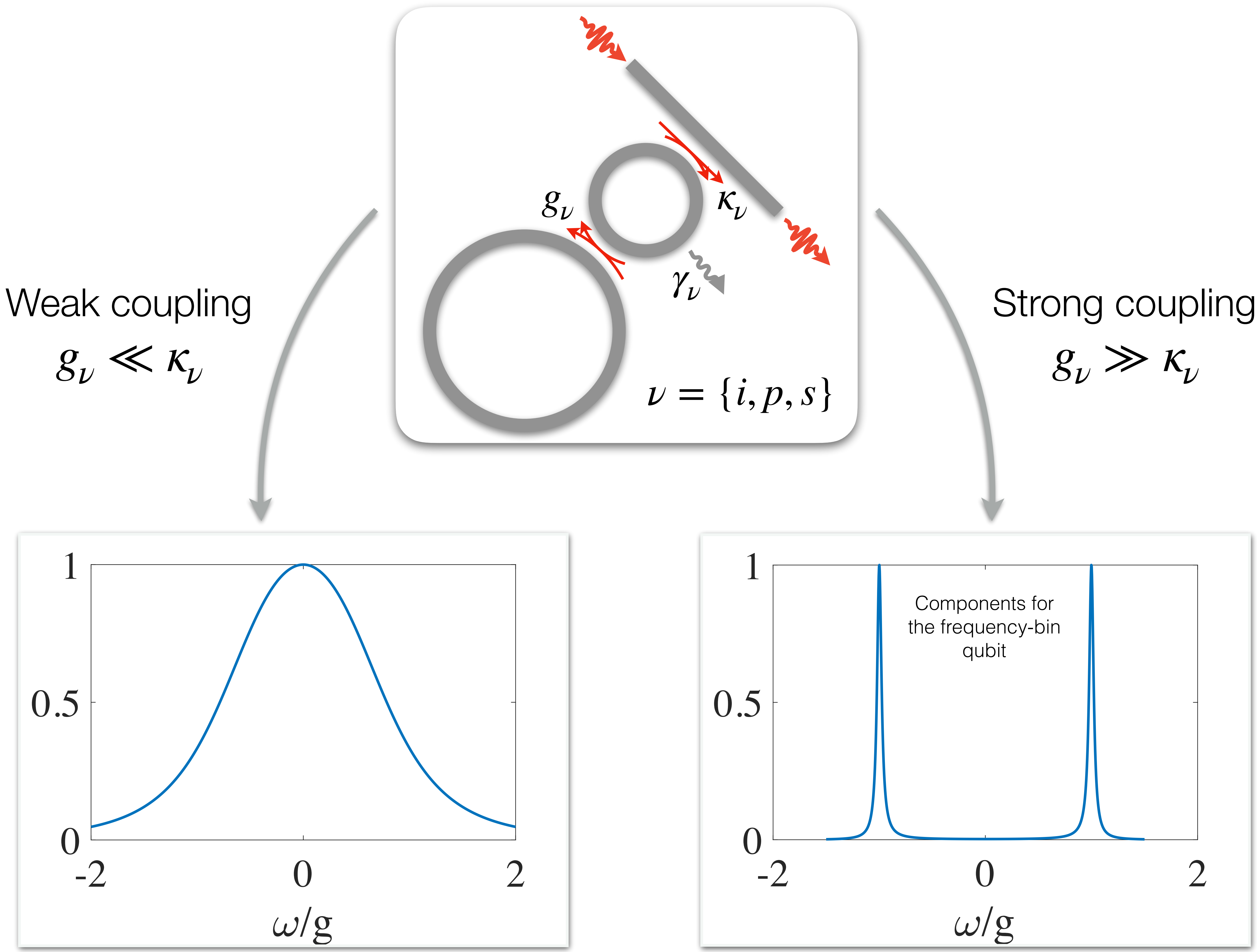
Single-qubit gate in the frequency domain:
H.-H. Lu, et al. Physical Review Letters 120, 030502 (2018)

Our approach: modulation of the pump field



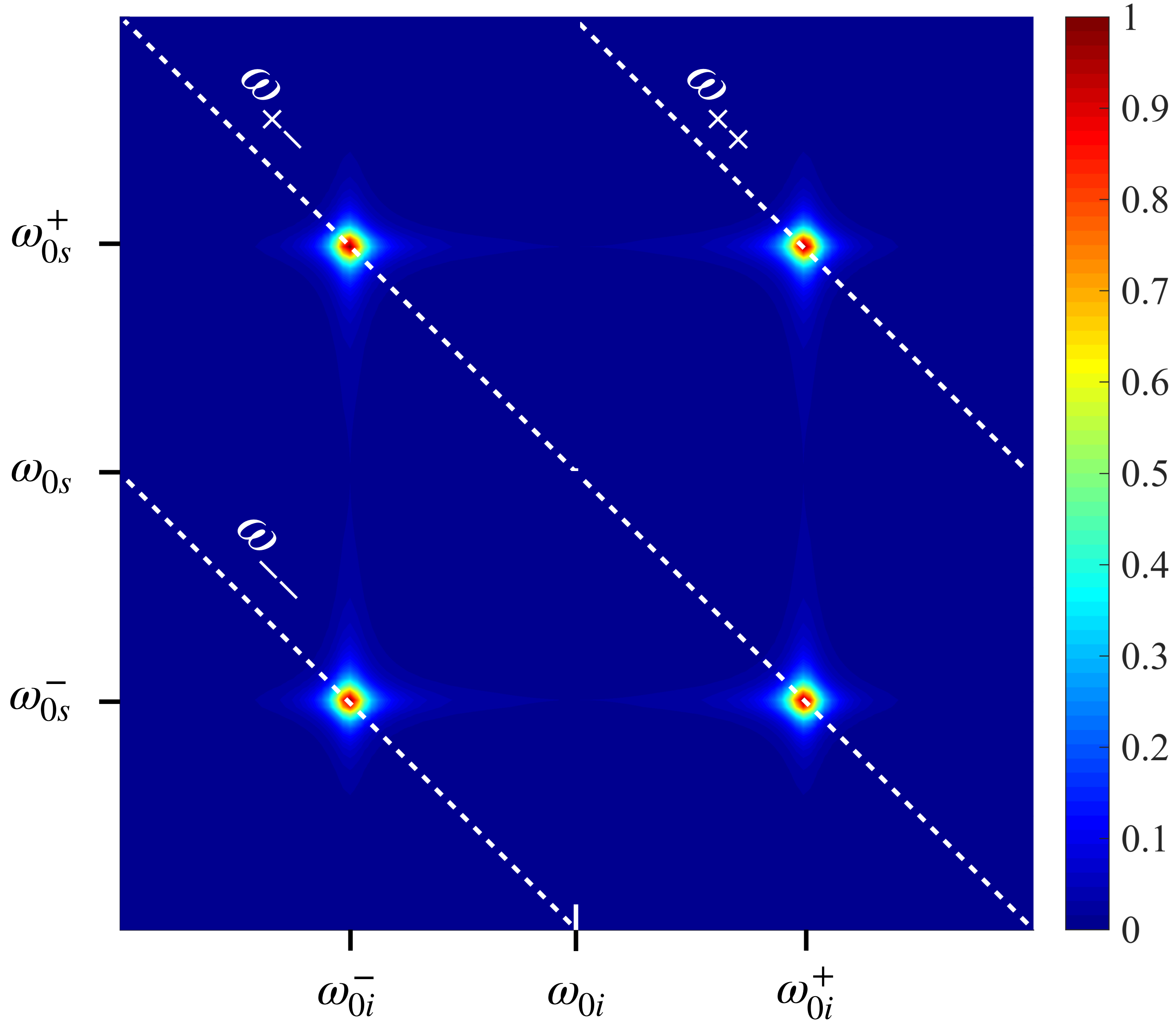
Highest possible heralding efficiency

Single-qubit gate in the frequency domain:
H.-H. Lu, et al. Physical Review Letters 120, 030502 (2018)



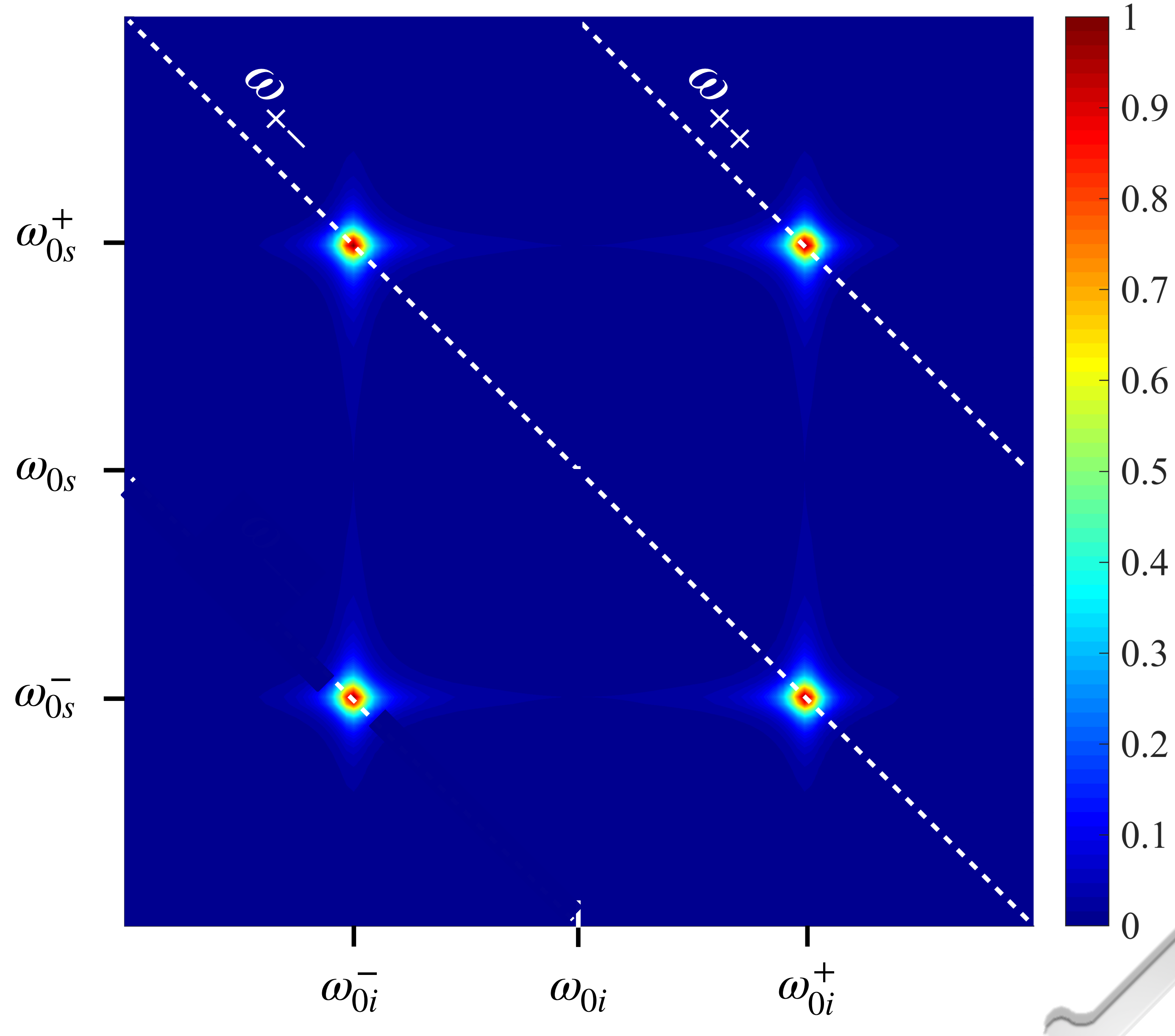
g_ν
κ_ν
γ_ν
$\nu = \{i, p, s\}$
Weak coupling
$g_\nu \ll \kappa_\nu$
Strong coupling
$g_\nu \gg \kappa_\nu$
1
0.5
0
-2
0
2
ω/g
Components for the frequency-bin qubit
1
0.5
0
-2
0
2
ω/g

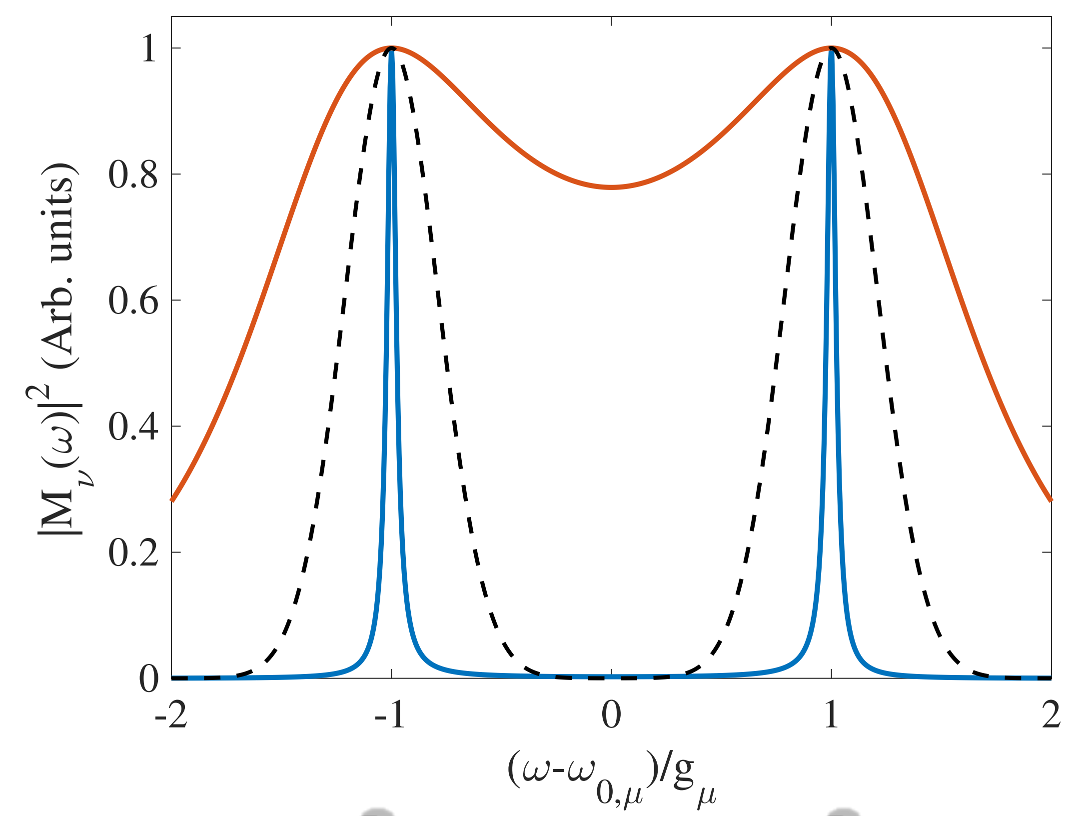
JSA: broadband pump field



$$M_\mu(\omega_\mu) = M_\mu^-(\omega_\mu) + M_\mu^+(\omega_\mu),$$

JSA: two-frequency pump field

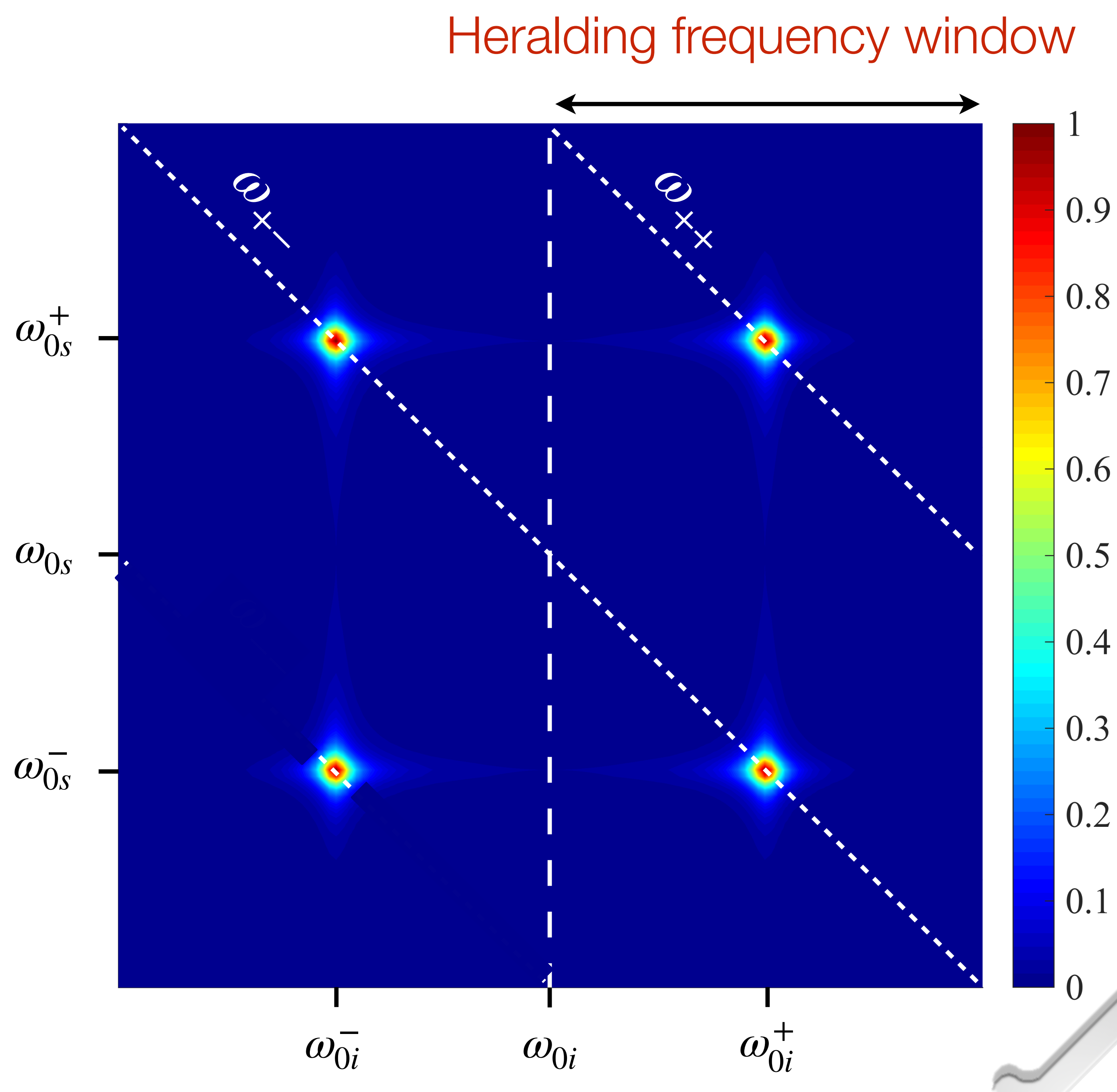


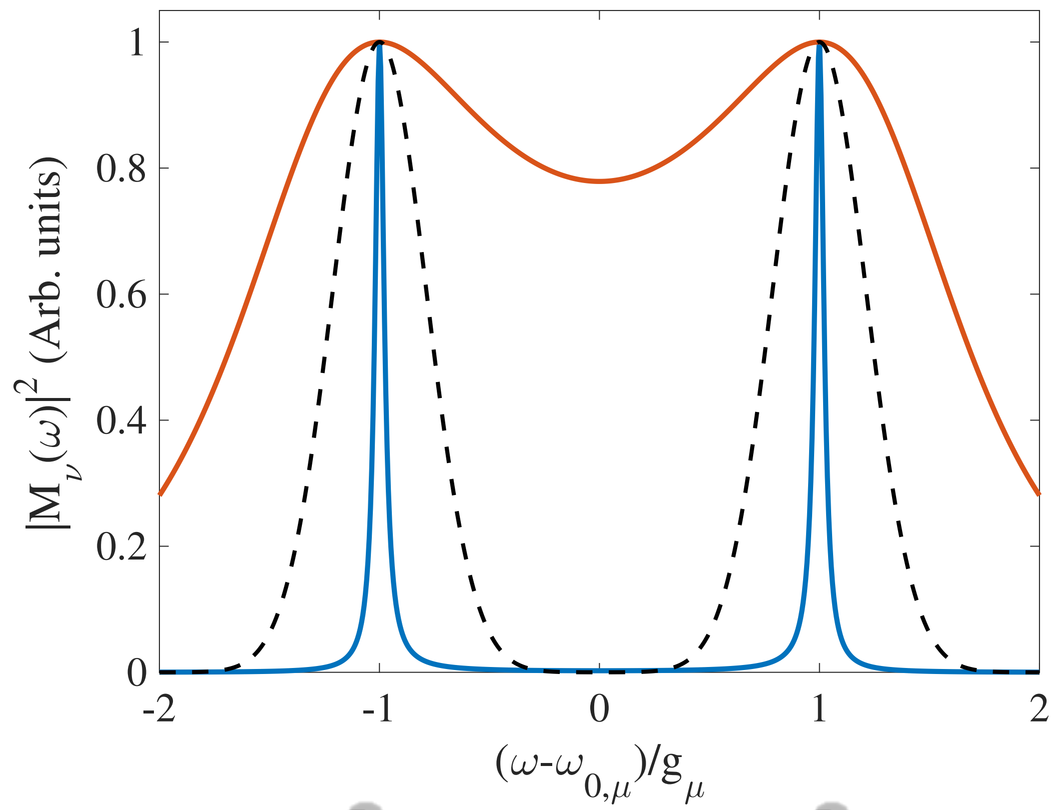


$$M_\mu(\omega_\mu) = M_\mu^-(\omega_\mu) + M_\mu^+(\omega_\mu),$$

$$\alpha(\omega_p) \sim \sqrt{A}\, e^{-\sigma(\omega_p-\omega_{+-})^2 - i\phi_a/2} + \sqrt{B}\, e^{-\sigma(\omega_p-\omega_{++})^2 - i\phi_b/2}$$

JSA: two-frequency pump field





$$|\psi_s\rangle = Ae^{-i\phi_a}|a\rangle + Be^{-i\phi_b}|b\rangle$$

$$|a\rangle = \int d\omega_s M_s^-(\omega_s) y_{out,s}^\dagger(\omega_s)|0\rangle$$

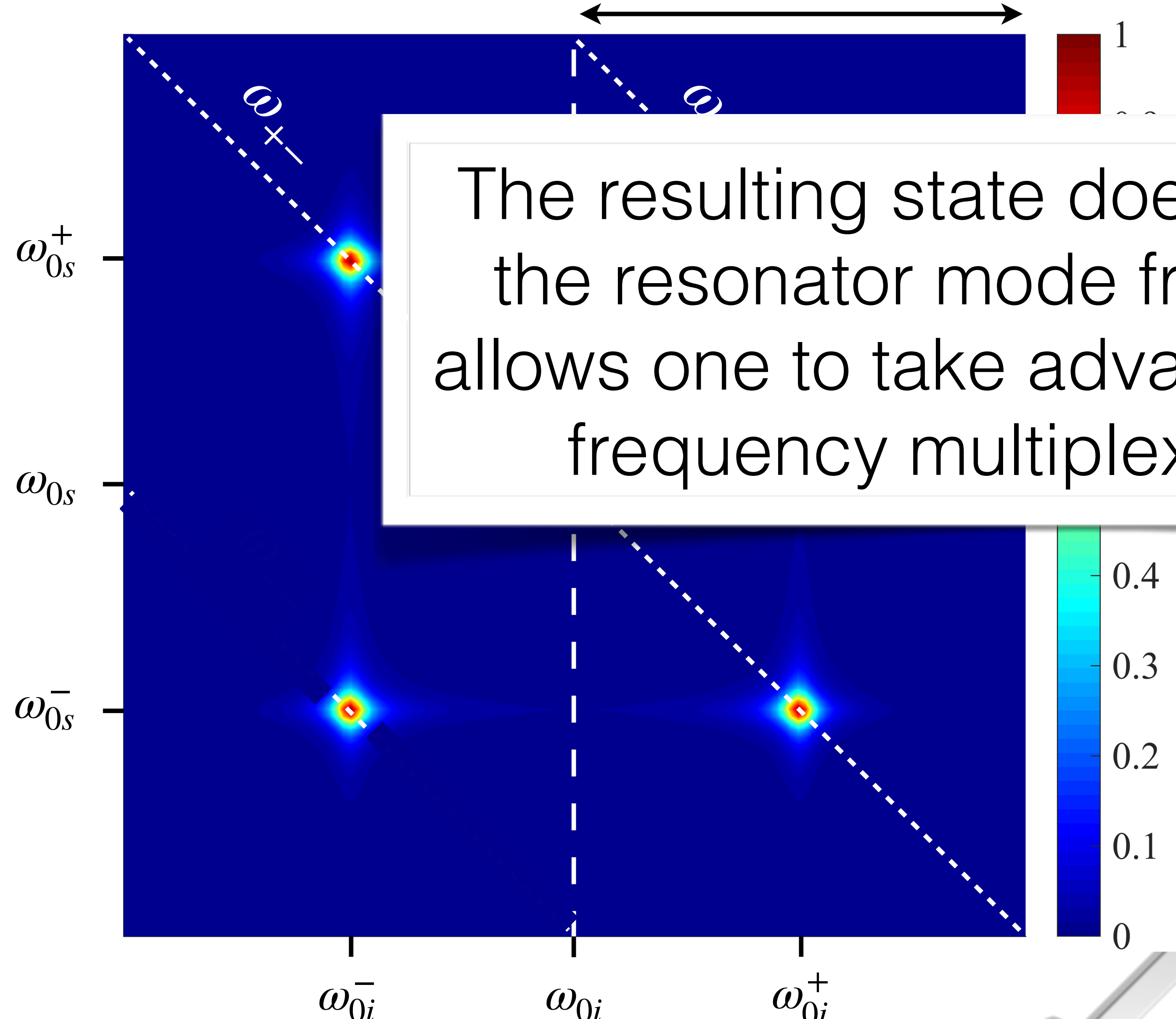
$$|b\rangle = \int d\omega_s M_s^+(\omega_s) y_{out,s}^\dagger(\omega_s)|0\rangle$$

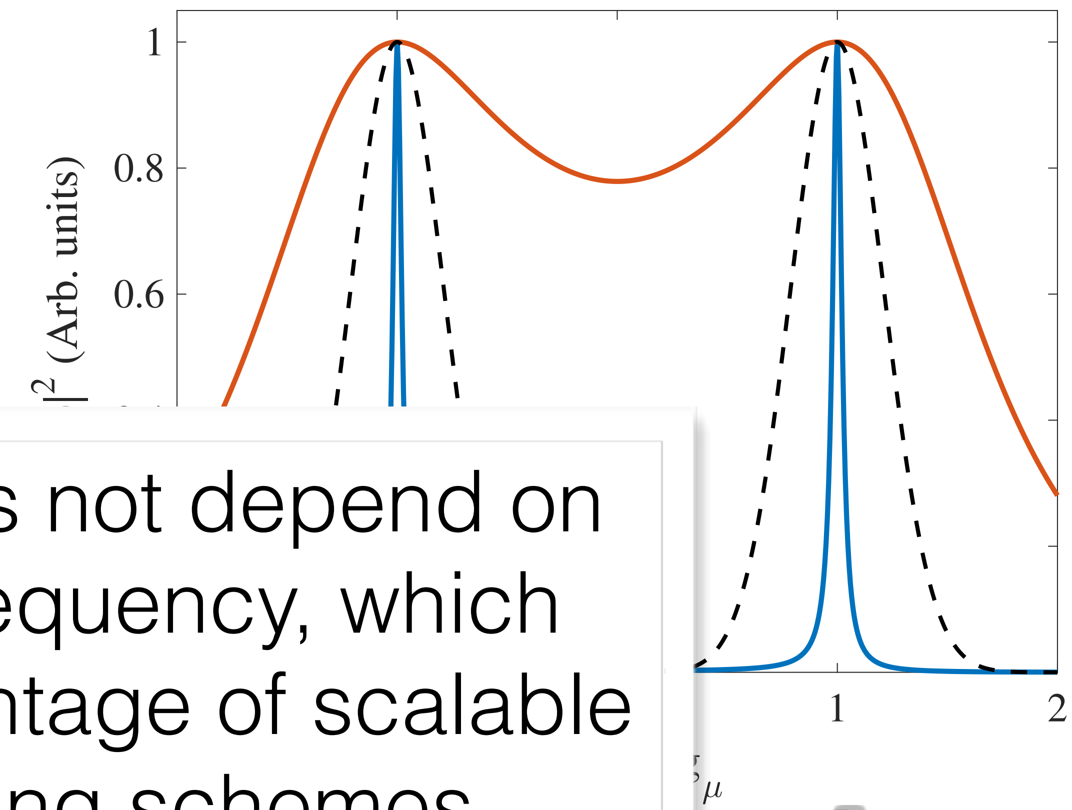
$$M_\mu(\omega_\mu) = M_\mu^-(\omega_\mu) + M_\mu^+(\omega_\mu),$$

$$\alpha(\omega_p) \sim \sqrt{A}\, e^{-\sigma(\omega_p-\omega_{+-})^2 - i\phi_a/2} + \sqrt{B}\, e^{-\sigma(\omega_p-\omega_{++})^2 - i\phi_b/2}$$

JSA: two-frequency pump field

Heralding frequency window





The resulting state does not depend on the resonator mode frequency, which allows one to take advantage of scalable frequency multiplexing schemes

$$|\psi_s\rangle = Ae^{-i\phi_a}|a\rangle + Be^{-i\phi_b}|b\rangle$$

$$|a\rangle = \int d\omega_s M_s^{-}(\omega_s) y_{out,s}^{\dagger}(\omega_s)|0\rangle$$

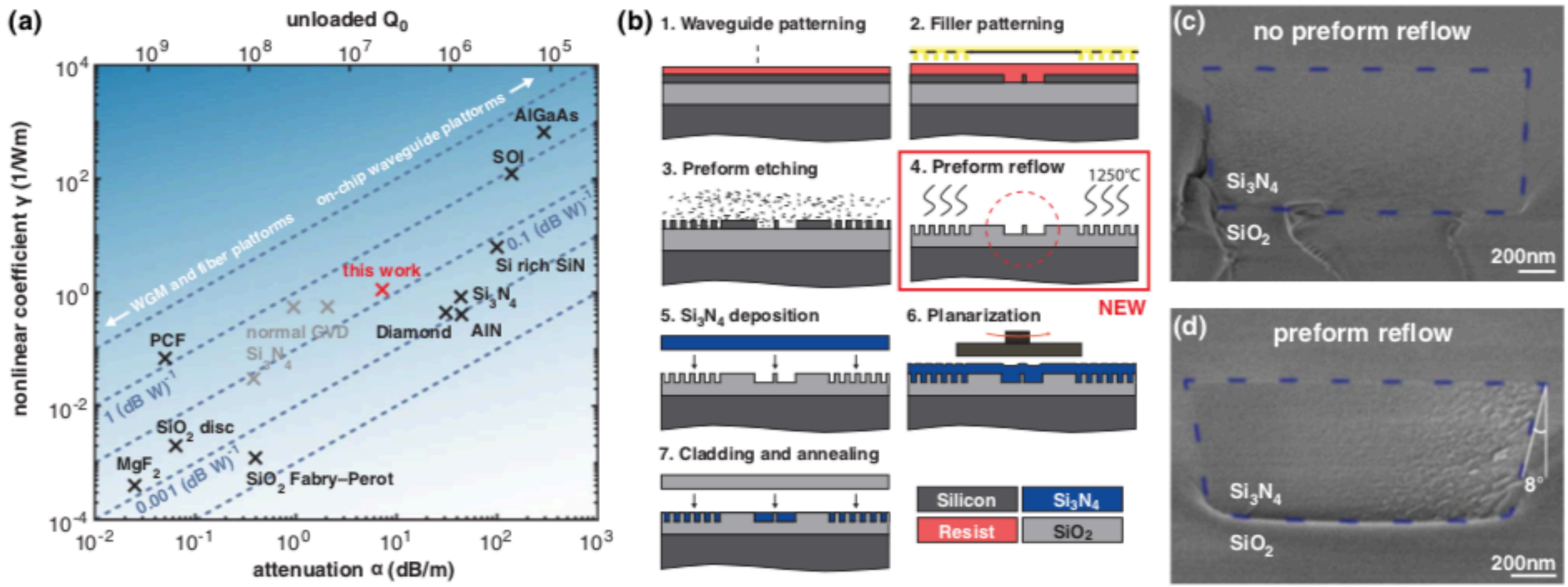
$$|b\rangle = \int d\omega_s M_s^{+}(\omega_s) y_{out,s}^{\dagger}(\omega_s)|0\rangle$$

$$M_\mu(\omega_\mu) = M_\mu^{-}(\omega_\mu) + M_\mu^{+}(\omega_\mu),$$

$$\alpha(\omega_p) \sim \sqrt{A}\, e^{-\sigma(\omega_p-\omega_{+-})^2 - i\phi_a/2} + \sqrt{B}\, e^{-\sigma(\omega_p-\omega_{++})^2 - i\phi_b/2}$$

Implementation issues

Silicon Nitride (Si_3N_4) ring resonators

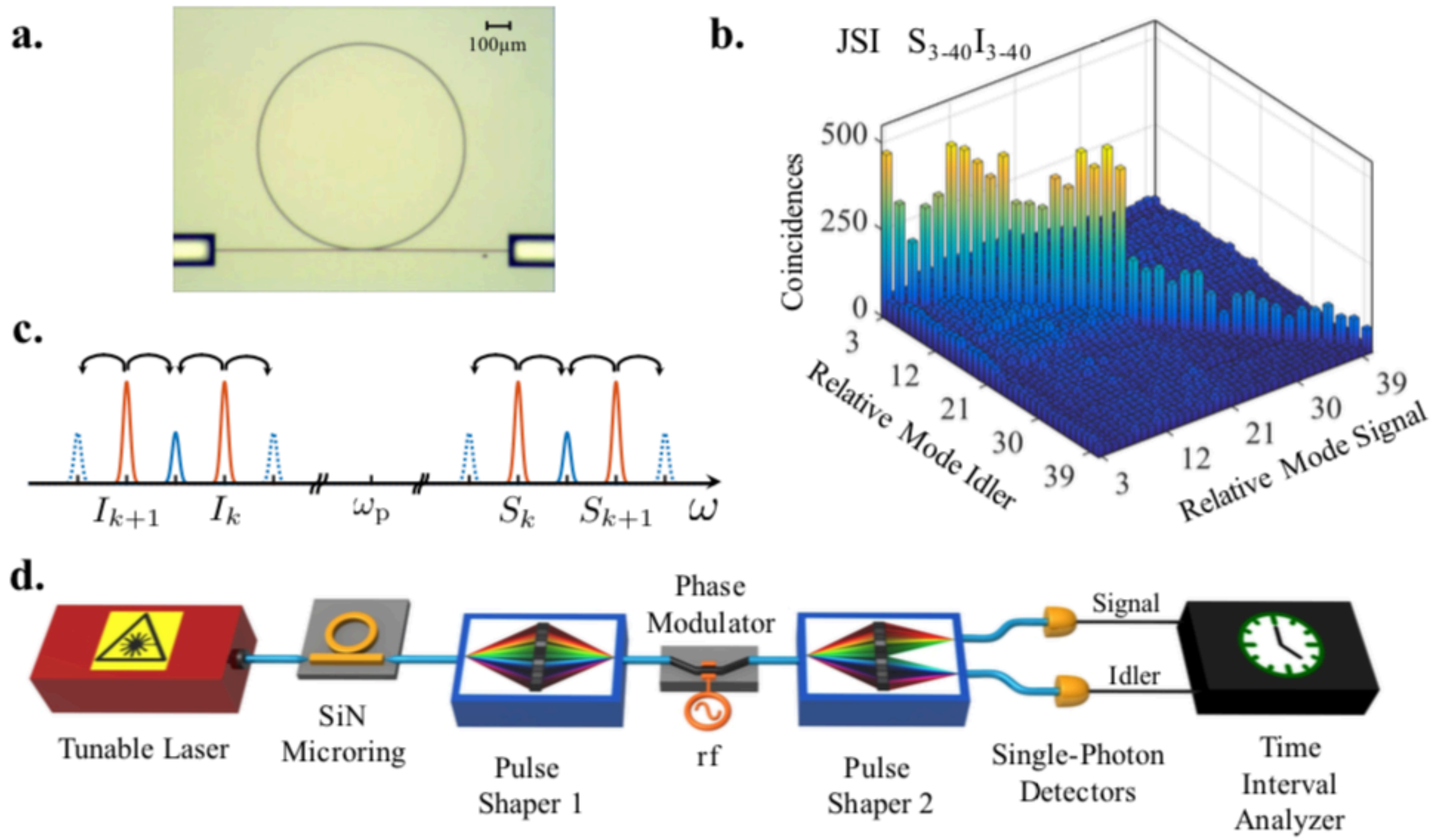


Losses ~ 0.01 dB/cm at 1.5 um

$Q > 1 \cdot 10^7$, $R = 230\ \mu m$, $F \sim 3000$

M.H.P. Pfeiffer, et al, Optica 5, 884 (2018)

Silicon Nitride (Si_3N_4) ring resonators



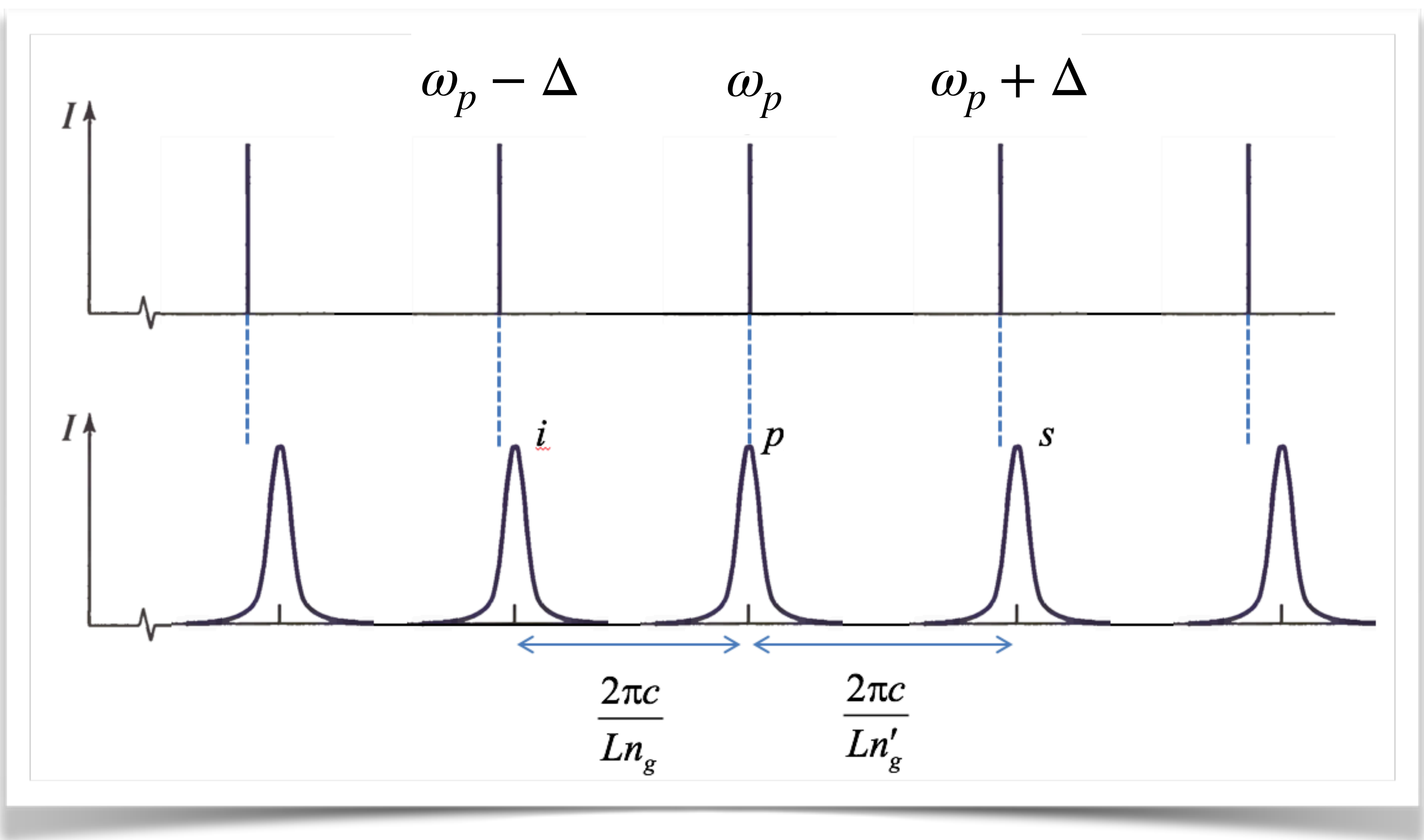
~ 40 entangled modes with frequency interval ~ 50 GHz

Q ~ 10^6, R = 500 μm

P. Imany, et al. Opt. Express 26, 1825 (2018)

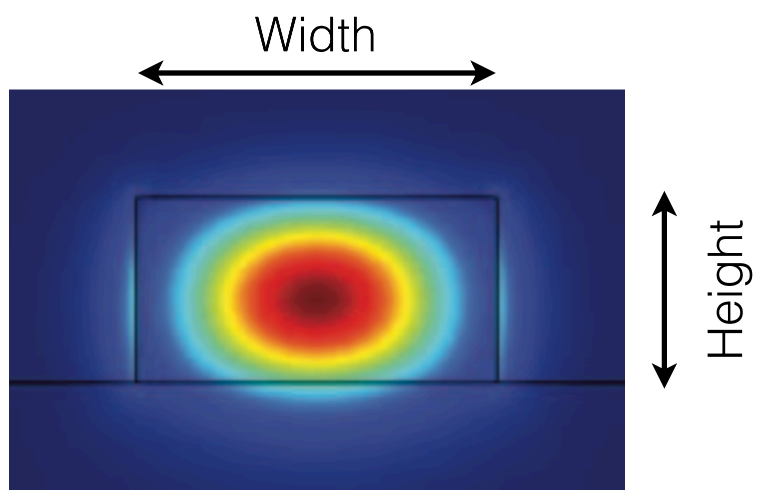
Group velocity dispersion: non-degenerate SFWM

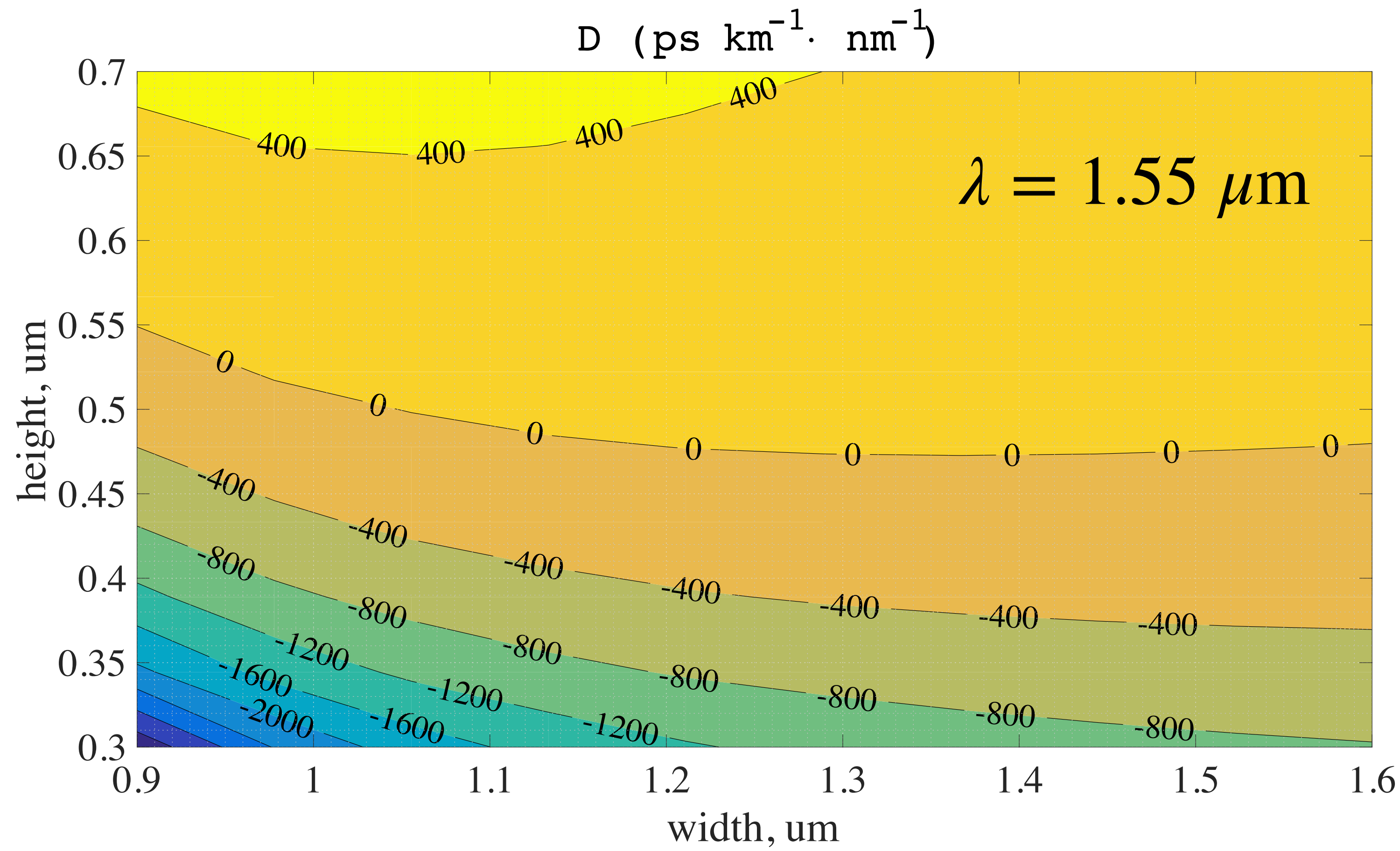
$$D \equiv \frac{1}{c}\frac{\partial n_g}{\partial \lambda} = -\frac{\lambda}{c}\frac{\partial^2 n}{\partial \lambda^2} = 0$$

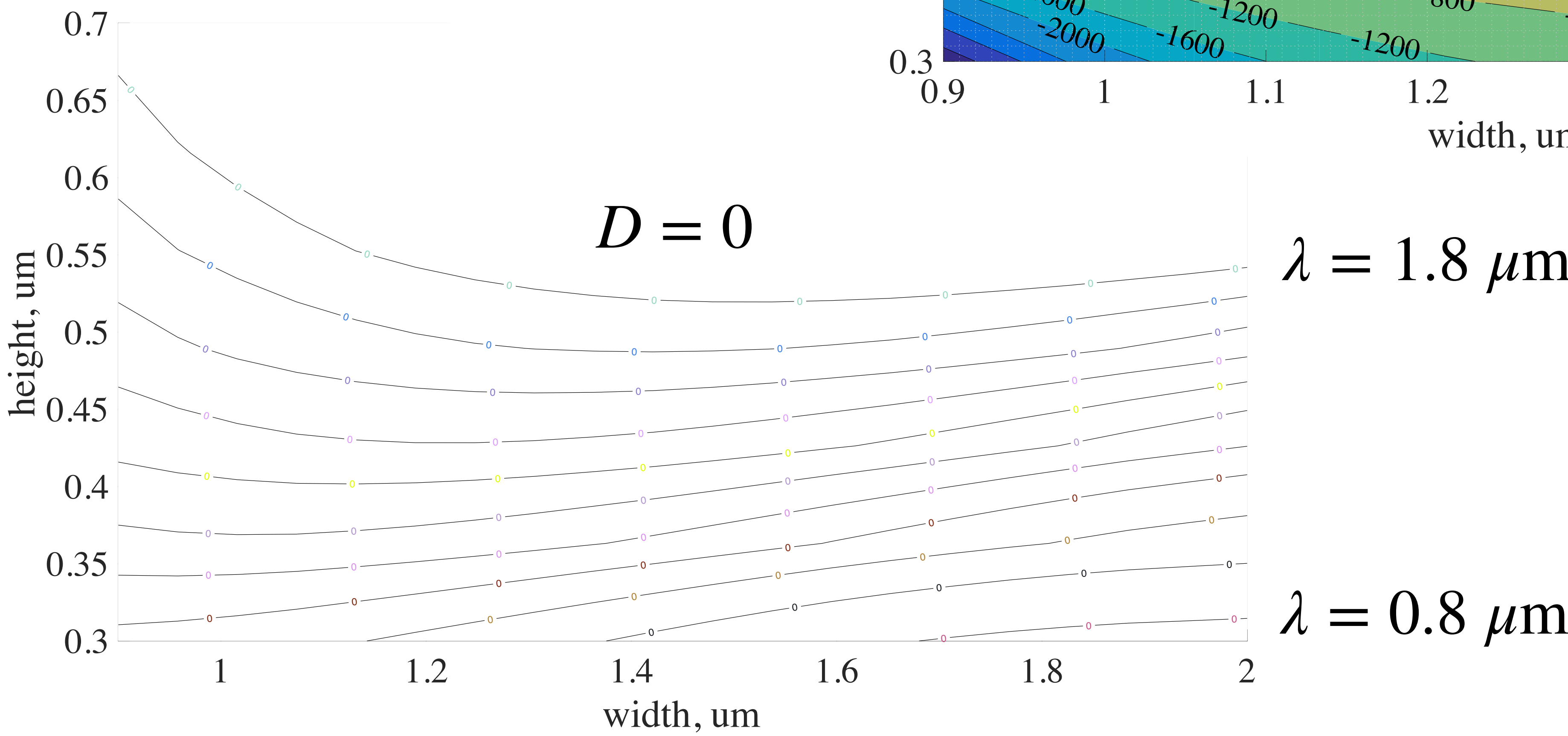


$$D = 0 \rightarrow n_g = n'_g$$

Waveguide design

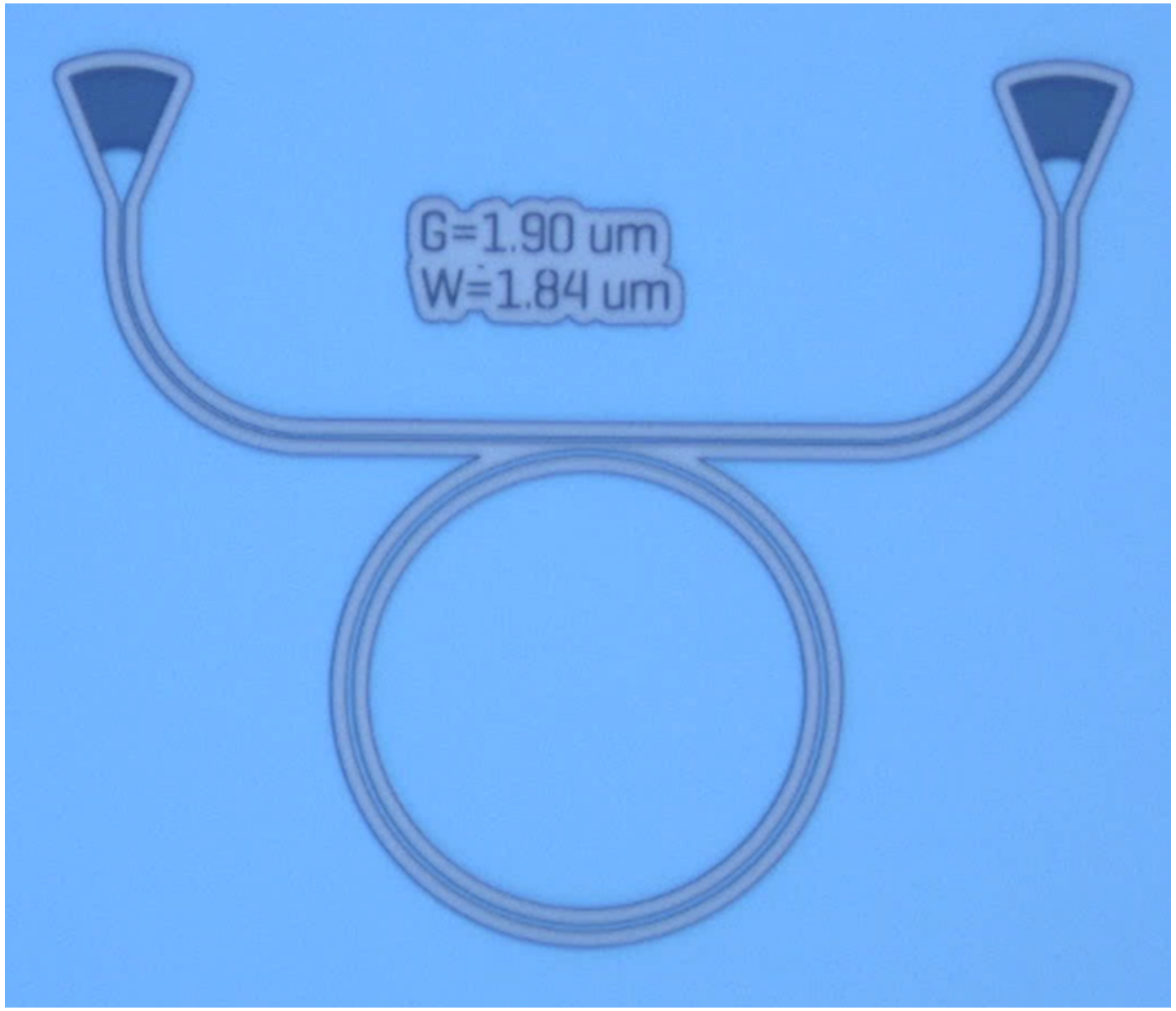


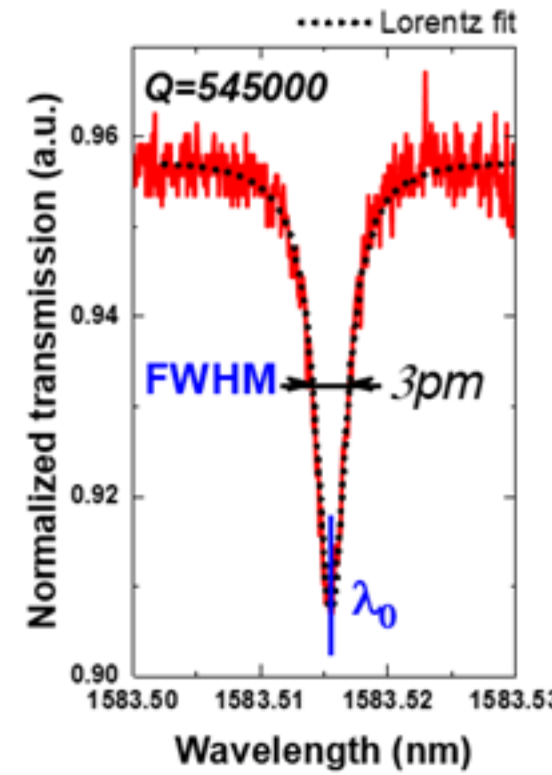




I.N. Chuprina, et al. // Quantum Electronics, 47, 887 (2017)

Experimental results



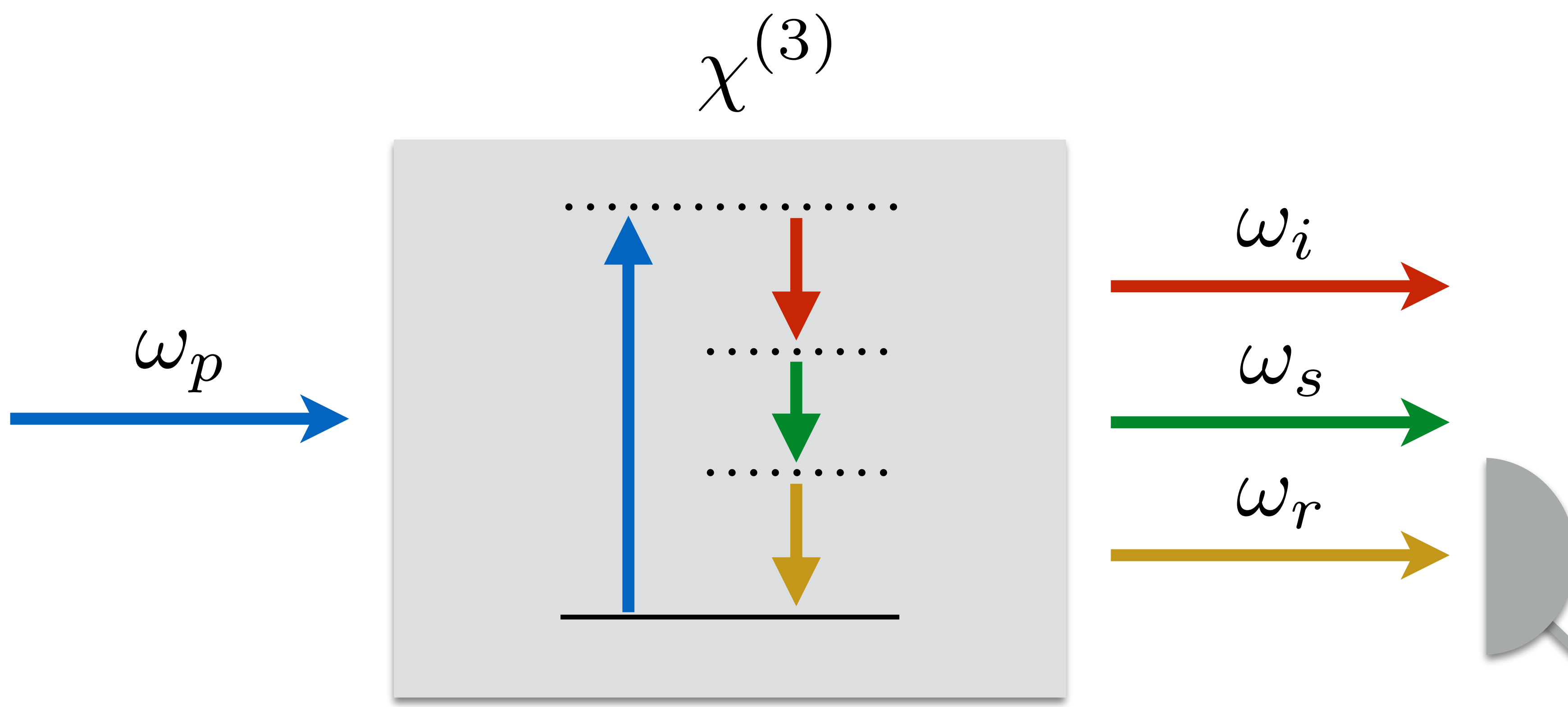


$R = 63.73\ \mu\mathrm{m}$

$W = 1.84\ \mu\mathrm{m}$ $\quad H = 450\ \mathrm{nm}$

by G.N. Gol'tsman et al., Moscow State Pedagogical University

Third-order spontaneous parametric down-conversion



$$|\psi\rangle = |0\rangle + \iint d\omega_s d\omega_i d\omega_r \, F(\omega_s, \omega_i, \omega_r)|\omega_s\rangle\omega_i\rangle\omega_r\rangle$$

$$\rho(\omega_s, \omega_i) = \int d\omega_r \langle\omega_r|\psi\rangle\langle\psi|\omega_r\rangle$$

Entangled photon triples

Heralded photon pairs

TOSPDC in crystals

A.A. Hnilo. PRA 71, 033820 (2005)
M.V. Chekhova, et al. PRA 72, 023818 (2005)
K. Bencheikh, et al. C.R. Phys. 8, 206220 (2007)
A. Dot, et al. PRA 85, 023809 (2012)
N.A. Borshchevskaya, et al. Laser Phys. Lett. 12, 115404 (2015)

Cavity enhancement of TOSPDC in bulk crystals

A.A. Kalachev, Y.Z. Fattakhova. Quant. Electron. 37, 1087 (2007)
N.A. Borshchevskaya, et al. Laser Phys. Lett. 12, 115404 (2015)

TOSPDC in fibers

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S. Richard, et al. Opt. Lett. 36, 3000 (2011)

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N.A. Borshchevskaya, et al. Laser Phys. Lett. 12, 115404 (2015)

emitted flux ~ 0.0001 s^{-1}
for Calcite of 0.1 mm length
and 10 W pump

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TOSPDC in fibers

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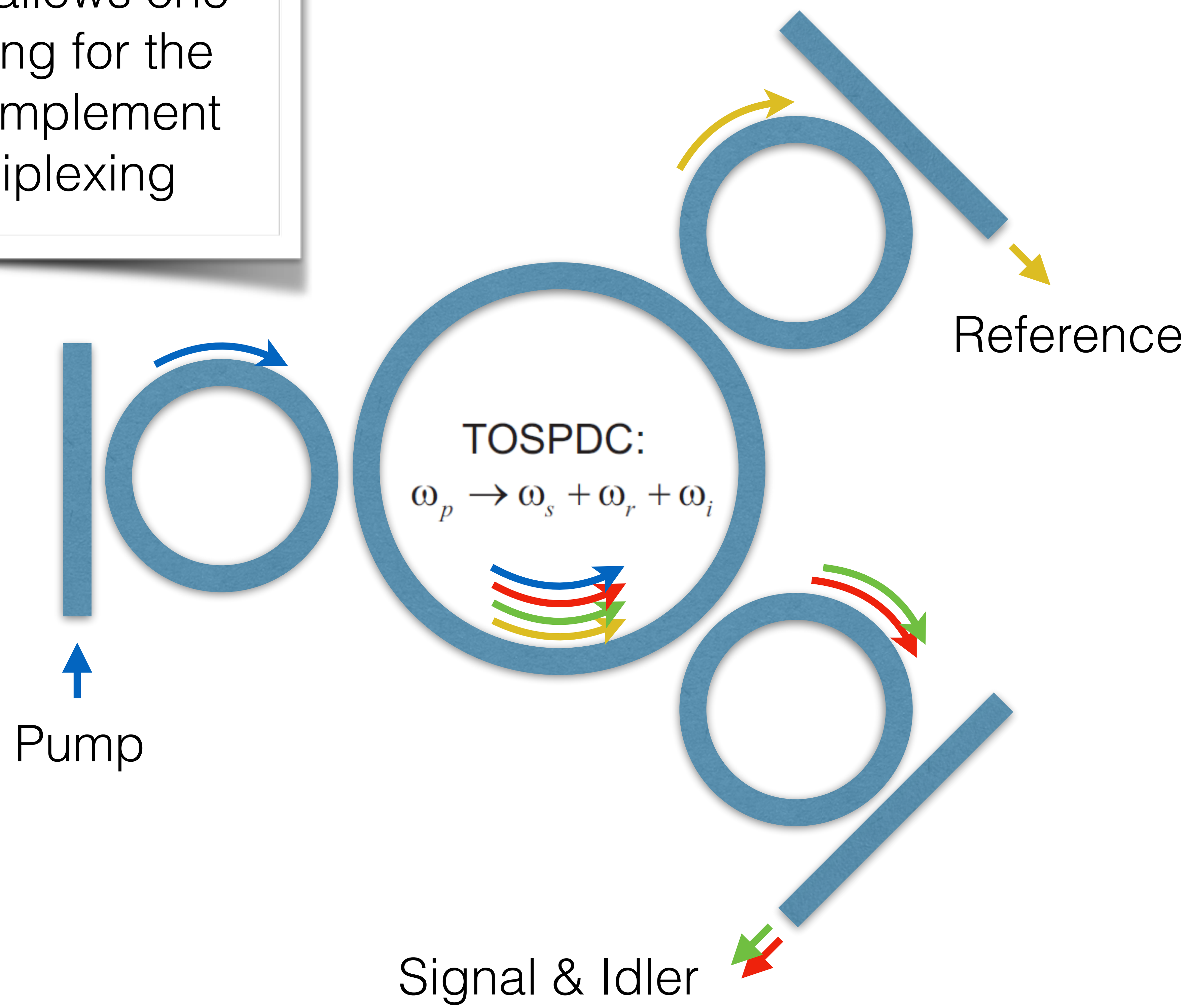
M. Corona, et al. Opt. Lett. 36, 190192 (2011)
S. Richard, et al. Opt. Lett. 36, 3000 (2011)

emitted flux ~ 2 s^{-1}
for MNF of 10 cm length
and 100 mW pump

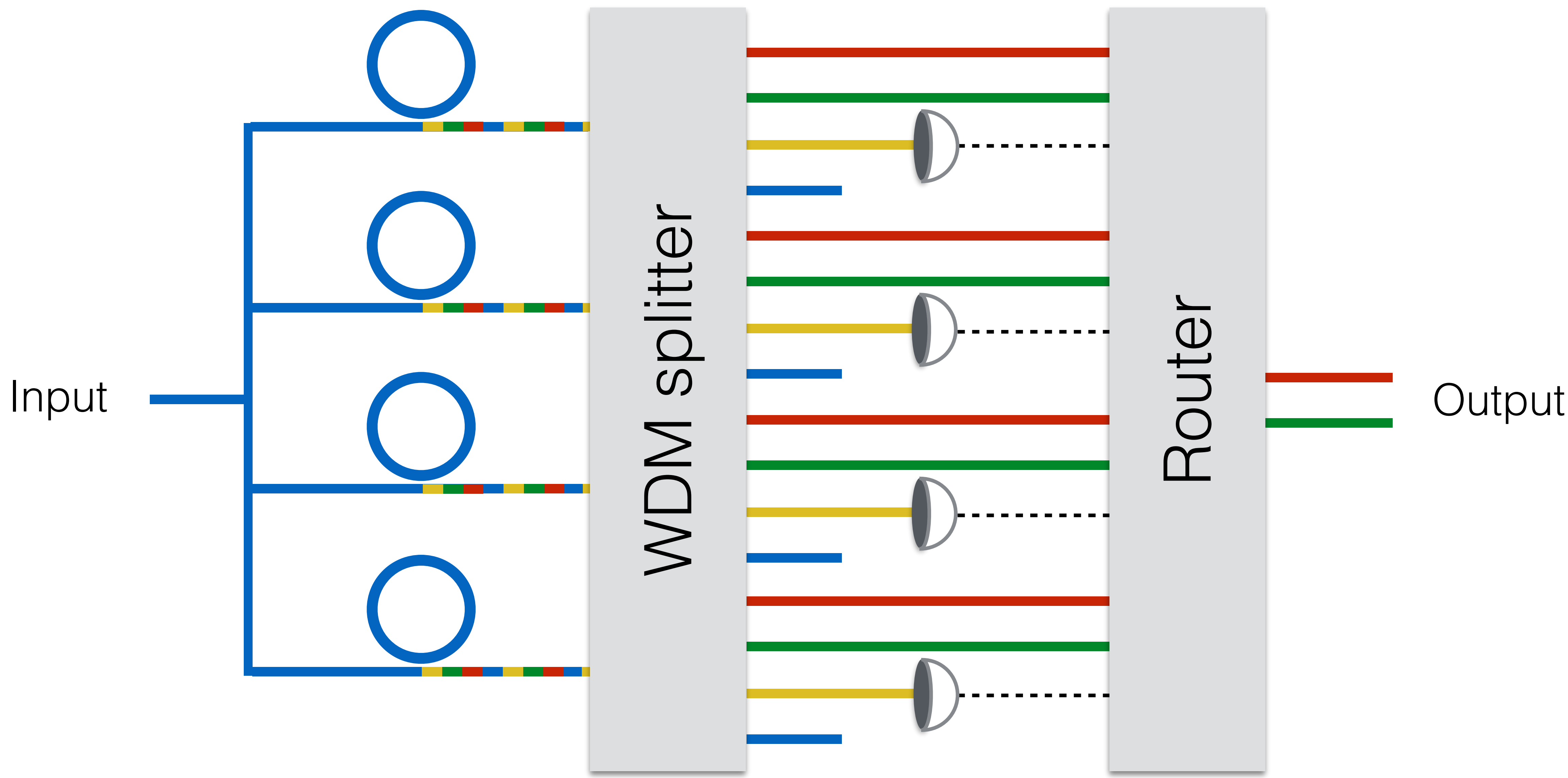
Bandwidth ~ 10 nm →
Low spectral brightness

The model

Resonant coupling allows one to avoid overcoupling for the triphoton field and implement frequency demultiplexing



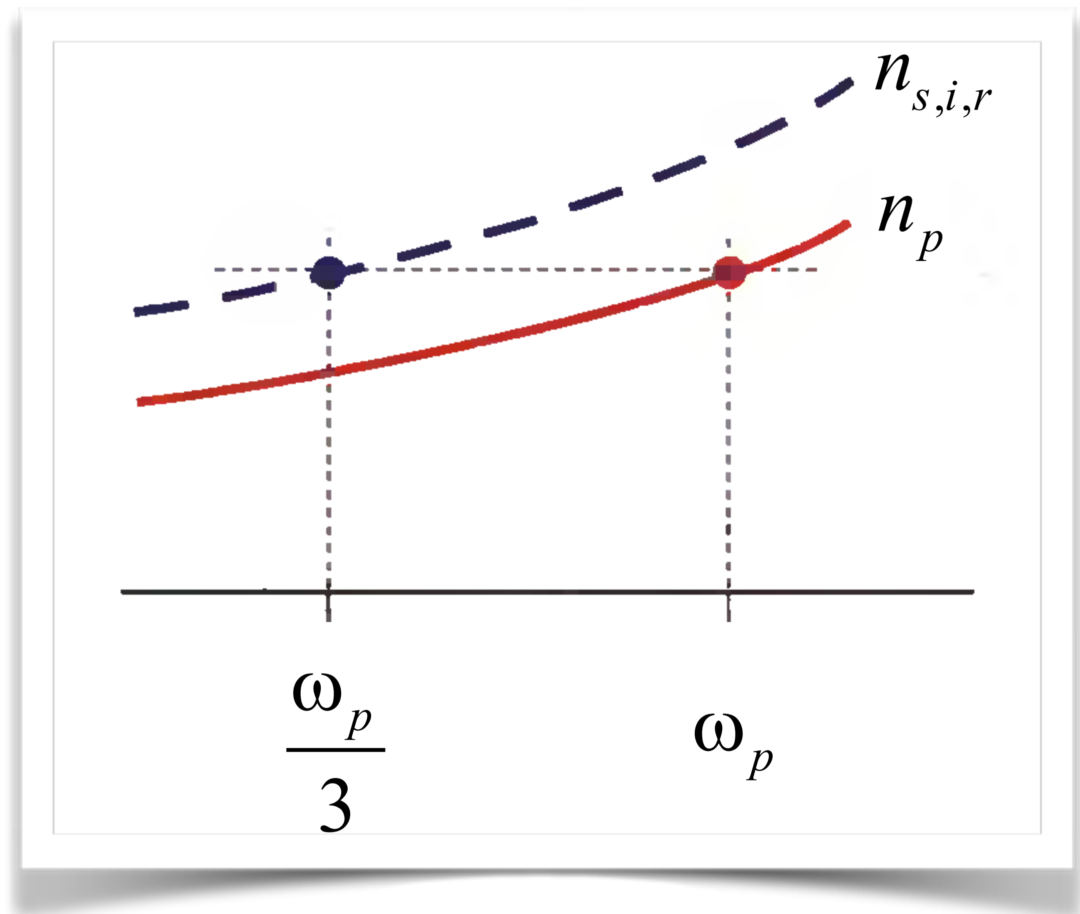
Heralded photon pair source with spatial multiplexing



Dispersion: phase matching conditions

$$\Delta\beta_0 = \beta(\omega_{p0}) - \beta(\omega_{i0}) - \beta(\omega_{s0}) - \beta(\omega_{r0}) = 0$$

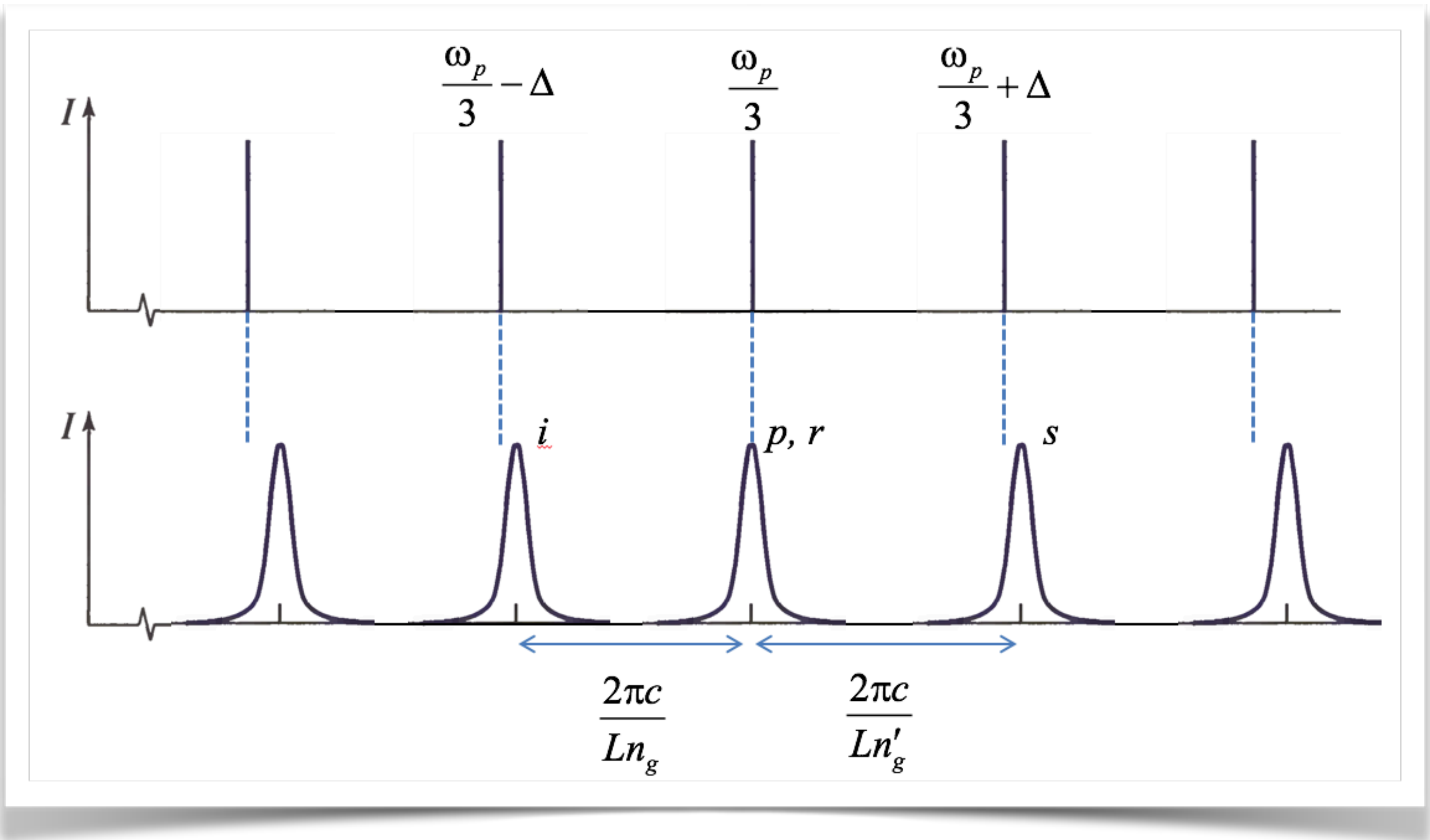
$$\beta_\nu(\omega) = \omega n_\nu(\omega)/c$$



To achieve phase matching with such a large frequency difference, different spatial modes for the pump and triphoton fields can be used

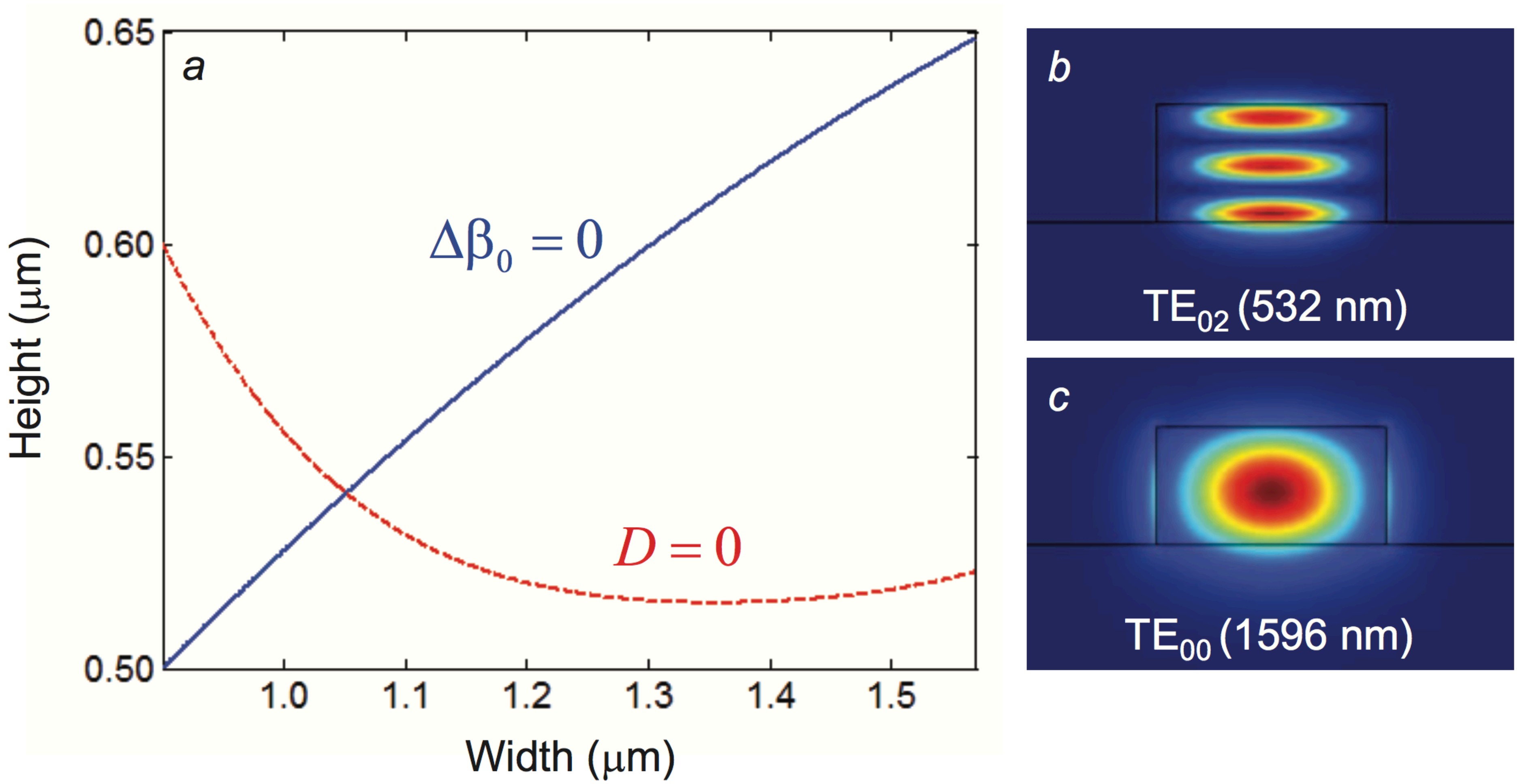
Group velocity dispersion: non-degenerate TOSPDC

$$D \equiv \frac{1}{c}\frac{\partial n_g}{\partial \lambda} = -\frac{\lambda}{c}\frac{\partial^2 n}{\partial \lambda^2} = 0$$



$$D = 0 \rightarrow n_g = n'_g$$

Silicon Nitride (Si_3N_4) ring resonators



$$D \equiv (-\lambda/c) d^2 n/d\lambda^2 \qquad \Delta\beta_0 = \beta(\omega_{p0}) - \beta(\omega_{i0}) - \beta(\omega_{s0}) - \beta(\omega_{r0})$$

$$\gamma = 1.84 \ \mathrm{m}^{-1}\mathrm{W}^{-1}$$

$$F = F_p = 1000$$

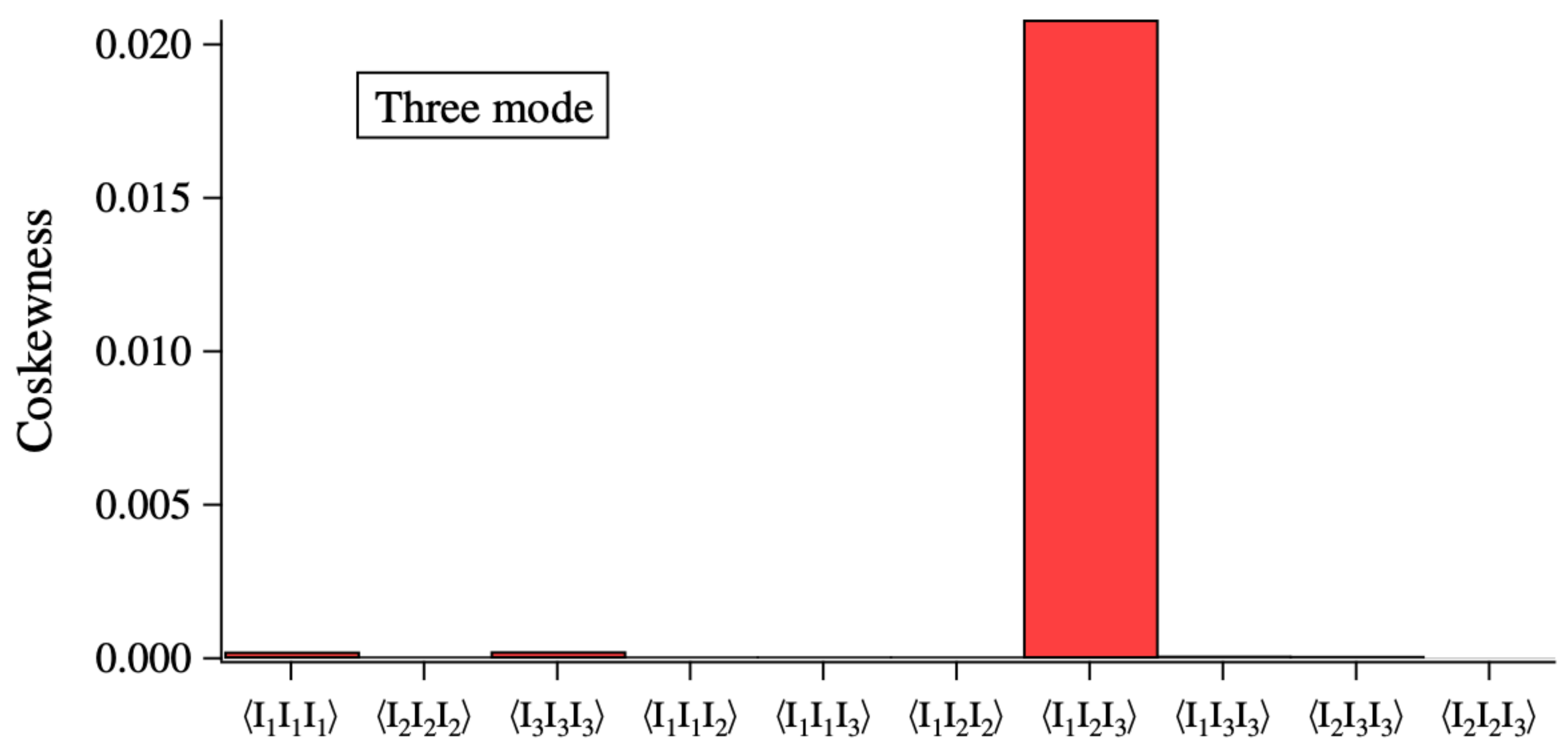
$$P = 100 \ \mathrm{mW}$$

emitted flux ~ $0.84 \ \mathrm{s}^{-1}$

Bandwidth ~ 1 GHz →
Spectral brightness is 10^4 times larger than for MNF case

First experiment in a superconducting parametric cavity

SPDC	Combinations	Frequency [GHz] Pump	Mode 1	Mode 2	Mode 3	Effective Hamiltonians
Single-mode	$f_{p1} = 3 \times f_1$	12.6	4.2	$\cdots$	$\cdots$	$\hat{H}_{1M} = \hbar g(\hat{a}_1^3 + \hat{a}_1^{\dagger 3})$
Two-mode	$f_{p2} = 2 \times f_1 + f_2$	14.5	4.2	6.1	$\cdots$	$\hat{H}_{2M} = \hbar g(\hat{a}_1^2 \hat{a}_2 + \hat{a}_1^{\dagger 2} \hat{a}_2^\dagger)$
Three-mode	$f_{p3} = f_1 + f_2 + f_3$	17.8	4.2	6.1	7.5	$\hat{H}_{3M} = \hbar g(\hat{a}_1 \hat{a}_2 \hat{a}_3 + \hat{a}_1^\dagger \hat{a}_2^\dagger \hat{a}_3^\dagger)$



C.W.S. Chang et al. // Phys. Rev. X, 10, 011011 (2019)

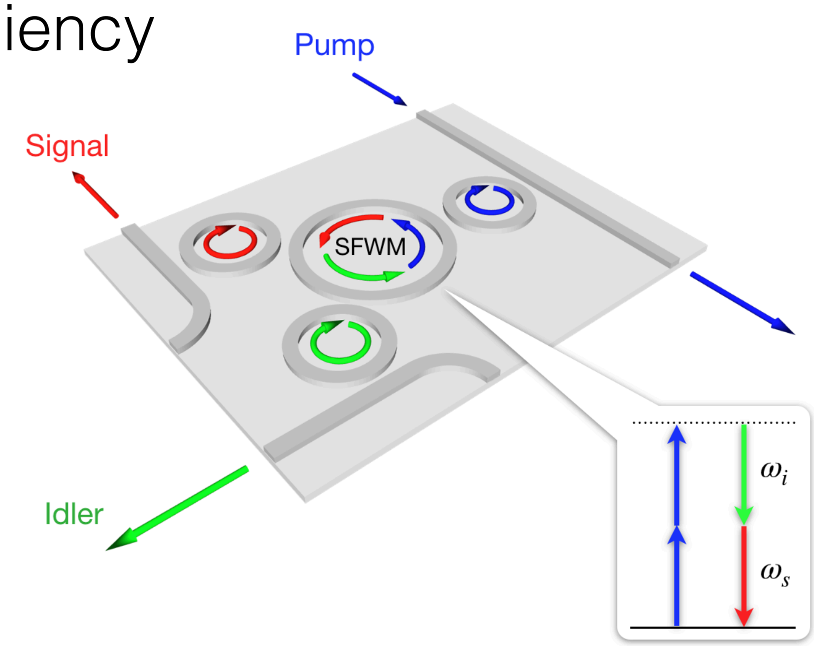
Conclusion

- Photonic molecules provide wide possibilities for engineering the absorption and dispersion properties in the transmission spectrum, which is useful for improving properties of heralded single-photon sources

- It is shown that microring resonators are promising systems for observing third order SPDC and developing heralded sources of photon pairs
  M. Akbari, A.K.. // Laser Phys. Lett. 13, 115204 (2016)

- Optimal coupling parameters that provide maximum purity of the heralded photons for a given pump linewidth are determined.
  I.N. Chuprina, et al. // Laser Phys. Lett. 15, 105104 (2018)

- A scheme of heralded source of frequency-bin qubits is developed such that highest possible heralding efficiency can be achieved and frequency multiplexing is naturally involved
  I.N. Chuprina, A.K. // Phys. Rev. A, 100, 043843 (2019)



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