Inflation and reheating in the early Universe

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Experimenal data in Particle Physics

- We know the initial states of particles before interaction,
  use photons, electrons, positrons, protons, neutrons, ions, neutrinos...
- Then they collide and we measure the particles in the final state
- Thus we learn about interaction
- Each experiment may be repeated:
  – with the same facility
  – building a copy in the same or other place
  – constructing similar devise
  . . .
  And results must be the same
  . . . on average within QM   need many collisions

  theory predicts distributions

Experimenal data in Cosmology and Astrophysics

- Each experiment may be unique (unrepeatable):
  – observe only one Universe
  – (so far) registered only one SN explosion
  – might observe only one magnetic monopole (?)
  – can study only one star
  – (so far) can directly investigate only one planet
  ...
- we register photons, neutrinos, gravitational waves, electrons, positrons, protons, nuclei,
  but only photons, neutrinos and gravitational waves can point at the source
- Can not directly check the model of sources
- Can not directly check the media in between

Outline

“Natural” units in particle physics

$$\hbar = c = k_B = 1$$

measured in GeV: energy E, mass M, temperature T

$m_p = 0.938$ GeV, $\quad 1\ \mathrm{K} = 8.6 \times 10^{-14}$ GeV

measured in GeV^{-1}: time t, length L

$1\ \mathrm{s} = 1.5 \times 10^{24}\ \mathrm{GeV}^{-1}, \quad 1\ \mathrm{cm} = 5.1 \times 10^{13}\ \mathrm{GeV}^{-1}$

Gravity (General Relativity): $V(r) = -G\frac{m_1 m_2}{r}$ $\qquad [G] = M^{-2}$

$$M_{\mathrm{Pl}} = 1.2 \times 10^{19}\ \mathrm{GeV} = 22\ \mu\mathrm{g} \qquad G \equiv \frac{1}{M_{\mathrm{Pl}}^2}$$

"Natural" units in cosmology

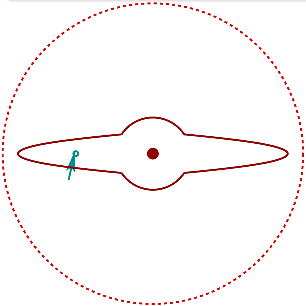
$$1\ \text{Mpc} = 3.1 \times 10^{24}\ \text{cm}$$

$1\ \text{AU} = 1.5 \times 10^{13}\ \text{cm}$ — mean Earth-to-Sun distance

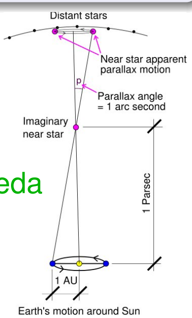
$1\ \text{ly} = 0.95 \times 10^{18}\ \text{cm}$ — distance light travels in one year

$1\ \text{yr} = 3.16 \times 10^{7}\ \text{s}$

$1\ \text{pc} = 3.3\ \text{ly} = 3.1 \times 10^{18}\ \text{cm}$ — distance to object which has a parallax angle of one arcsec

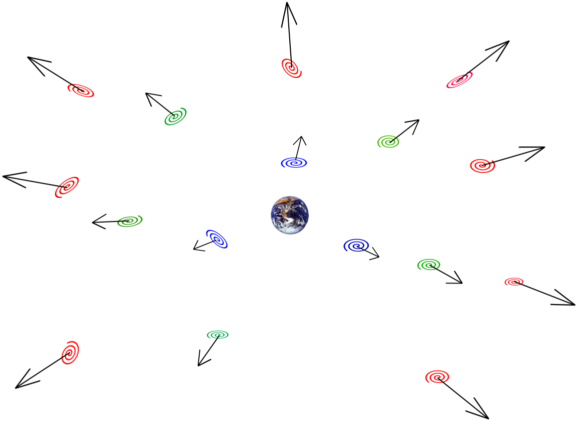


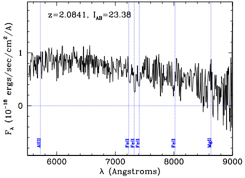
100 AU — Solar system size
1.3 pc — nearest-to-Sun stars
1 kpc — size of dwarf galaxies
50 kpc — distance to dwarves
0.8 Mpc — distance to Andromeda
1-3 Mpc — size of clusters
15 Mpc — distance to Virgo



Universe is expanding $\qquad \lambda_{\text{abs.}}/\lambda_{\text{em.}} \equiv 1 + z$

Doppler redshift Z of light





$L \propto a(t) \quad \longrightarrow \quad n \propto a^{-3}$

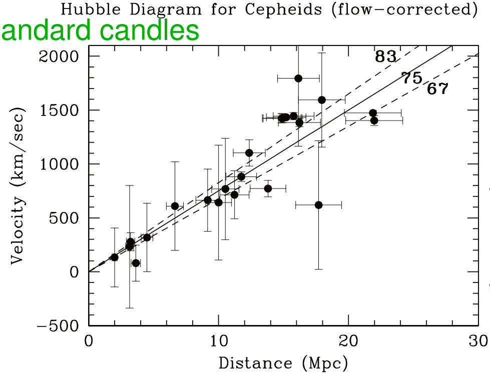
Hubble parameter $H(t) = \frac{\dot{a}(t)}{a(t)}$

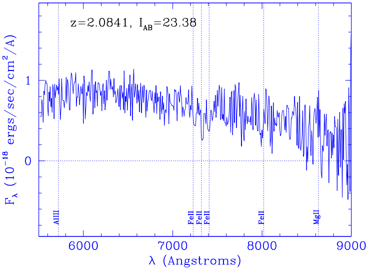
Expansion: redshift z $\qquad \lambda_{\text{abs.}}/\lambda_{\text{em.}} \equiv 1 + z$

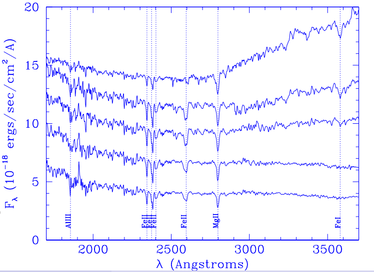
$z \ll 1$ Hubble law : $z = H_0 r$

$$H_0 \simeq 70\ \frac{\text{km}}{\text{s}\cdot\text{Mpc}} \simeq \frac{2\times 10^{-4}}{\text{Mpc}}$$

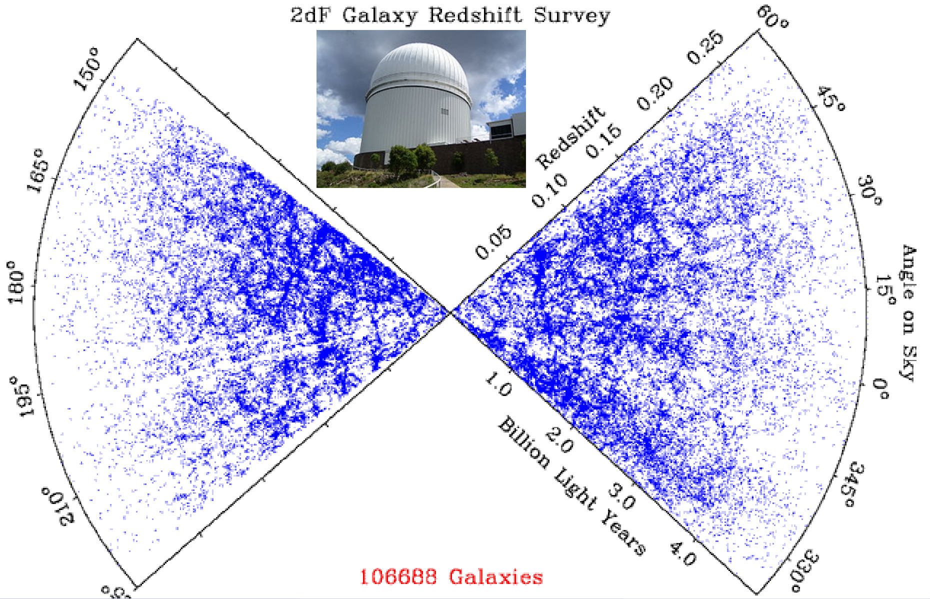
standard candles







Universe is homogeneous and isotropic



The Universe: age & geometry & energy density

$$[H_0] = L^{-1} = t^{-1}$$

time scale: $t_{H_0} = H_0^{-1} \approx 14 \times 10^9$ yr — age of our Universe

spatial scale: $l_{H_0} = H_0^{-1} \approx 4.3 \times 10^3$ Mpc $\approx 10^{28}$ cm — size of the visible Universe

t_{H_0} is in agreement with various observations

homogeneity and isotropy in 3d: — if exact

flat, spherical or hyperbolic — R^3, S^3 or H^3

Observations: "very" flat — $R_{curv} > 30 \times l_{H_0}$

order-of-magnitude estimate: — $1/l_u \sim GM_U/l_U^2 \sim G\rho_0 4\pi/3 l_{H_0}^3/l_{H_0}^2$

flat Universe

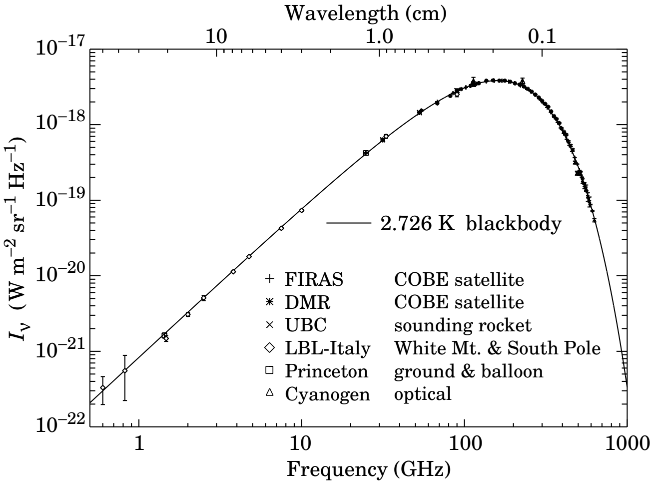
$$\rho_c = \frac{3}{8\pi} H_0^2 M_{\rm Pl}^2 \approx 0.53 \times 10^{-5} \frac{\rm GeV}{\rm cm^3} \longrightarrow$$ 5 protons in each 1 m^3

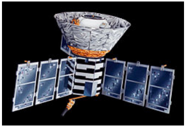
Universe is occupied by “thermal” photons

the spectrum (shape and normalization!) is thermal

$T_0 = 2.726$ K

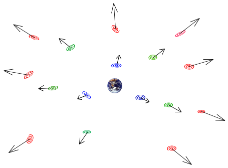
$n_\gamma = 411$ cm^{-3}





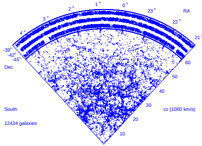
Conclusions from observations

The Universe is
homogeneous, isotropic, hot and expanding...



- interval between events gets modified

$$\Delta s^2 = c^2\,\Delta t^2 - a^2(t)\,\Delta \mathbf{x}^2$$

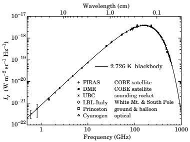


- in GR expansion is described by the Friedmann equation

$$\left(\frac{\dot a}{a}\right)^2 = H^2(t) = \frac{8\pi}{3}\,G\rho_{\text{density}}^{\text{energy}}$$

$$\rho_{\text{density}}^{\text{energy}} = \rho_{\text{matter}} + \rho_{\text{radiation}} + \dots$$

$$\rho_{\text{matter}} \propto 1/a^3(t)\,,\;\; \rho_{\text{radiation}} \propto 1/a^4(t)\,,\;\; \rho_{\text{curvature}} \propto 1/a^2(t)$$



- in the past
  the matter density was higher,
  our Universe was “hotter”,
  and was filled with electromagnetic plasma

$a(t)$ reveals the composition of the present Universe

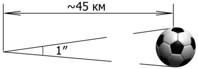
$\Delta s^2 = c^2\,\Delta t^2 - a^2(t)\,\Delta\vec{x}^2 \;\rightarrow\; ds^2 = g_{\mu\nu}dx^\mu dx^\nu$ Light propagation changes...

How do we check it? by measuring distance L to an object!

- Measuring angular size θ of an object of known size d

single-type galaxies

$$\theta = \frac{d}{L}$$



- Measuring angular size $\theta(t)$ corresponding to physical size $d(t)$ with known evolution

– BAO in galaxy distribution
– lensing of CMB anisotropy

$$\theta(t) = \frac{d(t)}{L}$$

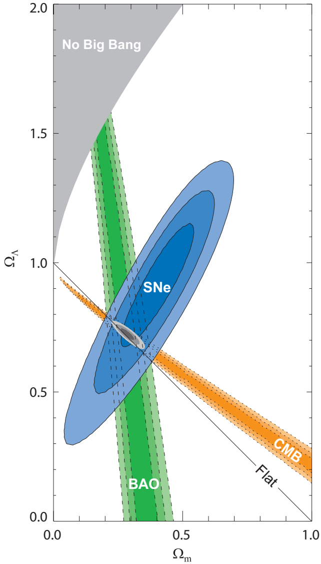
- Measuring brightness J of an object of known luminosity F

"standard candles"

$$J = \frac{F}{4\pi L^2}$$

In the expanding Universe all these laws get modified

Astrophysical and cosmological data are in agreement



$$\left(\frac{\dot a}{a}\right)^2 = H^2(t) = \frac{8\pi}{3} G\rho_{\text{density}}^{\text{energy}}$$

$$\rho_{\text{density}}^{\text{energy}} = \rho_{\text{radiation}} + \rho_{\text{matter}}^{\text{ordinary}} + \rho_{\text{matter}}^{\text{dark}} + \rho_\Lambda$$

$$\rho_{\text{radiation}} \propto 1/a^4(t) \propto T^4(t)\,, \quad \rho_{\text{matter}} \propto 1/a^3(t)$$

$$\rho_\Lambda = \text{const}$$

$$\frac{3H_0^2}{8\pi G} = \rho_{\text{density}}^{\text{energy}}(t_0) \equiv \rho_c \approx 0.53 \times 10^{-5}\,\frac{\text{GeV}}{\text{cm}^3}$$

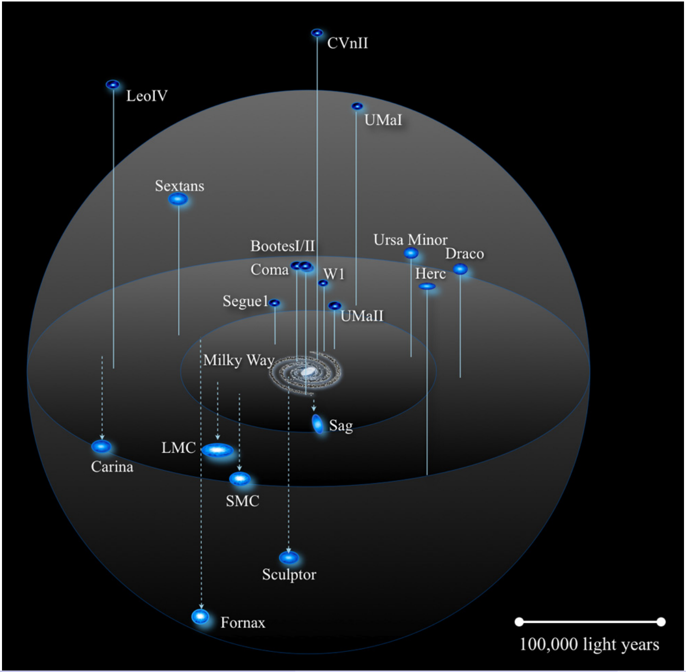
radiation: $\Omega_\gamma \equiv \frac{\rho_\gamma}{\rho_c} = 0.5 \times 10^{-4}$

Baryons (H, He): $\Omega_B \equiv \frac{\rho_B}{\rho_c} = 0.05$

Neutrino: $\Omega_\nu \equiv \frac{\sum \rho_{\nu_i}}{\rho_c} < 0.01$

Dark matter: $\Omega_{DM} \equiv \frac{\rho_{DM}}{\rho_c} = 0.27$

Dark energy: $\Omega_\Lambda \equiv \frac{\rho_\Lambda}{\rho_c} = 0.68$


CVnII
LeoIV
UMaI
Sextans
BootesI/II
Ursa Minor
Draco
Coma
W1
Herc
Segue1
UMaII
Milky Way
Sag
LMC
Carina
SMC
Sculptor
Fornax
100,000 light years

Dark Matter Properties $\quad p = 0$

(If) particles:

1. stable on cosmological time-scale
2. nonrelativistic long before RD/MD-transition

   $(z \simeq 3000,\ T = 0.8\,\text{eV})$
3. (almost) collisionless
4. (almost) electrically neutral

If were in thermal equilibrium: $M_X \gtrsim 1$ keV

If not: for bosons

$\lambda = 2\pi/(M_X v_X)$, in a galaxy $v_X \sim 0.5 \cdot 10^{-3} \longrightarrow M_X \gtrsim 3 \cdot 10^{-22}$ eV

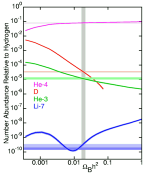
for fermions

Pauli blocking: $M_X \gtrsim 750$ eV

$$f(\mathbf{p},\mathbf{x}) = \frac{\rho_X(\mathbf{x})}{M_X} \cdot \frac{1}{\left(\sqrt{2\pi} M_X v_X\right)^3} \cdot e^{-\frac{\mathbf{p}^2}{2M_X^2 v_X^2}} \Bigg|_{\mathbf{p}=0} \leq \frac{g_X}{(2\pi)^3}$$

Present knowledge about the past: back to 2-3 MeV

past stages		observables
deceleration/acceleration	$\ddot{a} = 0$	SN Ia, CMB, clusters
reionization	$\gamma + H \to p + e$	CMB, quasars, stars
recombination	$p + e \to \gamma + H^*$	CMB, BAO
RD/MD equality	$\rho_{\text{matter}} = \rho_{\text{radiation}}$	CMB, BAO
nucleosynthesis	$p + n \to D + \gamma,\ etc$	cold gas clouds
neutrino decoupling	$\nu_e + n \to p + e$	cold gas clouds



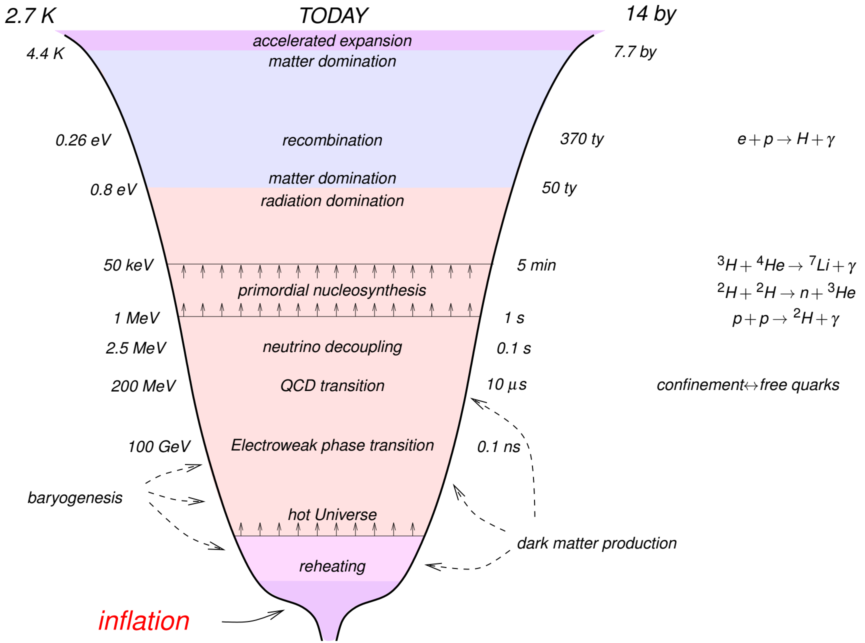
$$H^2 \propto \rho_\gamma + \rho_\nu$$

New Physics in Cosmology: any energy scales. . .

Cosmology constrains the time-scale, rather than energy-scale

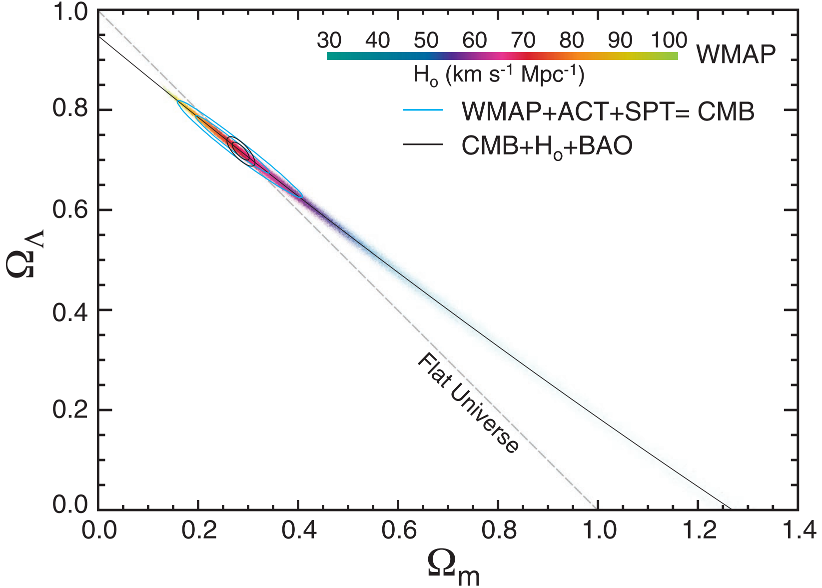
$$\Gamma \sim H \propto T^2/M_{\rm Pl}$$

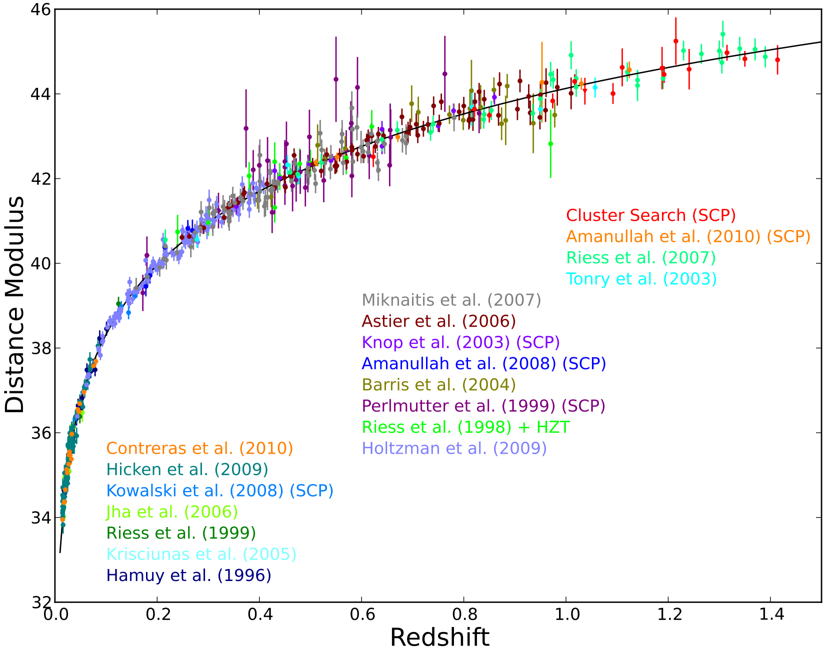
- Dark matter (if particles) — be produced by $T \gg 1\,\mathrm{eV}$
- Dark energy — be present by $T \gg 5\,\mathrm{K}$
- Baryon asymmetry — be generated by $T \gg 1\,\mathrm{MeV}$


2.7 K
TODAY
14 by
accelerated expansion
4.4 K
matter domination
7.7 by
0.26 eV
recombination
370 ty
$e + p \to H + \gamma$
matter domination
0.8 eV
radiation domination
50 ty
50 keV
5 min
$^3H + {}^4He \to {}^7Li + \gamma$
primordial nucleosynthesis
$^2H + {}^2H \to n + {}^3He$
1 MeV
1 s
$p + p \to {}^2H + \gamma$
2.5 MeV
neutrino decoupling
0.1 s
200 MeV
QCD transition
10 μs
confinement↔free quarks
100 GeV
Electroweak phase transition
0.1 ns
baryogenesis
hot Universe
dark matter production
reheating
inflation

Fit to cosmological data. . . 7 years ago

1212.5225




Distance Modulus
Redshift
Cluster Search (SCP)
Amanullah et al. (2010) (SCP)
Riess et al. (2007)
Tonry et al. (2003)
Miknaitis et al. (2007)
Astier et al. (2006)
Knop et al. (2003) (SCP)
Amanullah et al. (2008) (SCP)
Barris et al. (2004)
Perlmutter et al. (1999) (SCP)
Riess et al. (1998) + HZT
Holtzman et al. (2009)
Contreras et al. (2010)
Hicken et al. (2009)
Kowalski et al. (2008) (SCP)
Jha et al. (2006)
Riess et al. (1999)
Krisciunas et al. (2005)
Hamuy et al. (1996)
0.0 0.2 0.4 0.6 0.8 1.0 1.2 1.4
32 34 36 38 40 42 44 46

Inhomogeneities from CMB & LSS: propagation in expanding Universe

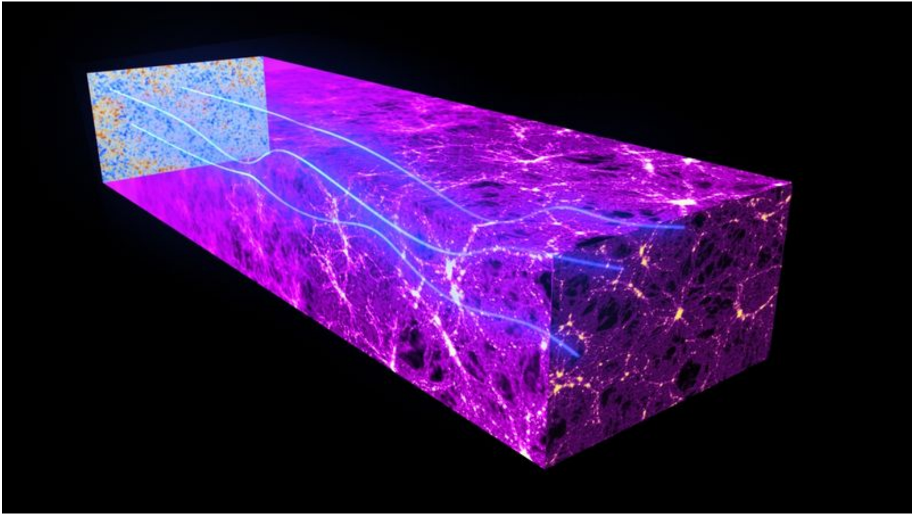


Figure 1: The Current Tension in the Determination of H_o

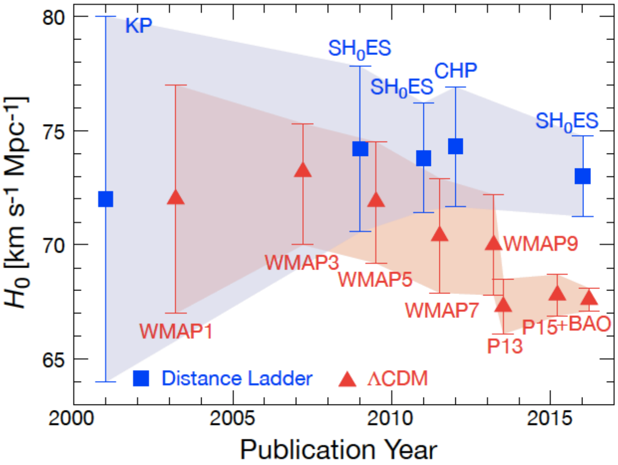
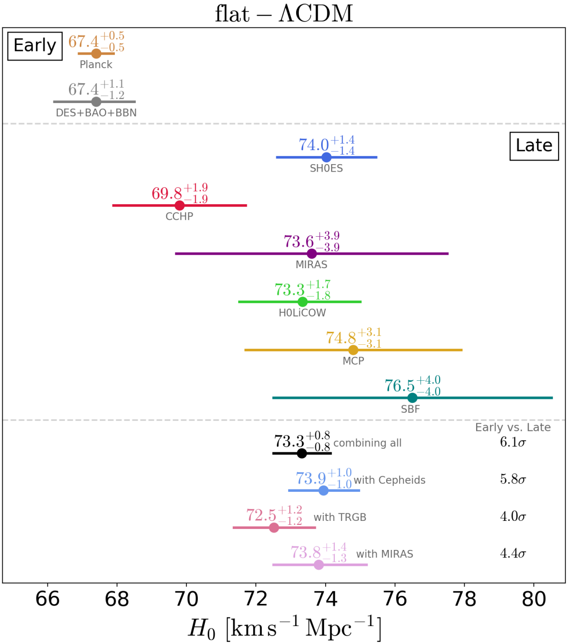


Figure 1: Recent values of H_o as a function of publication date since the Hubble Key Project (adapted from Beaton et al. 2016). Symbols in blue represent values of H_o determined in the nearby universe with a calibration based on the Cepheid distance scale

1706.02739


flat − ΛCDM
Early
67.4 +0.5 −0.5
Planck
67.4 +1.1 −1.2
DES+BAO+BBN
Late
74.0 +1.4 −1.4
SH0ES
69.8 +1.9 −1.9
CCHP
73.6 +3.9 −3.9
MIRAS
73.3 +1.7 −1.8
H0LiCOW
74.8 +3.1 −3.1
MCP
76.5 +4.0 −4.0
SBF
Early vs. Late
73.3 +0.8 −0.8 combining all
6.1σ
73.9 +1.0 −1.0 with Cepheids
5.8σ
72.5 +1.2 −1.2 with TRGB
4.0σ
73.8 +1.4 −1.3 with MIRAS
4.4σ
66 68 70 72 74 76 78 80
H0 [km s−1 Mpc−1]

1907.10625

FLRW metric for flat space $g_{\mu\nu}$

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = dt^2 - a^2(t) dl^2$$

$$H(t) = \frac{\dot{a}(t)}{a(t)}$$

coordnate distance: $l = \int dl$

physical distance at the moment t: $L(t) = a(t) \times l = a(t) \times \int dl$

Photons in the expanding Universe

$$S = -\frac{1}{4}\int d^4x\sqrt{-g}g^{\mu\nu}g^{\lambda\rho}F_{\mu\lambda}F_{\nu\rho}$$

$dt = ad\eta$ conformally flat metric

$$ds^2 = dt^2 - a^2(t)\delta_{ij}dx^i dx^j \longrightarrow ds^2 = a^2(\eta)[d\eta^2 - \delta_{ij}dx^i dx^j]$$

$$S = -\frac{1}{4}\int d^4x\,\eta^{\mu\nu}\eta^{\lambda\rho}F_{\mu\lambda}F_{\nu\rho}\,, \qquad A_\mu^{(\alpha)} = e_\mu^{(\alpha)}\mathrm{e}^{ik\eta - i\mathbf{kx}}\,, \quad k = |\mathbf{k}|$$

$$\Delta x = 2\pi/k\,, \quad \Delta\eta = 2\pi/k$$

$$\lambda(t) = a(t)\Delta x = 2\pi\frac{a(t)}{k}\,, \quad T = a(t)\Delta\eta = 2\pi\frac{a(t)}{k}$$

Redshift and the Hubble law $\lambda_0 = \lambda_i \frac{a_0}{a(t_i)} \equiv \lambda_i (1 + z(t_i))$

$$\mathbf{p}(t) = \frac{\mathbf{k}}{a(t)}\,, \quad \omega(t) = \frac{k}{a(t)}$$

for not very distant objects $\quad H_0 \equiv \dot{a}_0/a_0$

$$a(t_i) = a_0 - \dot{a}(t_0)(t_0 - t_i) = a_0[1 - H_0(t_0 - t_i)]$$

$$z(t_i) = H_0(t_0 - t_i) = H_0 r\,, \qquad z \ll 1$$

similar reddening for other relativistic particles

$\mathbf{p} = \frac{\mathbf{k}}{a(t)}$ is true for massive particles as well

Friedmann equation for the present Universe

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} G(\rho_{\mathrm{M}} + \rho_{rad} + \rho_{\Lambda})$$

$$\rho_c \equiv \frac{3}{8\pi G} H_0^2$$

$$\rho_c = \rho_{\mathrm{M},0} + \rho_{rad,0} + \rho_{\Lambda,0} = \rho_c = 0.5 \cdot 10^{-5} \frac{\mathrm{GeV}}{\mathrm{cm}^3} ,$$

$$\Omega_X \equiv \frac{\rho_{X,0}}{\rho_c}$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} G\rho_c \left[\Omega_{\mathrm{M}} \left(\frac{a_0}{a}\right)^3 + \Omega_{rad} \left(\frac{a_0}{a}\right)^4 + \Omega_{\Lambda}\right]$$

Examples of realistic cosmological solutions

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}G\rho$$

dust: $p = 0$ singular at $t = t_s$

$$\rho = \frac{\text{const}}{a^3}\,, \quad a(t) = const \cdot (t - t_s)^{2/3}\,, \quad \rho(t) = \frac{\text{const}}{(t - t_s)^2}$$

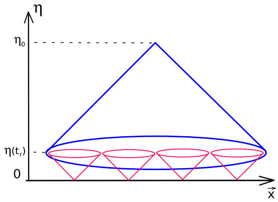
$$t_s = 0\,, \quad H(t) = \frac{\dot{a}}{a}(t) = \frac{2}{3t}\,, \quad \rho = \frac{3}{8\pi G}H^2 = \frac{1}{6\pi G}\frac{1}{t^2}$$

Cosmological (particle) horizon $l_H(t)$

distance covered by photons emitted at $t = 0$

the size of causally-connected region —
the size of the visible part of the Universe

in conformal coordinates:
$ds^2 = 0 \longrightarrow |d\mathbf{x}| = d\eta$
coordinate size of the horizon equals
$\eta(t) = \int d\eta$



$$l_H(t) = a(t)\eta(t) = a(t)\int_0^t \frac{dt'}{a(t')}$$

dust — **horizon problem**

$$l_H(t) = 3t = \frac{2}{H(t)}\,, \qquad \text{while} \qquad L_{phys} \propto a(t) \propto t^{2/3}$$

Present size of the recombination horizon

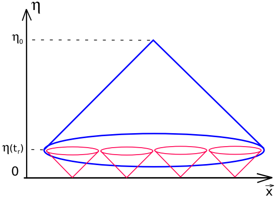
matter domination: $l_{\mathrm{H},r} = 2H_r^{-1}$

$$H_r^2 = \frac{8\pi}{3} G\rho_{\mathrm{M}}(t_r) = \frac{8\pi}{3} G\rho_{\mathrm{M},0}\left(\frac{a_0}{a_r}\right)^3 = \frac{8\pi}{3} G\rho_c \Omega_{\mathrm{M},0}(1+z_r)^3 \,.$$

at recombination: $l_{\mathrm{H},r} = \frac{2}{H_0\sqrt{\Omega_{\mathrm{M}}}} \frac{1}{(1+z_r)^{3/2}}$

today: $l_{\mathrm{H},r}(t_0) = l_{\mathrm{H},r} \times \frac{a_0}{a_r} = \frac{2}{H_0\sqrt{\Omega_{\mathrm{M}}}} \frac{1}{\sqrt{1+z_r}}$

$$\frac{l_{H_0}}{l_{\mathrm{H},r}(t_0)} \sim \sqrt{1+z_r} \simeq 30$$



Examples of realistic cosmological solutions

radiation: $p = \frac{1}{3}\rho$ singular at $t = t_s$

$$\rho = \frac{\text{const}}{a^4}\,, \quad a(t) = const \cdot (t - t_s)^{1/2}\,, \quad \rho(t) = \frac{\text{const}}{(t - t_s)^2}$$

$$t_s = 0\,, \quad H(t) = \frac{\dot{a}}{a}(t) = \frac{1}{2t}\,, \quad \rho = \frac{3}{8\pi G}H^2 = \frac{3}{32\pi G}\frac{1}{t^2}$$

$$l_H(t) = a(t)\int_0^t \frac{dt'}{a(t')} = 2t = \frac{1}{H(t)}\,.$$

If thermal equilibrium $T = \text{const}/a$

$$\rho = \frac{\pi^2}{30} g_* T^4$$

Entropy conservation: adiabatic expansion

$$\nabla_\mu T^{\mu 0} = 0 \quad \longrightarrow \quad \dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0$$

the equation of state

$$p = p(\rho)$$

many-component fluid, in case of thermal equilibrium

other equations

$$-3d(\ln a) = \frac{d\rho}{p+\rho} = d(\ln s)$$

entropy of cosmic primordial plasma is conserved in a comoving frame

$sa^3 = \text{const}$ entropy problem

Flatness problem

- Take non-flat 3-dim manifold (general case)
- Curvature contribution to the total energy density behaves as $\rho_{curv}(t) \propto 1/a^2(t)$
- Then at present:

$$0.01 > \Omega_{curv} = \frac{\rho_{curv}(t_0)}{\rho_c} \sim 10^{-4} \times \frac{\rho_{curv}(t_0)}{\rho_{rad}(t_0)} = 10^{-4} \times \frac{a^2(t_0)}{a^2(t_*)} \frac{\rho_{curv}(t_*)}{\rho_{rad}(t_*)}$$

$$\sim 10^{-4} \times \frac{T_*^2}{T_0^2} \frac{\rho_{curv}(T_*)}{\rho_{tot}(T_*)}$$

- For hypothetical Planck epoch $T_* \sim M_{Pl} \sim 10^{19}$ GeV one gets

$$0.01 > \Omega_{curv} \sim 10^{60} \times \frac{\rho_{curv}(M_{Pl})}{\rho_{tot}(M_{Pl})}$$

$R_{curv}/R_{Plankc} > 10^{31}$ enormously huge original manifold !!

Initial condition problems:

- horizon
- entropy
- curvature
- singularity...
- heavy relics...
- ...

Examples of realistic cosmological solutions

vacuum: $T_{\mu\nu} = \rho_{vac}\eta_{\mu\nu}$ $\quad$ $p = -\rho$

$$S_\Lambda = -\Lambda \int \sqrt{-g} d^4x \, .$$

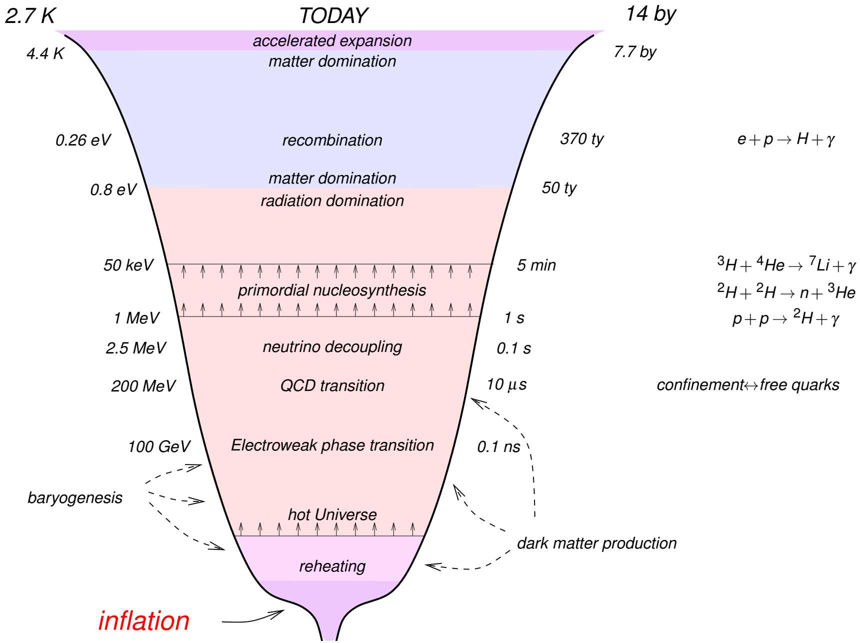
$$a = \mathrm{const} \cdot \mathrm{e}^{H_{\mathrm{dS}} t} \, , \quad H_{\mathrm{dS}} = \sqrt{\frac{8\pi}{3} G\rho_{vac}}$$

de Sitter space: space-time of constant curvature

$$ds^2 = dt^2 - \mathrm{e}^{2H_{dS}t} d\mathbf{x}^2$$

$\ddot{a} > 0$, no initial singularity

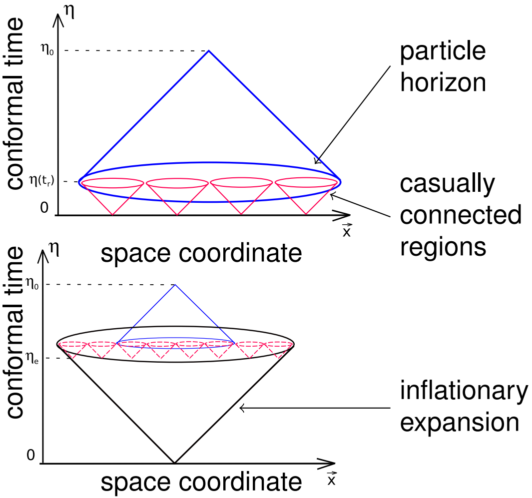
no cosmological horizon: $l_{\mathrm{H}}(t) = \mathrm{e}^{H_{dS}t} \int_{-\infty}^{t} dt' \mathrm{e}^{-H_{dS}t'} = \infty$



2.7 K
TODAY
14 by
4.4 K
accelerated expansion
matter domination
7.7 by
0.26 eV
recombination
370 ty
$e + p \to H + \gamma$
matter domination
0.8 eV
radiation domination
50 ty
50 keV
5 min
$^3H + {}^4He \to {}^7Li + \gamma$
primordial nucleosynthesis
$^2H + {}^2H \to n + {}^3He$
1 MeV
1 s
$p + p \to {}^2H + \gamma$
2.5 MeV
neutrino decoupling
0.1 s
200 MeV
QCD transition
$10\ \mu s$
confinement↔free quarks
100 GeV
Electroweak phase transition
0.1 ns
baryogenesis
hot Universe
dark matter production
reheating
inflation

Inflationary solution of Hot Big Bang problems

- no initial singularity in dS space
- all scales grow exponentially, including the radius of the 3-sphere
  the Universe becomes exponentially flat
- any two particles are at exponentially large distances
  no heavy relics
  no traces of previous epochs!
- no particles in post-inflationary Universe
  to solve entropy problem we need post-inflationary reheating



Inflation: general remarks

- Simplest variant

$$H^2 = \frac{8\pi}{3M_{Pl}^2}\rho_\Lambda = \text{const}\,,\ \Rightarrow\ a(t) \propto e^{Ht}$$

is not suitable: inflation must not last for ever!

- Universe has to reheat after! T_{reh}

$$\rho_e \gtrsim (3\,\text{MeV})^4\,,\ \text{and better}\ \rho_e \gtrsim (100\,\text{GeV})^4\,,$$

- How long? Horizon problem:
present size of the horizon at the end of inflation

$$l_{H,e}(t_0) = a_0 \int_{t_{Pl}}^{t_e} \frac{dt}{a(t)} = a_0 \int_{t_{Pl}}^{t_e} \frac{da}{a^2}\frac{1}{H} \sim \frac{a_0}{a(t_{Pl})}\cdot\frac{1}{H(t_{Pl})}$$

Solution to the horizon problem:

$$1 \lesssim \frac{l_{H,e}(t_0)}{l_{H,0}} \sim \frac{a_0}{a(t_{Pl})}\frac{H_0}{H(t_{Pl})} = \frac{a_0}{a(t_{reh})}\frac{a_{reh}}{a(t_e)}\frac{a(t_e)}{a(t_{Pl})}\cdot\frac{H_0}{H(t_{Pl})}$$

Introducing the number of e-foldings

$$N_e^{tot} = \ln\frac{a(t_e)}{a(t_{Pl})}\,,\quad N_e^{tot} = \int_{t_{Pl}}^{t_e} dt\, H(t) \sim H_e\cdot\Delta t_{infl}$$

For relativistic particles $\rho \propto T^4 \propto 1/a^4 \ \Rightarrow\ a_0/a(t_{reh}) \sim T_{reh}/T_0$

Inflation: general remarks

- How long? Solution to the horizon problem:

$$1 \lesssim \frac{l_{H,e}(t_0)}{l_{H,0}} \Rightarrow N_e^{tot} \gtrsim \log \frac{T_0}{H_0} + \ln \frac{a(t_e)}{a_{reh}} + \ln \frac{H(t_{Pl})}{T_{reh}} \simeq 50 - 60$$

  Inflation lasts not less than (accepting $H^2 \sim \rho / M_{Pl}^2$)

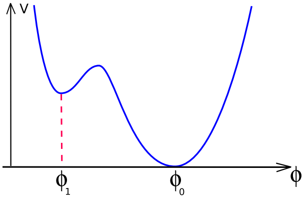
$$\Delta t_{infl} \sim N_e^{tot} / H_e \sim 10^{-11}\,\mathrm{c} \cdot \left(\frac{1\ \mathrm{TeV}}{T_{reh}} \right)^2$$

  we must reheat the Universe then!

- In realistic models $N_e^{tot} \ggg 100$!!!
  Inflatinary stage may be short, but expansion is enormous!

Inflatinary stage: simplest models

“Old inflation” by Guth



does not work in fact!

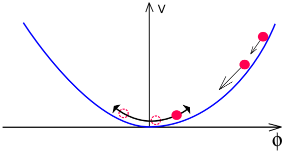
starts from a hot stage
and ends up in a false vacuum

reheating due to percollations

However: for sufficiently long inflationary stage requires
$\Gamma < H^4_{infl}$

hence the bubbles never collide!

“Chaotic inflation”



needs superplanckian field values!

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

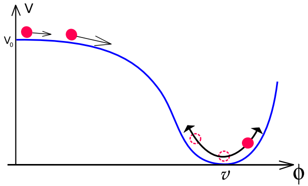
$$p = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

$$\varepsilon = \frac{M_{Pl}^2}{16\pi}\left(\frac{V'}{V}\right)^2 , \; \eta = \frac{M_{Pl}^2}{8\pi}\frac{V''}{V} ,$$

$V(\phi) \propto \phi^n \Rightarrow \varepsilon, \eta \sim M_{Pl}^2/\phi^2 \ll 1$ $\leftarrow$ slow-roll conditions

“New inflation”



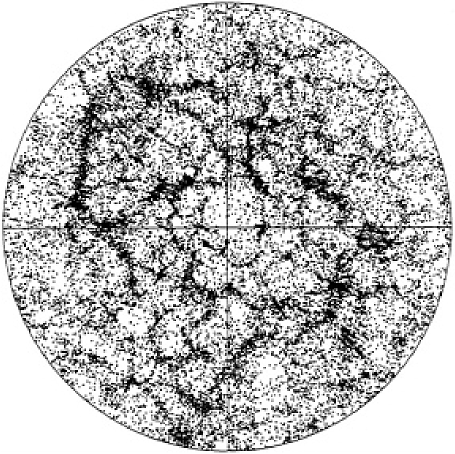
Initial condition is very specific!

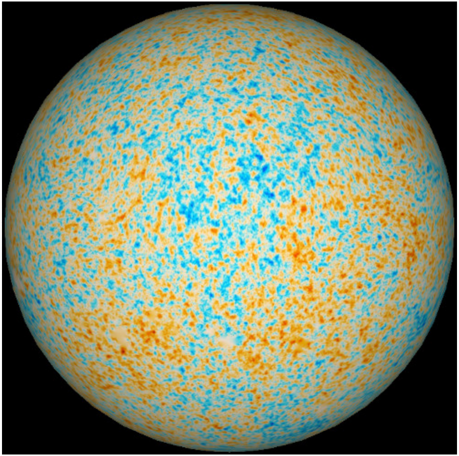
$$H^2 = \frac{8\pi}{3M_P^2} V(\phi) , \quad a(t) \propto e^{Ht}$$

and we require

$$V(\phi) < M_{Pl}^4$$

Inhomogeneous Universe





Large Scale Structure

CMB anisotropy

Small inhomogeneities in the expanding Universe

matter perturbations (perfect fluid approximation)

$$T_0^0 \to \rho(t) + \delta\rho(\eta,\mathbf{x}), \quad T_i^0 \to \partial_i v(\eta,\mathbf{x}), \quad T_j^i \to \delta p(\eta,\mathbf{x})$$

gravitational perturbations (scalar and tensor modes)

$$ds^2 = a^2(\eta)\left[(1+2\Phi(\eta,\mathbf{x}))\,d\eta^2 - (1+2\Psi(\eta,\mathbf{x}))\,\mathbf{dx}^2 - h_{ij}^{TT}(\eta,\mathbf{x})dx^i dx^j\right]$$

Equations for linear perturbations, $\delta\rho/\rho \equiv \delta \ll 1$, $\Phi \ll 1$, etc

$$R_{\mu\nu} + \frac{1}{2}Rg_{\mu\nu} = 8\pi G T_{\mu\nu} \;\; \to \;\; \ldots$$

$$\nabla_\mu T^{\mu\nu} = 0 \;\; \to \;\; \ldots$$

These inhomogeneities (matter perturbations)

originate from the initial matter density (scalar) perturbations

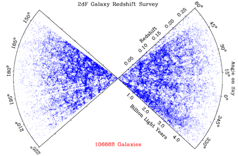
$\delta\rho/\rho \sim \delta T/T \sim 10^{-4}$, which are

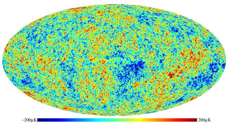
adiabatic $\qquad \delta\left(\frac{n_B}{s}\right) = \delta\left(\frac{n_{DM}}{s}\right) = \delta\left(\frac{n_L}{s}\right)$

Gaussian $\qquad \langle \frac{\delta\rho}{\rho}(\mathbf{k})\frac{\delta\rho}{\rho}(\mathbf{k}')\rangle \propto \left(\frac{\delta\rho}{\rho}(\mathbf{k})\right)^2 \times \delta(\mathbf{k}+\mathbf{k}')$

flat spectrum $\qquad \langle\left(\frac{\delta\rho}{\rho}(\mathbf{x})\right)^2\rangle = \int_0^\infty \frac{\mathbf{dk}}{\mathbf{k}}\mathscr{P}_S(\mathbf{k}) \qquad \mathscr{P}_S(\mathbf{k}) \approx \text{const}$

LSS and CMB $\qquad \mathscr{P}_S \equiv A_S \times \left(\frac{k}{k_*}\right)^{n_s-1} \qquad A_S \approx 2.5\times 10^{-9}\,, \quad n_S \approx 0.97$





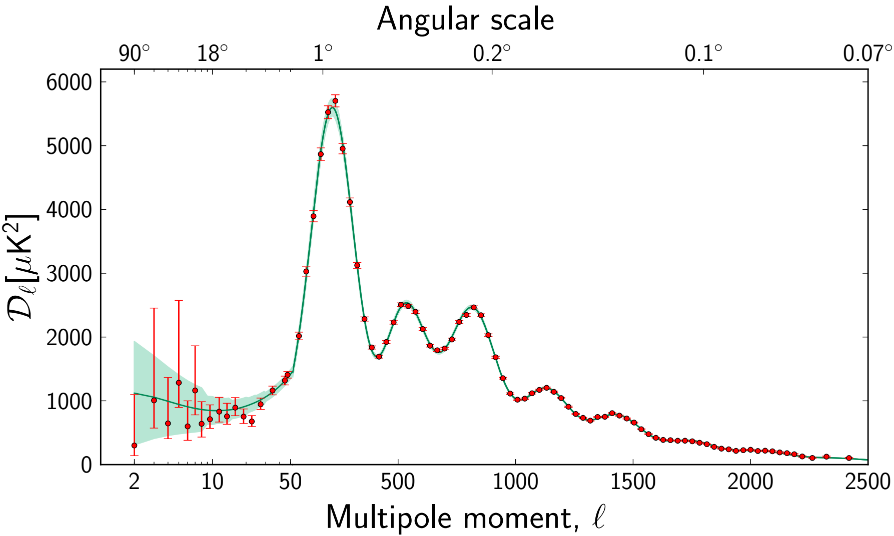
Mode evolution

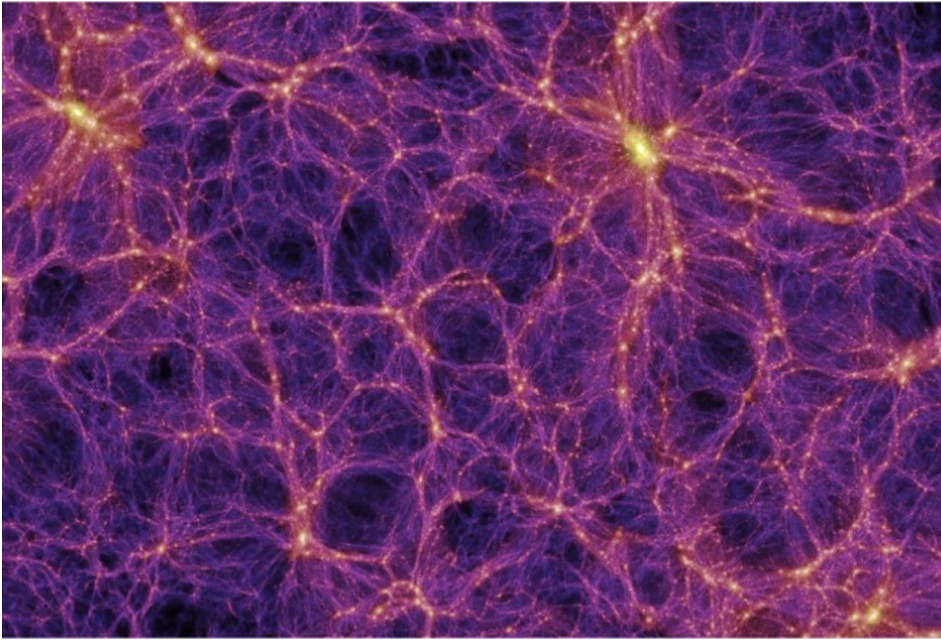
- Amplitude remains constant, while superhorizon, e.g. $k/a < H$
- Subhorizon Inhomogeneities of DM start to grow at MD-stage, $\delta\rho_{CDM}/\rho_{CDM} \propto a$ from $T \approx 0.8\,\text{eV}$
  Smaller objects (first stars, dwarf galaxies) are first to form
- Subhorizon Inhomogeneities of baryons join those of DM only after recombination, $\delta\rho_{CDM}/\rho_{CDM} \propto a$ from $T_{rec} \approx 0.25\,\text{eV}$
- at recombination $\delta\rho_B/\rho_B \sim \delta T/T \sim 10^{-4}$ and would grow only by a factor $T_{rec}/T_0 \sim 10^3$ without DM
- Subhorizon Inhomogeneities of photons $\delta\rho_\gamma/\rho_\gamma$ oscillate with constant amplitude at RD and with decreasing amplitude at MD, thus we can measure $T_{RD/MD}/T_{rec}$
- Phase of oscillations decoupled after recombination depends on the wave-length, recombination time and sound speed

$$\delta\rho_\gamma/\rho_\gamma \propto \cos\left(k\int_0^{t_r}\frac{v_s\,dt}{a(t)}\right) = cos(kl_{sound})$$

- $$\delta T(\theta,\varphi) = \sum a_{lm} Y_{lm}(\theta,\varphi)\,, \qquad \langle a^*_{lm} a_{lm}\rangle = C_l \equiv 2\pi\mathscr{D}_l/(l(l+1))$$

CMB measurements (Planck) $H_0, \Omega_{DM}, \Omega_B, \Omega_\Lambda, \Delta_{\mathscr{R}}, n_s$





Unexpected bonus: generation of perturbations

- Quantum fluctuations of wavelength λ of a free massless field φ have an amplitude of $\delta\varphi_\lambda \simeq 1/\lambda$
- In the expanding Universe: $\lambda \propto a$

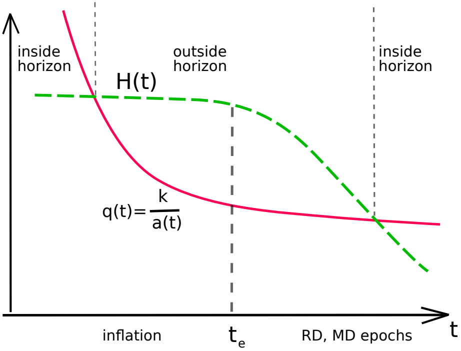
  inflation: $l_H \sim 1/H = \text{const}$, so modes “exit horizon”
  Ordinary stage: $l_H \sim 1/H \propto t$, $l_H/\lambda \nearrow$, modes “enter horizon”

Evolution at inflation

- inside horizon: $\lambda < l_H$
  $\lambda \propto a \Rightarrow$ $\delta\varphi_\lambda \propto 1/\lambda \propto 1/a$

- outside horison: $\lambda > l_H$
  $\lambda \propto a \Rightarrow$ $\delta\varphi_\lambda = \text{const} = H_{infl}$!!!

- got “classical” fluctuations:
  $\delta\varphi_\lambda = \delta\varphi_\lambda^{quantum} \times e^{N_e}$



Power spectrum of perturbations

In the Minkowski space-time:

- fluctuations of a free quantum field φ are gaussian its power spectrum is defined as

$$\int_0^\infty \frac{dq}{q}\,\mathscr{P}_\varphi(q) \equiv \langle \varphi^2(x) \rangle = \int_0^\infty \frac{dq}{q}\,\frac{q^2}{(2\pi)^2}$$

  We define amplitude as $\delta\varphi(q) \equiv \sqrt{\mathscr{P}_\varphi} = q/(2\pi)$

- In the expanding Universe momenta $q = k/a$ gets redshifted
- Cast the solution in terms $\phi(\mathbf{x},t) = \phi_c(t) + \varphi(\mathbf{x},t)$, $\varphi(\mathbf{x},t) \propto \mathrm{e}^{\pm i\mathbf{kx}}\,\varphi(\mathbf{k},t)$
  φ solves the equation

$$\ddot{\varphi} + 3H\dot{\varphi} + \frac{k^2}{a^2}\,\varphi + V''(\phi_c)\varphi = 0$$

- $q = k/a \gg H$ $\Rightarrow$ as in Minkowski space-time
- $q = k/a \ll H$ $\Rightarrow$ for inflaton $\varphi = \mathrm{const}$
- Matching at t_k: $q(t_k) = k/a(t_k) = H(t_k) \equiv H_k$ gives

$$\delta\varphi(q) = \frac{H_k}{2\pi} \;\Rightarrow\; \mathscr{P}_\varphi(q) = \frac{H_k^2}{(2\pi)^2}$$

amplification $H_k/q = \mathrm{e}^{N_e(k)}$!!!

$H_k \approx \mathrm{const} = H_{infl}$ hence (almost) flat spectrum

Transfer to matter perturbations: simple models

Illustration: Local delay(advance)δt in evolution due to impact of $\delta\phi$ of all modes with $\lambda > H$:

$$\delta\phi = \dot{\phi}_c\,\delta t\,, \quad \delta\rho \sim \dot{\rho}\,\delta t$$

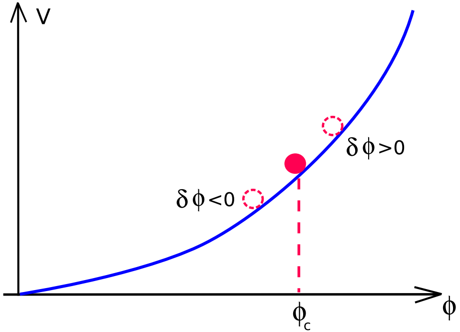
at the end of inflation $\dot{\rho} \sim -H\rho$, then

$$\frac{\delta\rho}{\rho} \sim \frac{H}{\dot{\phi}_c}\,\delta\phi$$

Hence, $\delta\rho/\rho$ is also gaussian.
Power spectrum of scalar perturbations

$$\mathscr{P}_{\mathscr{R}}(k) = \left(\frac{H^2}{2\pi\,\dot{\phi}_c}\right)^2\,,$$

everything is calculated at $t = t_k$: $H = k/a$



Analogously for the tensor perturbations: each of the two polarizations of the gravity waves solves the free scalar field equation!

$$\mathscr{P}_T(k) = \frac{16}{\pi}\,\frac{H_k^2}{M_{Pl}^2}$$

To the leading order no k-dependence: both spectra are "flat" (scale-invariant)!

Inflaton parameters and spectral parameters

- Observation of CMB anisotropy gives $\delta T/T$

$$\frac{\delta T}{T} \sim \frac{\delta\rho}{\rho} \;\Rightarrow \Delta_{\mathscr{R}} \equiv \sqrt{\mathscr{P}_{\mathscr{R}}} = 5\times 10^{-5}$$

- These are so-called adiabatic perturbations!
  Other possibles (isocurvature) modes (e.g. $\delta T = 0$, but $\delta n_B/n_B \neq 0$) are not found.
- $\Delta_{\mathscr{R}} = 5\times 10^{-5} \Rightarrow$ fixes model paramaters, e.g.:

$$V(\phi) = \frac{\beta}{4}\phi^4 \to \lambda \sim 10^{-13}$$

With such a tiny coupling perturbations are obviously gaussian
So far confirmed by observations

Inflation & Reheating: simple realization with Higgs

$$\ddot{X}+3H\dot{X}+V'(X)=0$$

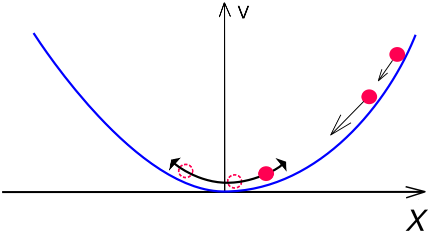
$X_e > M_{Pl}$

generation of scale-invariant scalar (and tensor) perturbations from exponentially stretched quantum fluctuations of X

$\delta\rho/\rho \sim 10^{-5}$ requires
$V=\beta X^4 : \beta \sim 10^{-13}$

reheating ? renormalizable?

the only choice: $\alpha H^\dagger H X^2$
"Higgs portal"



Chaotic inflation, A.Linde (1983)

larger α larger T_{reh}

quantum corrections $\propto \alpha^2 \lesssim \beta$

Inflaton parameters and spectral parameters

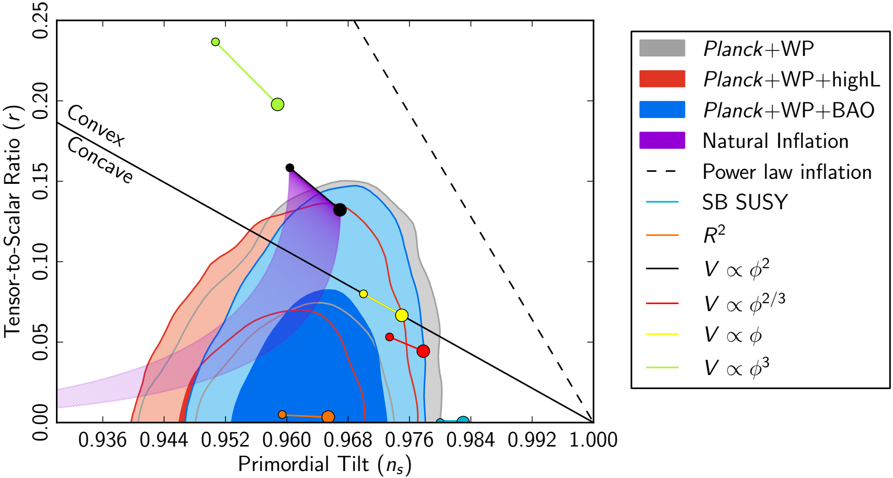
- In fact, spectra are a bit tilted, as H_{infl} slightly evolves

$$\mathscr{P}_{\mathscr{R}}(k) = A_{\mathscr{R}} \left(\frac{k}{k_*}\right)^{n_s - 1} , \qquad \mathscr{P}_T(k) = A_T \left(\frac{k}{k_*}\right)^{n_T} .$$

- Measure $\Delta_{\mathscr{R}}$ at present scales $q \simeq 0.002/\mathrm{Mpc}$, it fixes the number of e-foldings left N_e
- For tensor perturbations one introduces:

$$r \equiv \frac{\mathscr{P}_T}{\mathscr{P}_{\mathscr{R}}} = \frac{1}{\pi} \frac{M_{Pl}^2 V'^2}{V} = 16\,\varepsilon \;\; \to \;\; \frac{16}{N_e} \;\text{ for }\; \beta\phi^4$$

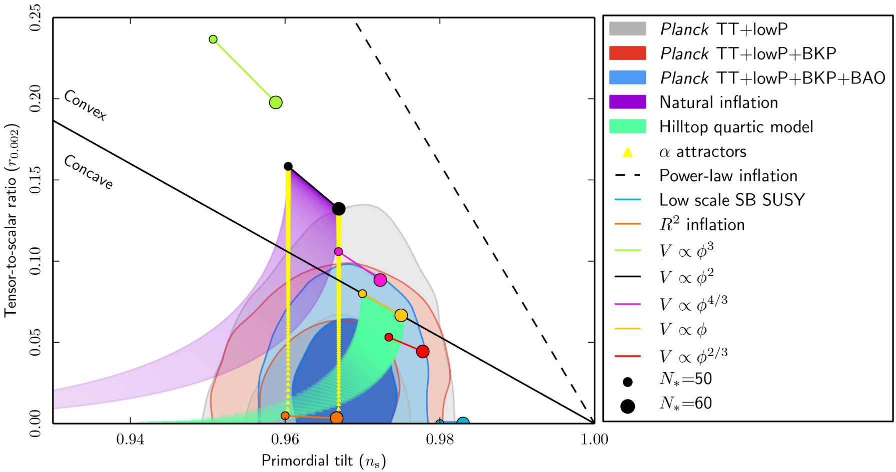
Planck analysis of cosmlogical data (2013)



1303.5062

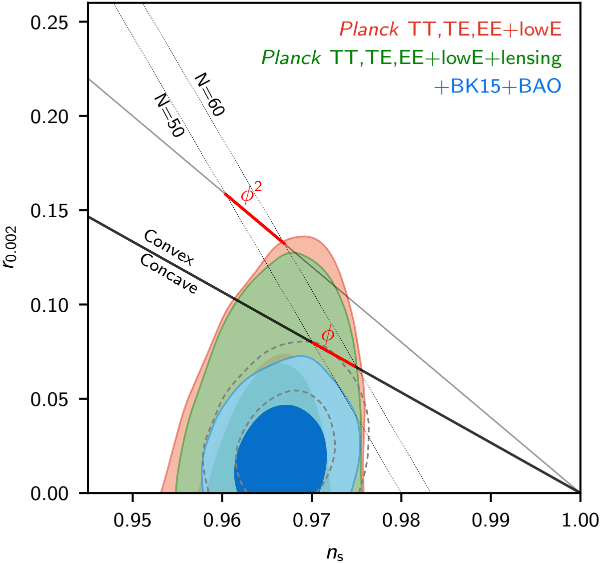
$N_e = 50 - 60$

Planck (2015)

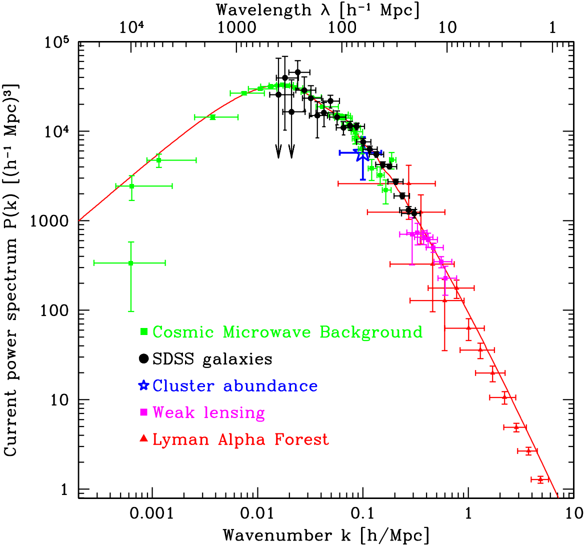


$$r = \frac{A_T}{A_S} \propto \frac{\dot{\phi}^2}{H^2\,M_{Pl}^2} \propto \left(\frac{V'}{V}\right)^2 \ll 1$$

Planck analysis of cosmlogical data (2018)



Actually we observe rather narrow range



Observable range:

$$\frac{k_{max}}{k_{min}} \sim 10^5$$

$$\Delta N_e \simeq 10$$

Small scales cannot describe:
for a long time in nonlinear regime

Reheating

After inflation we must produce particles
to enter the radiation dominating stage
i.e. we must reheat the Universe

inflaton couples to SM

- perturbative... e.g. decays:

$\phi \to hh$, reheating at $H = \Gamma$

- through oscillations induced by inflaton

time-dependent external force $F(t)$ or mass $m(t)$

— can be resonantly amplified !!
— most efficient: tachionic, when $m^2(t) < 0$

Particle production I

An elementary particle:

$$E = mc^2 \quad \longrightarrow \quad E^2 = k^2c^2 + m^2c^4$$

equation of motion

$$\ddot{\phi}(t,\mathbf{x}) - \Delta\phi(t,\mathbf{x}) + m^2\phi(t,\mathbf{x}) = 0 \qquad \phi \propto \mathrm{e}^{iEt+i\mathbf{kx}}$$

for particular 3-momenta looks as oscillator

$$\ddot{\phi}_k(t) + \left(\mathbf{k}^2 + m^2\right)\phi_k(t) = 0 \qquad \phi(t,\mathbf{x}) = \int d^3x\phi_k(t)\mathrm{e}^{i\mathbf{kx}}$$

Quantum physics:

even in vacuum (no particles) $\qquad \phi_k = \phi_k^{vac}(t) \neq 0 \qquad$!!

Particle production II

In the expanding Universe

$$\ddot{\phi}_k(t) + 3H(t)\,\dot{\phi}_k(t) + \left(\frac{\mathbf{k}^2}{a^2(t)} + m^2\right)\phi_k(t) = 0$$

interaction with inflaton $X(t)$, e.g. $X^2\phi^2$:

$$\ddot{\phi}_k(t) + 3H(t)\,\dot{\phi}_k(t) + \left(\frac{\mathbf{k}^2}{a^2(t)} + m^2 + X^2(t)\right)\phi_k(t) = 0$$

oscillator with time-dependent frequency can be excited if

— $\Omega_X \gg \Omega_{\phi_k}$ high-frequency (this case)

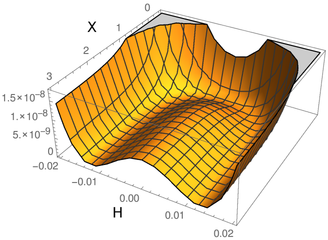
among other generic options

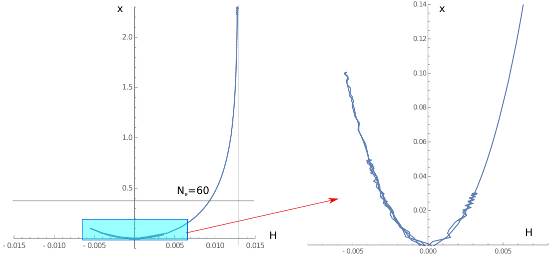
— at zero crossings, that is $\Omega_X^{eff} \simeq 0$ large field X

— at tachyonic time slots with $\Omega_X^{eff\,2} < 0$

Higgs & Scalaron

D.G., A.Tokareva 1807.02392





Scalar perturbations:

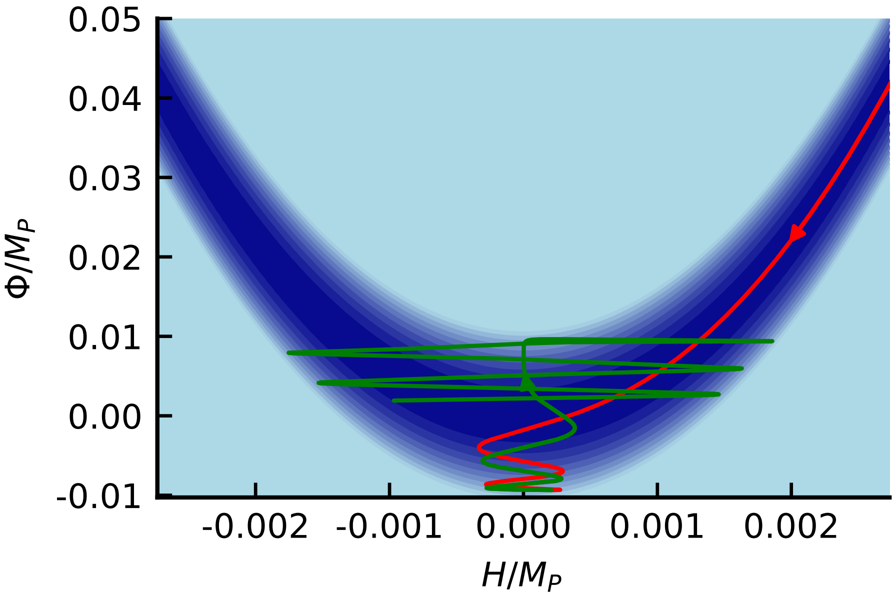
1701.07665

$$\beta + \frac{\xi^2}{\lambda} \simeq 2 \times 10^9$$

At small β like in the Higgs-inflation

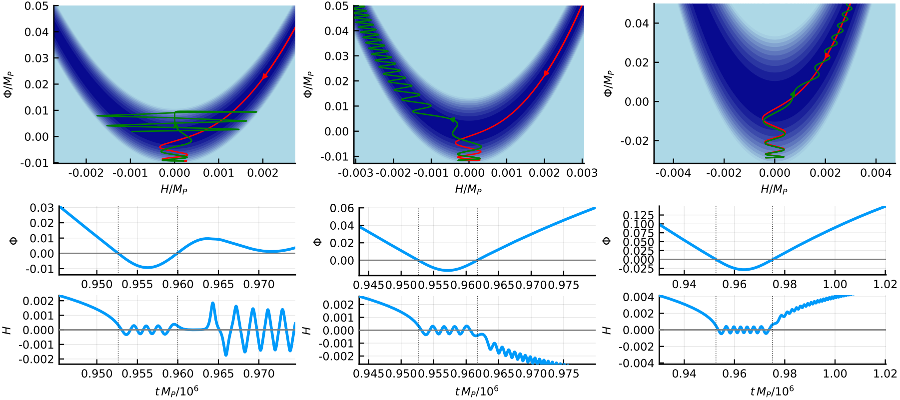
heavy scalaron is integrated out

$$\frac{\xi^2}{4\pi} < \beta < \frac{\xi^2}{\lambda} \quad \to \quad 5 \times 10^{13}\,\text{GeV} < m < 1.5 \times 10^{15}\,\text{GeV}$$


0.05
0.04
0.03
0.02
0.01
0.00
-0.01
Φ/M_P
-0.002
-0.001
0.000
0.001
0.002
H/M_P

Scalaron Φ and Higgs H evolution after inflation

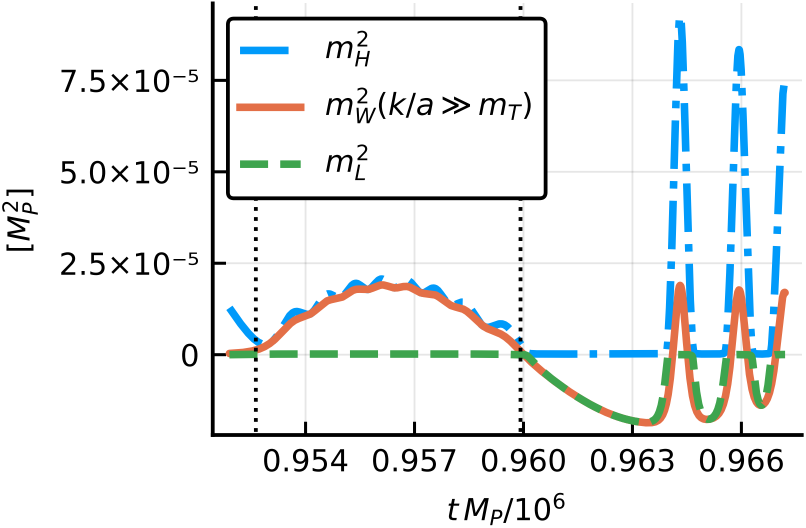
Homogeneous modes (mixing in kinetic sector), $\dot{\Phi} < 0$, $\dot{\Phi} > 0$



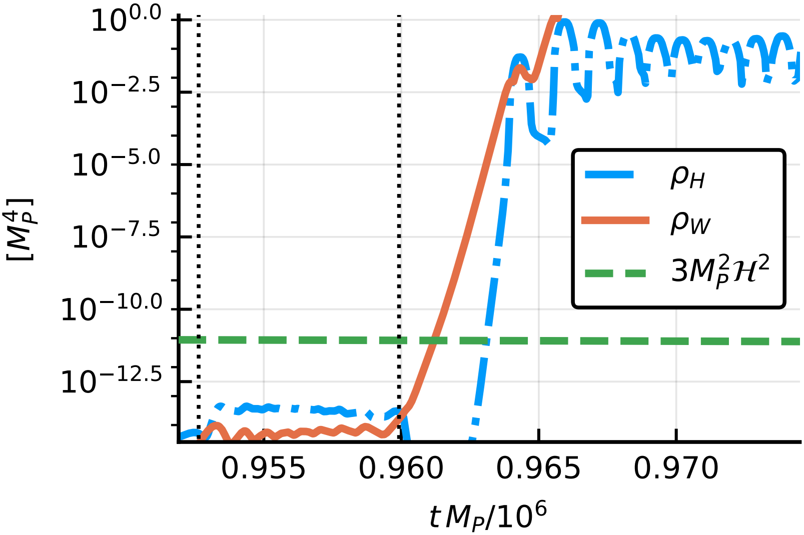
$\beta = 1.869 \times 10^6$ $\beta = 2.9 \times 10^6$ $\beta = 18 \times 10^6$

$$V(H,\Phi) = \frac{1}{4}\left(\lambda + \frac{\xi^2}{\beta}\right)H^4 + \frac{M_P^2}{6\beta}\Phi^2 - \frac{\xi M_P}{\sqrt{6}\beta}\Phi H^2 + \frac{7}{108\beta}\Phi^4 + \frac{\xi}{6\beta}\Phi^2 H^2 - \frac{M_P}{3\sqrt{6}\beta}\Phi^3$$

Numerical results: mass squared



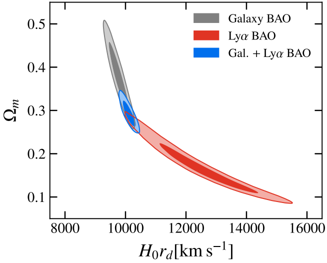
Numerical results: energy in perturbations

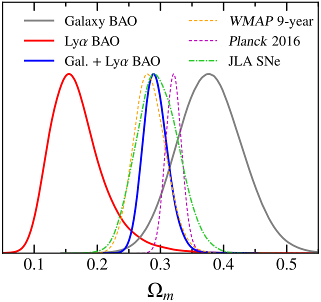


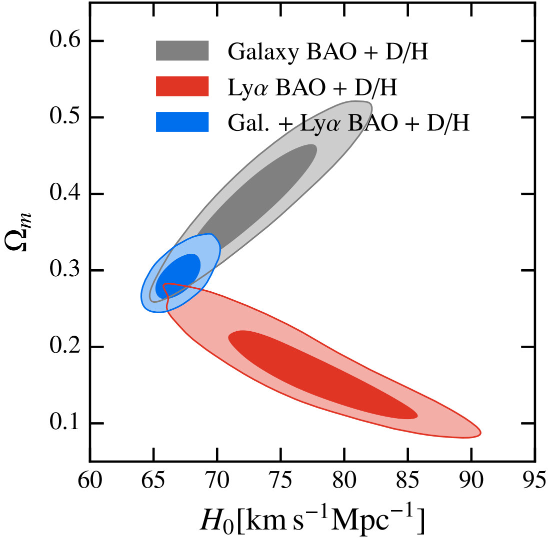
Backup slides

Impact of BAO: Galaxies vs Ly-α

1707.06547







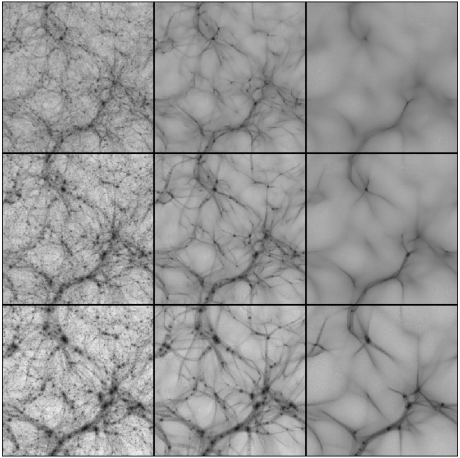
Missing satellites: free streaming or selfinteraction?

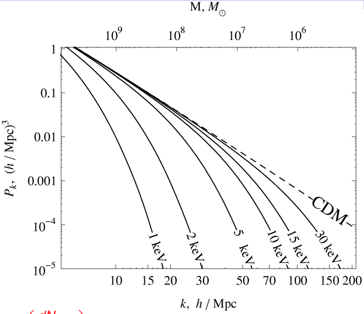
Missing satellites:

$$\frac{dN_{obj}}{d\ln M} \propto \frac{1}{M}$$

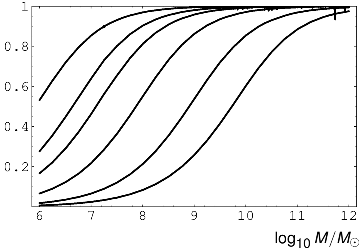
no-scale

100 instead of 1000

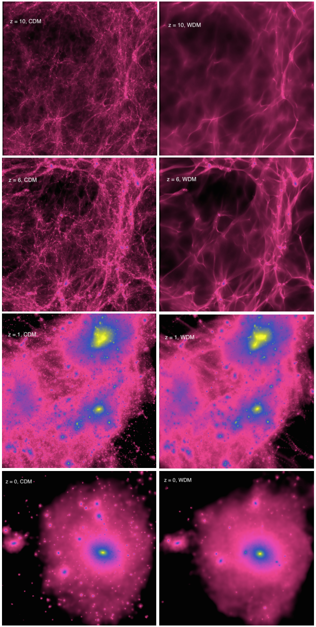


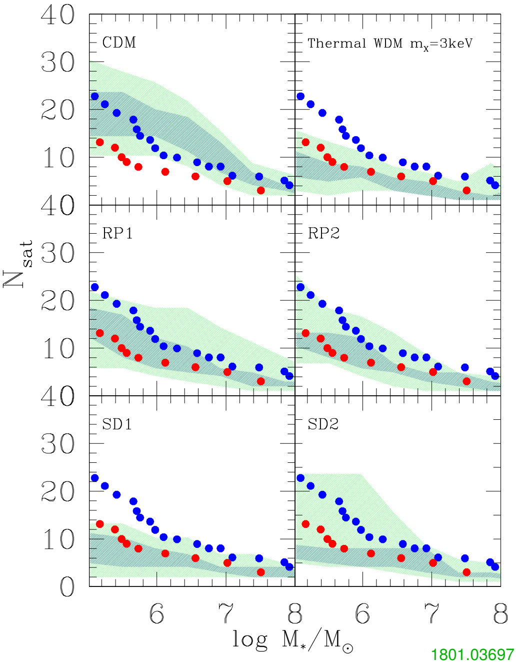


$$\left(\frac{dN_{obj}}{d\ln M}\right)_{WDM} / \left(\frac{dN_{obj}}{d\ln M}\right)_{CDM}$$



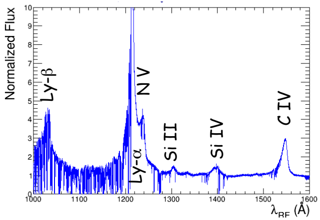
1801.03697 WDM $m = 3.3\,\mathrm{keV}$



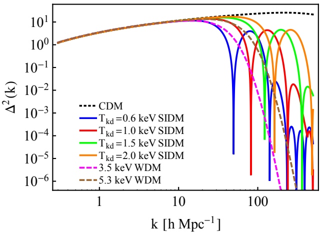


1801.03697

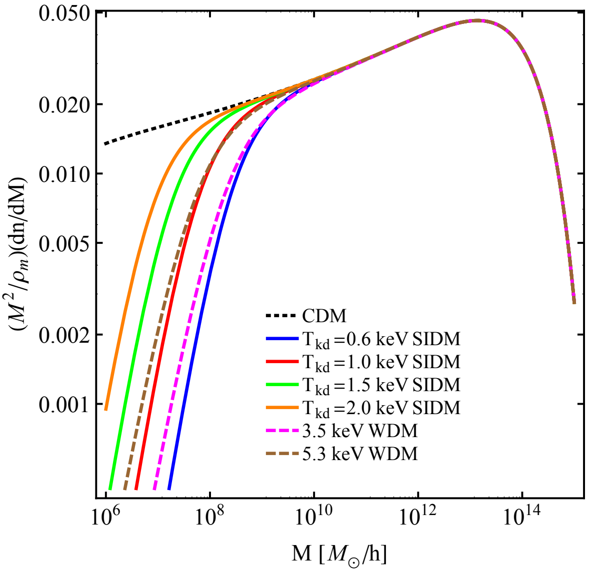
Missing satellites: dwarfs vs Ly-α



1702.03314

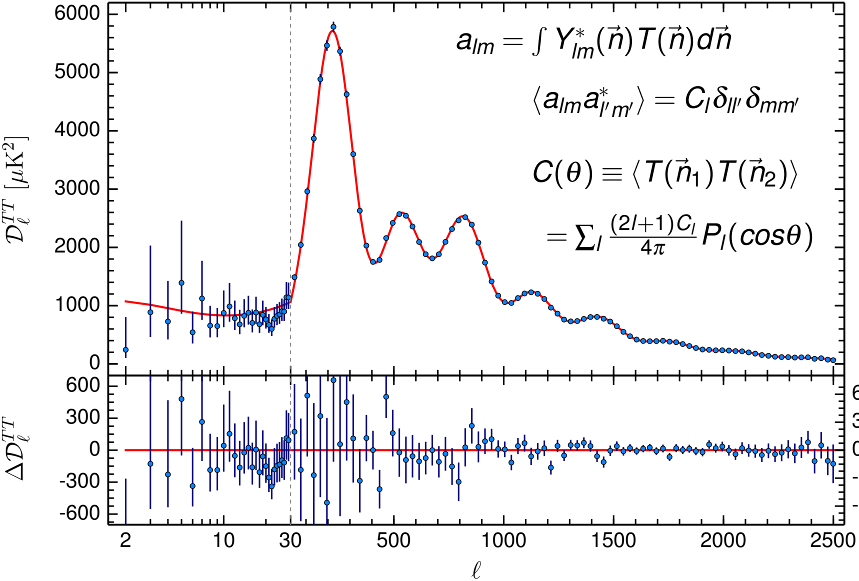


1709.09717



CMB anisotropy spectrum by Planck

1502.01582



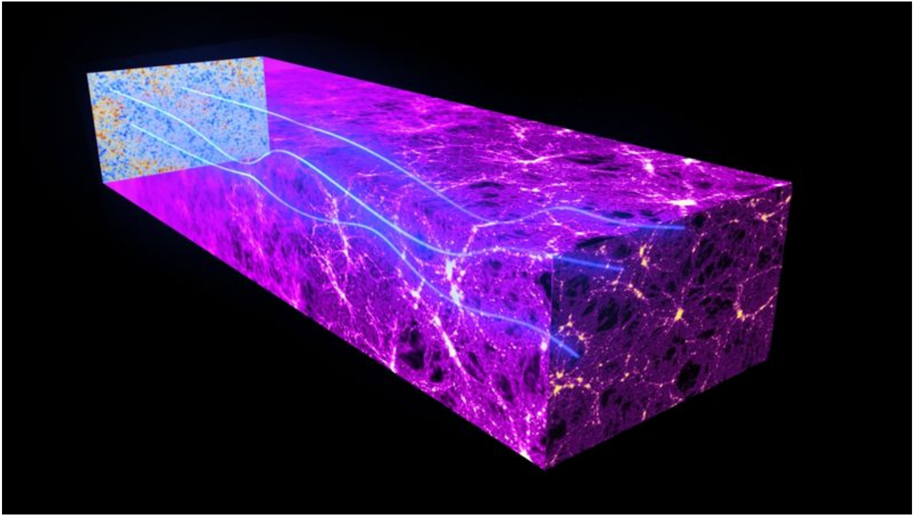
$$a_{lm} = \int Y^*_{lm}(\vec{n})T(\vec{n})d\vec{n}$$

$$\langle a_{lm}a^*_{l'm'}\rangle = C_l\delta_{ll'}\delta_{mm'}$$

$$C(\theta) \equiv \langle T(\vec{n}_1)T(\vec{n}_2)\rangle$$

$$= \sum_l \frac{(2l+1)C_l}{4\pi}P_l(cos\theta)$$

Initial or Induced: propagation in expanding Universe



Cold spot (Planck)

1502.01582