Framework
for Dynamically Consistent Parameterization
of Oceanic Mesoscale Eddies

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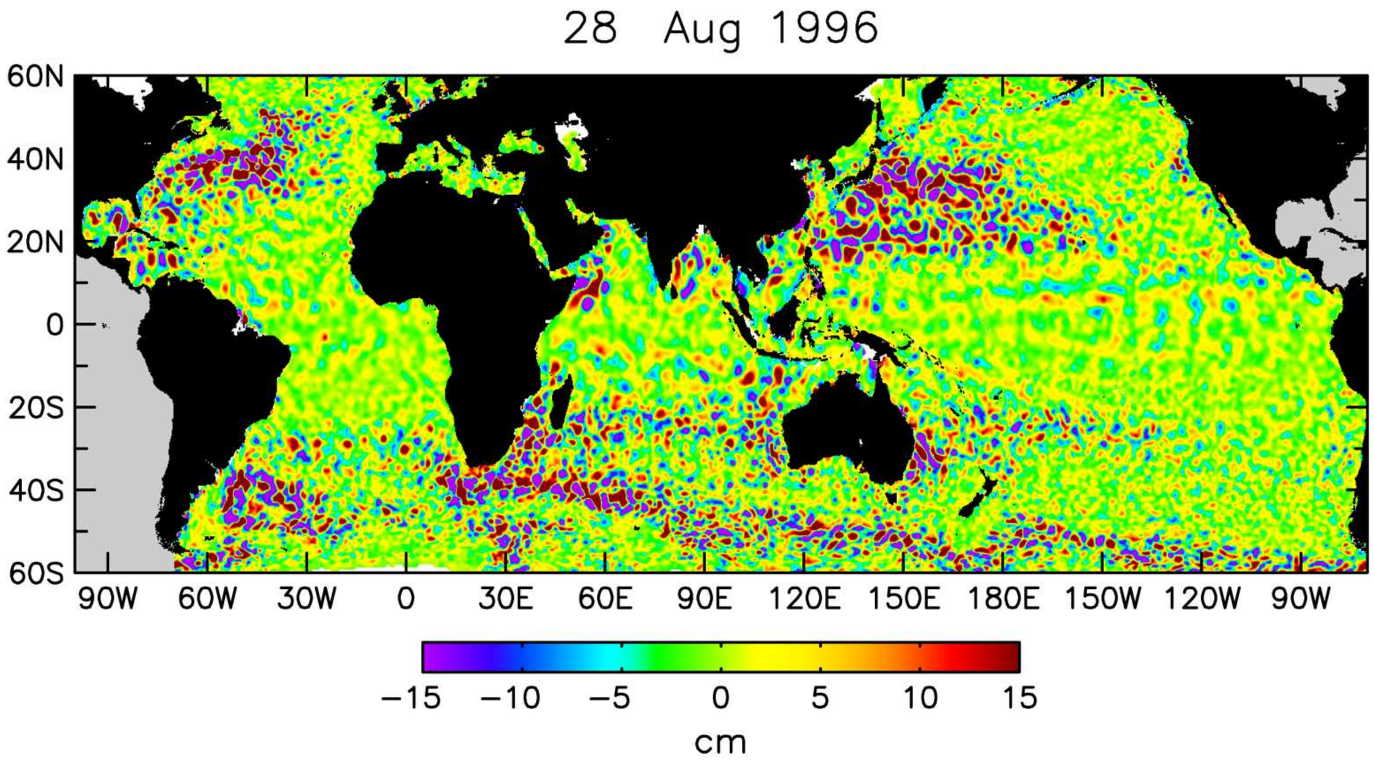
Results are in *Berloff (2015)*; extension is in *Berloff (2016)*.

Transient Mesoscale Oceanic Eddies



Snapshot of the observed SSH anomaly shows eddies all over the ocean

• Eddies have important *kinematical* and *dynamical* effects on the large-scale circulation.

— Ocean models must account for these effects either *directly* by brute-force computations or *indirectly* by simple models referred to as *"eddy parameterizations"*.

• Examples of successful parameterizations:

— *Gent-McWilliams* (for downgradient diffusion of buoyancy due to baroclinic instability);

— *Eddy viscosity* (for downgradient diffusion of momentum due to Reynolds stresses).

Eddy Diffusion: To be or not to be?

Diffusive eddy parameterizations are easier to deal with, provided proper relation between the eddy flux and mean gradient; for example, in the classical Reynolds decomposition case:

$$\overline{\mathbf{u}'\,q'} = -\mathbf{K}\,\nabla\overline{q} \qquad \Longrightarrow \qquad \frac{\partial\overline{q}}{\partial t} + \overline{\mathbf{u}}\cdot\nabla\overline{q} \;=\; \nabla\cdot(\mathbf{K}\,\nabla\overline{q}) + Q\,;$$

where overbars can be generalized to indicate the large-scale (i.e., space-time filtered) fields, and primes — to indicate the small-scale residual fluctuations.

• *Coarse-graining problem*: eddy/large-scale decomposition of the flow fields is not unique and should fit the purpose;

• *Fitting problem*: estimates of spatially inhomogeneous, anisotropic and nonstationary eddy diffusivity tensor $\mathbf{K}$ are data-constrained; and its Lagrangian estimates are even nonlocal;

• *Ill-posedness problem*: a non-zero eddy flux on the top of zero mean gradient; up-gradient eddy fluxes;

• *Complexity problem*: Transient eddy fluxes can be more important than the mean ones, resulting in highly transient and inhomogeneous $\mathbf{K}(t,\mathbf{x})$; also, non-uniqueness of $\mathbf{K}$.

• *Closure problem*: Relating $\mathbf{K}$ to the large-scale flow properties is problematic, mostly because of the nonlocal processes involved.

All of the above problems appear in the *western boundary currents* with their eastward jet extensions, thus, making them notoriously difficult for diffusive parameterizations.

Let me now pose specific eddy parameterization problem and explain the new ideas...

Double-Gyre Ocean Model

• Focus is on an idealized model of midlatitude ocean circulation with vigorous eddy dynamics despite a simple setup (steady wind forcing; square basin; flat bottom). Large Reynolds numbers are reached by solving with fine grid resolution.

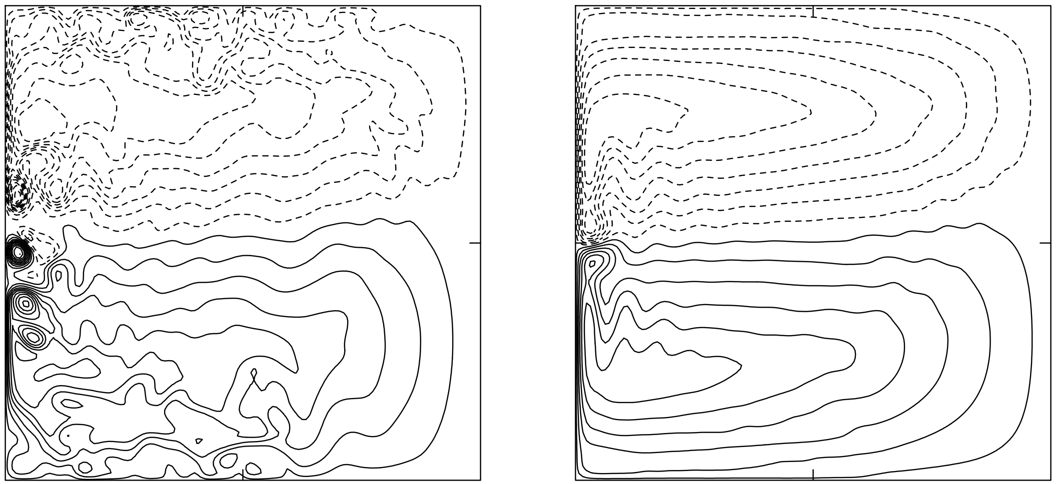
• Governing equations for the two-layer QGPV β-plane double-gyre configuration:

$$\begin{aligned} \frac{\partial q_1}{\partial t} + \mathbf{u}_1 \cdot \nabla q_1 + \beta v_1 &= \nu \nabla^4 \psi_1 + W \\ \frac{\partial q_2}{\partial t} + \mathbf{u}_2 \cdot \nabla q_2 + \beta v_2 &= \nu \nabla^4 \psi_2 - \gamma \nabla^2 \psi_2 \end{aligned} \qquad u_i = -\frac{\partial \psi_i}{\partial y}, \quad v_i = \frac{\partial \psi_i}{\partial x}$$

$$q_1 = \nabla^2 \psi_1 + S_1 (\psi_2 - \psi_1), \qquad q_2 = \nabla^2 \psi_2 + S_2 (\psi_1 - \psi_2)$$

• *Importance of eddy effects*: Eastward jet extension and its adjacent recirculation zones are not formed, if mesoscale eddies are not properly resolved.

instantaneous *time-mean*



Upper-ocean circulation of the double-gyre model with coarse grid ($1/4°$) and low Re.

Goal of this study: *To restore the main eddy effects with a fully closed parameterization*.

Eddy Forcing Components

• Dynamical effects of eddies on the large-scale circulation can be expressed by *eddy forcing (EF)*, which is obtained by flow decomposition into the *large-scale* (overbarred) and *eddy* (primed) components:

$$
\begin{aligned}
EF(t,\mathbf{x}) = \ & -\nabla\cdot\overline{\mathbf{u}}\,q' && (\textit{large-scale/eddy advection}) \\
& -\nabla\cdot\mathbf{u}'\,\overline{q} && (\textit{eddy/large-scale advection}) \\
& -\nabla\cdot\mathbf{u}'q' && (\textit{eddy/eddy advection})
\end{aligned}
$$

Eddy forcing can be further decomposed into the *time-mean* and *transient* components:

$$EF(t,\mathbf{x}) = \langle EF\rangle(\mathbf{x}) + EF'(t,\mathbf{x})$$

• Let's focus on the transient eddy forcing component $EF'(t,\mathbf{x})$. (first new idea!)
Eddy backscatter (part of the "inverse energy cascade") is a dynamical mechanism based on persistent correlations between EF' and the evolving large-scale flow. Despite being very important in the ocean, this mechanism remains poorly understood.

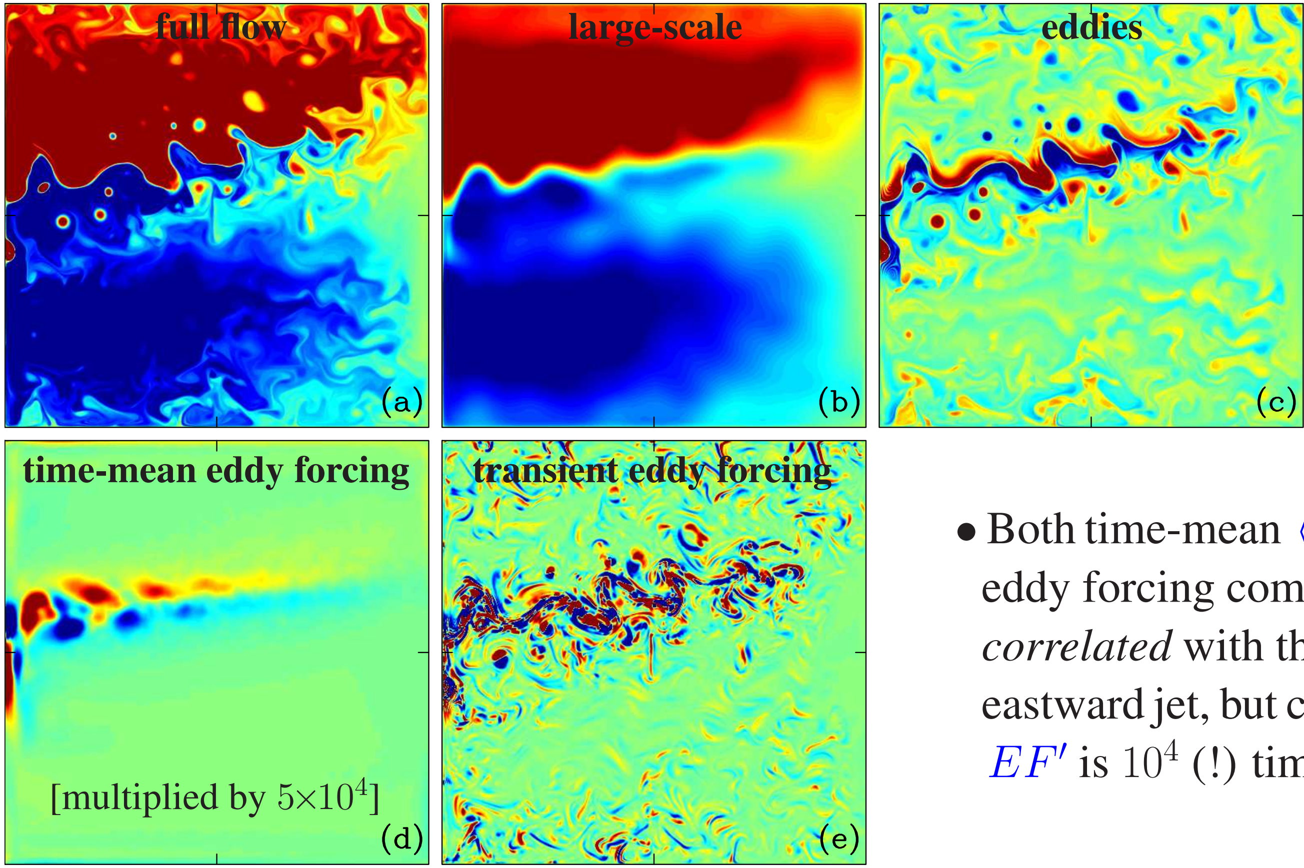
• NOTE: Classical *Reynolds decomposition* (into the *time-mean* state and *fluctuations*) is:

(a) *Useless* for eddy backscatter, because (by construction) its EF' is *degenerate* (i.e., completely decorrelated from the large-scale flow).

(b) *Misleading*, when the large-scale flow evolves far from the time-mean state.

Eddy Backscatter Mechanism

• Eastward jet extension of the western boundary currents is driven by the eddy backscatter mechanism (e.g., *Berloff 2005; Waterman and Jayne 2011*).



Large-scale and eddy fields are separated by spatial filtering and time averaging (shown are PV anomalies for some snapshot).

• Both time-mean $\langle EF \rangle$ and transient EF' eddy forcing components are *positively correlated* with the evolving large-scale eastward jet, but covariance of the jet with EF' is 10^4 (!) times larger.

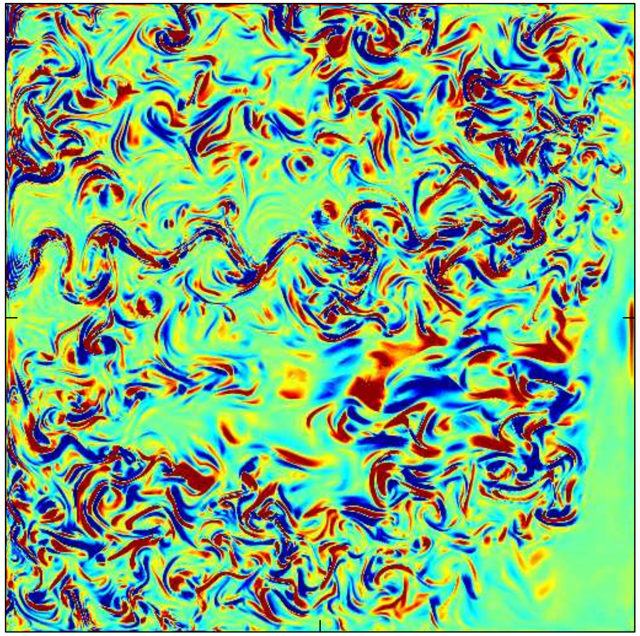
• We have to shift focus from modeling $\langle EF \rangle$ (e.g., by relating eddy fluxes to large-scale gradients) to modeling the backscatter effect of EF' either *directly* (e.g., by adding stochastic forcing) or *indirectly* (e.g., by adding cumulative effect of stochastic forcing).

Transient Eddy Forcing in Eddy-Resolving Ocean Gyres

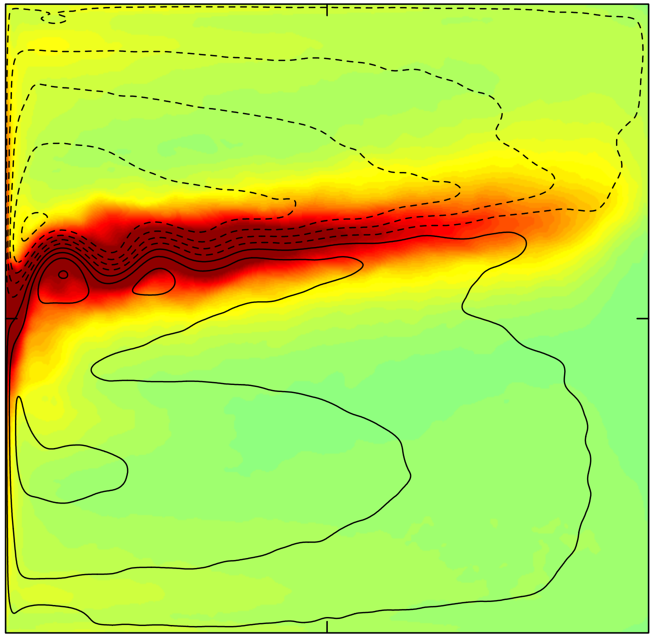
• Diagnozed $EF'(t,\mathbf{x})$ has spatio-temporal pattern characterized by weak variations of correlation length scale $L_{corr}(\mathbf{x})$ and median time scale $T_{med}(\mathbf{x})$, hence, it can be approximated (locally) as *elementary "plunger" forcing*: (second new idea!)

$$F(t,x,y;\,x_0,y_0) \sim A(x_0,y_0)\,\sin\left(\frac{2\pi\,t}{T_{med}}\right)\,\cos\left(\frac{\pi}{2}\,\frac{r}{L_{corr}}\right), \qquad r=\sqrt{(x-x_0)^2+(y-y_0)^2} < L_{corr}$$

upper-ocean snapshot of EF



standard deviation of EF, mean flow



• Local, linear-dynamics, eddy-resolving responses to elementary "plungers" can provide the relation between eddy forcing and large-scale flow pattern. (third new idea!)

Single-Layer Green's Function

Let's consider the linearized, conservative single-layer dynamics with time-periodic δ-forcing:

$$\frac{\partial}{\partial t}\Big(\nabla^2\psi - S\,\psi\Big) + \beta\,\frac{\partial\psi}{\partial x} \;=\; \delta(\mathbf{x})\,e^{-i\omega_0 t}$$

Let's look for solution in the form $G = \tilde{G}\,\exp\left\{-i\left(\frac{\beta x}{2\omega_0} + \omega_0 t\right)\right\}$ and obtain:

$$\nabla^2\tilde{G} + \gamma^2\,\tilde{G} = \frac{i}{\omega_0}\,\delta(\mathbf{x})\,, \qquad \gamma^2 = \left(\frac{\beta}{2\omega_0}\right)^2 - S\,.$$

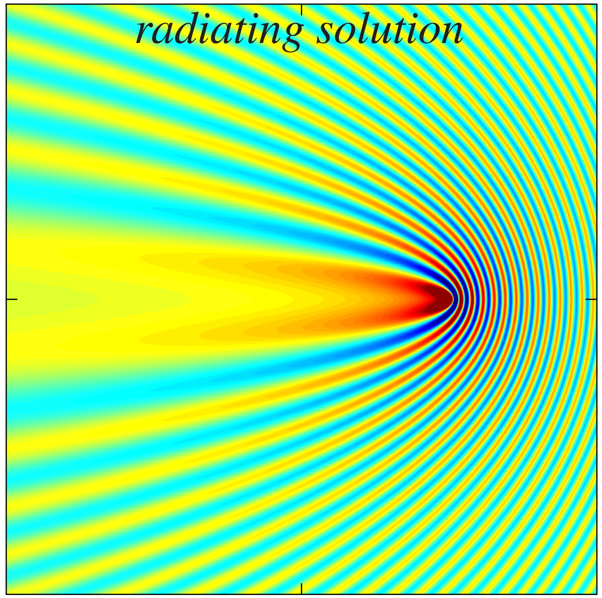
This equation can be solved in terms of special functions, and the solution is either *radiating*:

$$\gamma^2 > 0\;:\; G(t,x,y) \sim H_0^{(2)}(\gamma r)\,\exp\left\{-i\left(\frac{\beta x}{2\omega_0} + \omega_0 t\right)\right\},$$

or *trapped*:

$$\gamma^2 < 0\;:\; G(t,x,y) \sim K_0^{(2)}(\gamma r)\,\exp\left\{-i\left(\frac{\beta x}{2\omega_0} + \omega_0 t\right)\right\},$$

depending on the deformation radius $S^{-1/2}$ involved.



- Solution doesn't radiate, if $S \neq 0$ and forcing frequency ω_0 is larger than some cutoff value.

— Baroclinic EF in the gyres has ω_{med} that is way too large for the baroclinic radiation, nevertheless such EF generates radiating response — is there a paradox?

Plunger-Induced Dynamics in the Presence of Stratification and Background Flow

• *Stratified fluid.* In the absence of background flow, the barotropic and baroclinic modes are decoupled $\implies$ each mode of the solution is given by its own Green's function.

• *Spatially distributed plunger.* Solution can be obtained as a convolution of Green's functions.

In the presence of a vertically sheared background flow, the vertical modes become *coupled*, and the analytic solution for the Green's function is unknown...

• *Background flow effect* is the most important, because it provides basis for a closure.

In the two-layer case with *zonal* background flow (U_1, U_2), the equations to solve are:

$$-i\omega_0 \left(\nabla^2 \tilde{G}_1 - S_1 \left(\tilde{G}_1 - \tilde{G}_2\right)\right) + \beta_1 \frac{\partial \tilde{G}_1}{\partial x} + U_1 \frac{\partial}{\partial x} \left(\nabla^2 \tilde{G}_1 - S_1 \left(\tilde{G}_1 - \tilde{G}_2\right)\right) = F_1(x, y)$$

$$-i\omega_0 \left(\nabla^2 \tilde{G}_2 - S_2 \left(\tilde{G}_2 - \tilde{G}_1\right)\right) + \beta_2 \frac{\partial \tilde{G}_2}{\partial x} + U_2 \frac{\partial}{\partial x} \left(\nabla^2 \tilde{G}_2 - S_2 \left(\tilde{G}_2 - \tilde{G}_1\right)\right) = 0$$

where $\beta_1 = \beta + S_1(U_1 - U_2)$ and $\beta_2 = \beta + S_2(U_2 - U_1)$.

Let's Fourier transform these equations:

$$-i\left(kU_1 + \omega_0\right)\Big[-\left(k^2 + l^2 + S_1\right)\tilde{g}_1 + S_1\,\tilde{g}_2\Big] - i\,\beta_1 k\,\tilde{g}_1 = f_1(k, l)$$

$$-i\left(kU_2 + \omega_0\right)\Big[-\left(k^2 + l^2 + S_2\right)\tilde{g}_2 + S_2\,\tilde{g}_1\Big] - i\,\beta_2 k\,\tilde{g}_2 = 0\,,$$

These equations can be written in the matrix form

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} \tilde{g}_1 \\ \tilde{g}_2 \end{pmatrix} = \begin{pmatrix} if_1 \\ 0 \end{pmatrix} ,$$

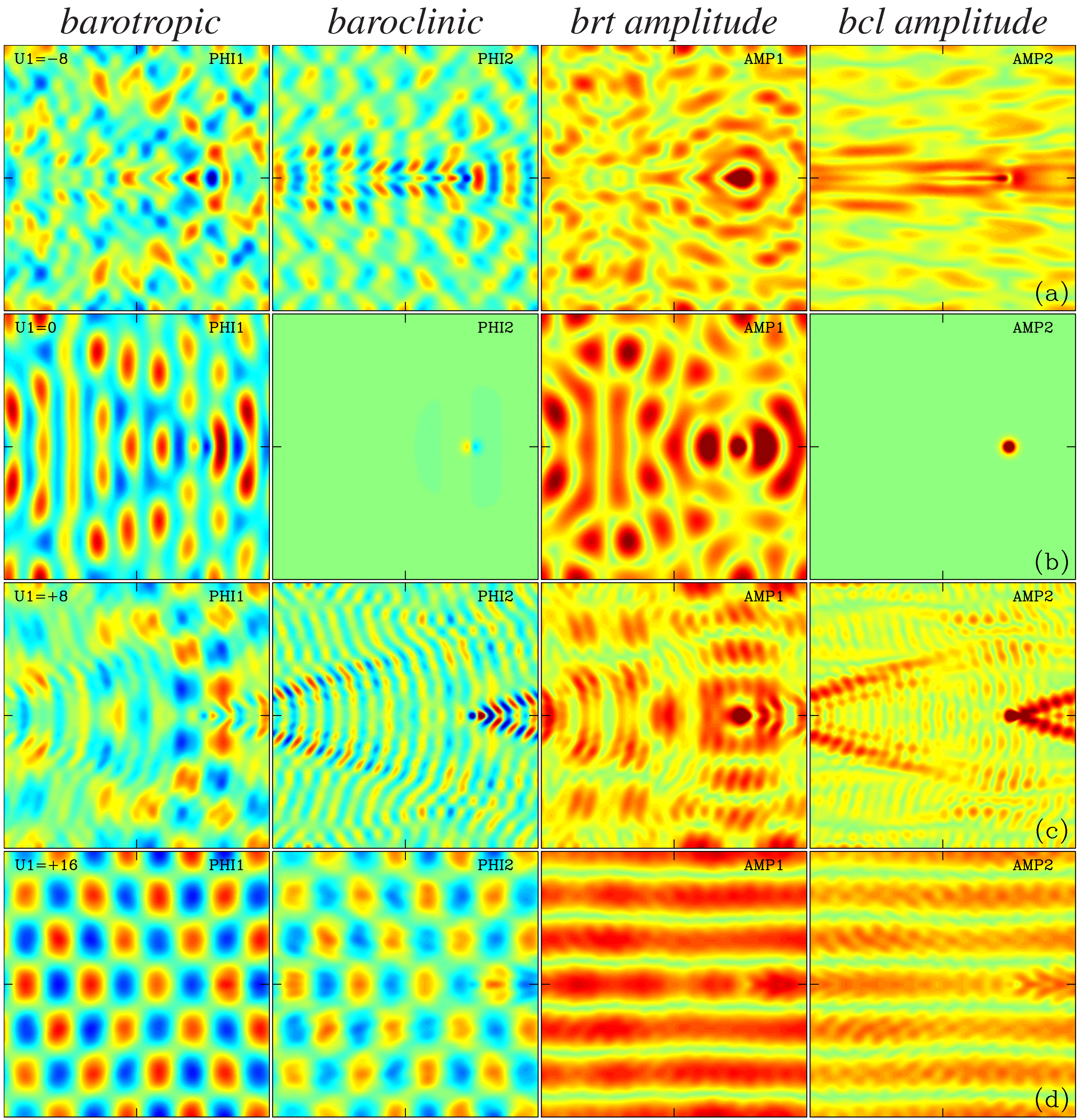
solved for each (k, l) and inverted back to the physical space.

• Multi-layer extension of the problem is straightforward.

• Generalizations to arbitrary background flows and boundary conditions are not straightforward but can be done.

Let's take a look at typical plunger-induced solutions for spatially uniform, zonal vertical shears...

Plunger-Induced Solutions



Shown are the real components and complex amplitudes...

- Background flow effects:

(1) *Baroclinic delocalization.*

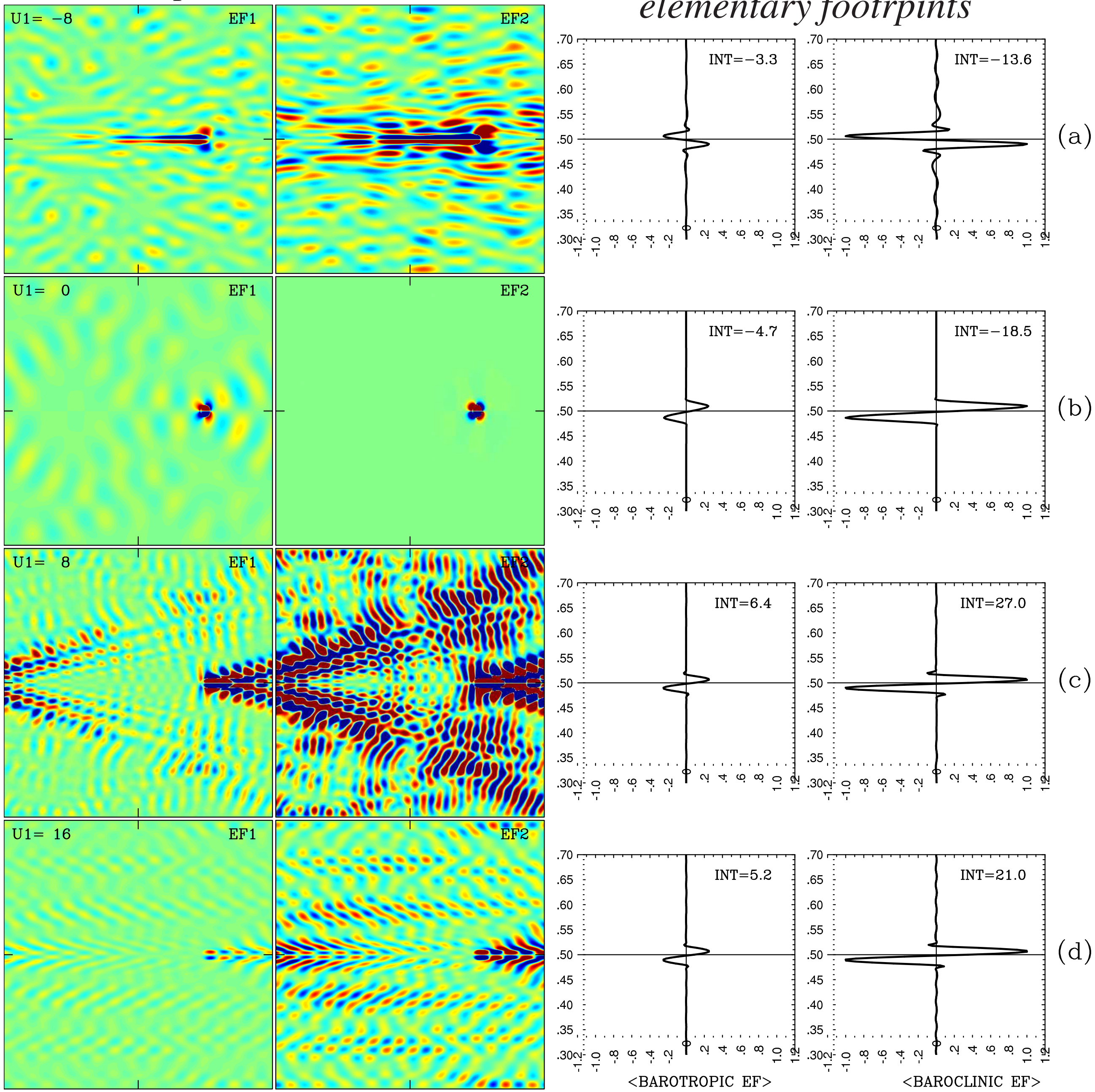
(2) *Baroclinic amplification.*

- Let's introduce the concept of *"footprint"*, which is the time-mean nonlinear self-interaction of the plunger-induced solution.

Plunger-Induced Footprints

barotropic *baroclinic*

elementary footrpints



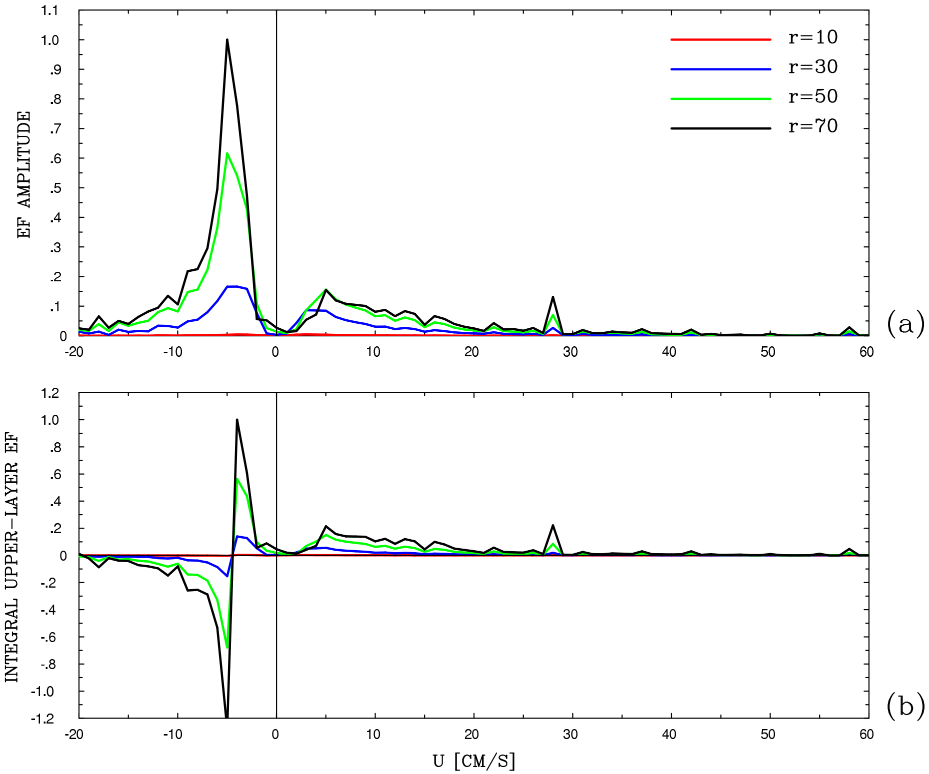
- Zonally and temporally averaged footprint is called: *"elementary footprint"*
- It describes rearrangement of PV by transient forcing
- It strongly depends on $U(z)$ and other parameters of the problem
- It is characterized by the amplitude and half-basin integral
- It can be coarse-grained for parameterization

Important Properties of Footprints

• *No positive-definiteness "curse" as with eddy diffusivity*: Footprints can rearrange PV (and any other properties) both down and up its gradient.

• *Very strong dependence on the background flow — basis for closure*: Footprints are most intense for intermediate values of background shear.

• Footprint amplitude increases with L_{corr}, T_{med}, and Reynolds number.

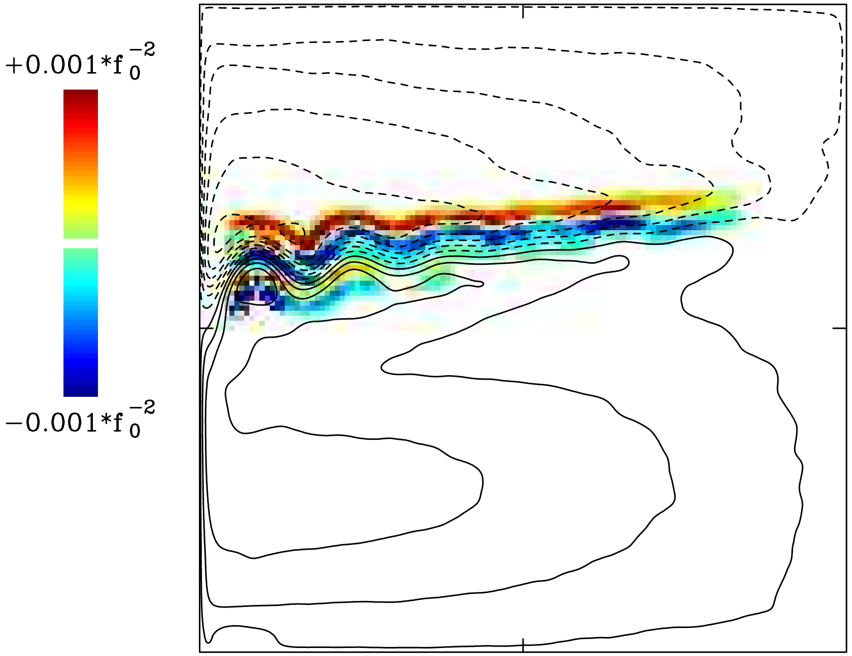


Dependence of elementary footprint properties on the background shear and radius of the plunger

Implementation Algorithm for Parameterization

- *Scaling*: Footprint comes from the linear solution $\Longrightarrow$ its plunger amplitude has to be scaled by some large-scale property. Scaling by PV anomaly flux $|\overline{\mathbf{u}}\,\overline{q}|$ was implemented.

- *Cumulative correction field*, obtained by summing up elementary footprints all around the basin, is added as *extra forcing* to the coarse-grid model and *evolves with the flow solution*.



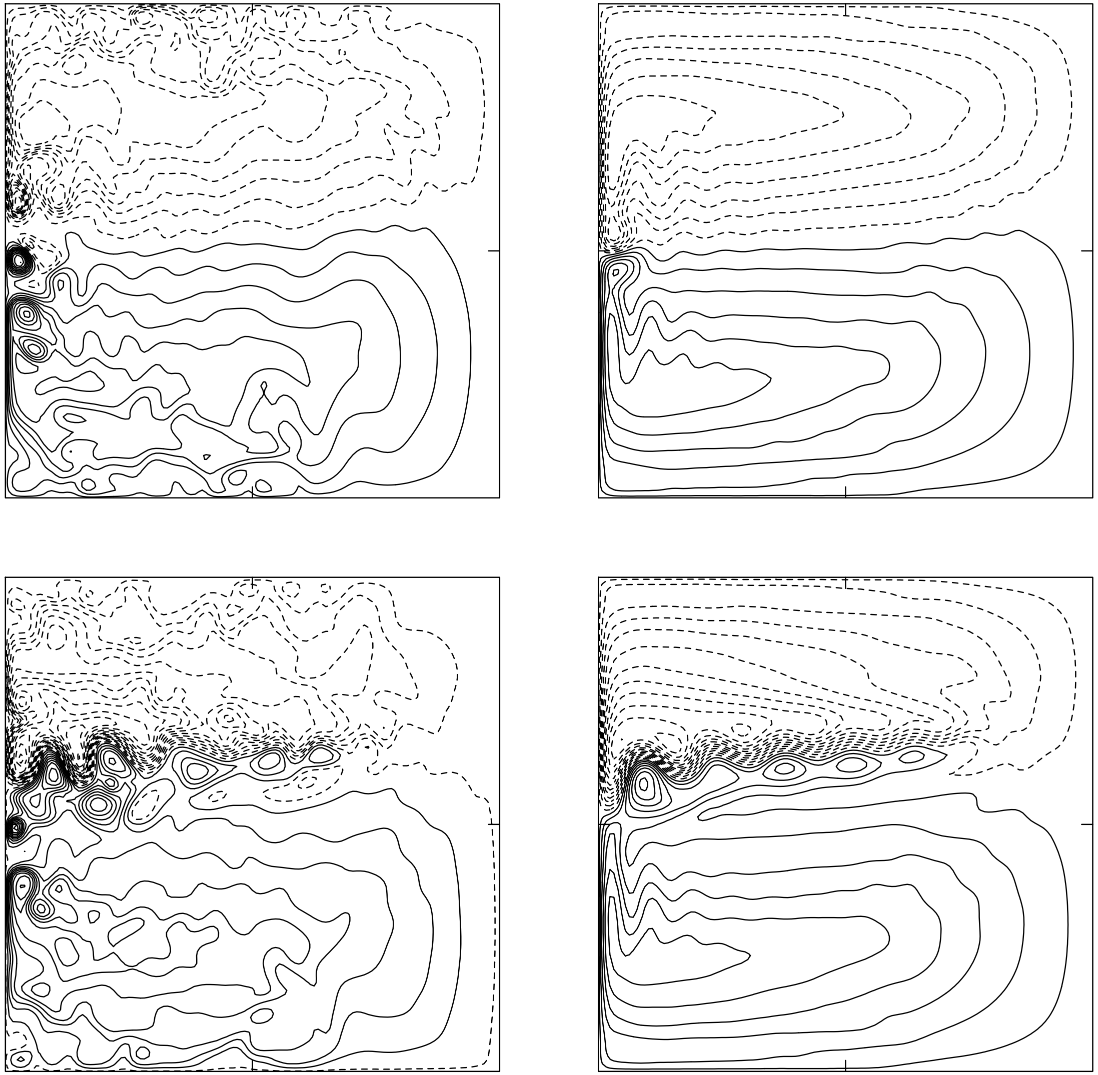
- *Cumulative correction field (color) for the time-mean reference flow (contours) must enhance the eastward jet.*

Let's take a look at fully parameterized solutions of the coarse-grid ocean model...

Effect of the Eddy Parameterization

- Eastward jet and its adjacent recirculation zones recover!

instantaneous *time-mean*



Coarse-grid model solution

Parameterized model solution

Summary of Results

• *New framework for eddy parameterization is proposed and implemented.*

• This framework builds on three new ideas:

(1) *Backscatter:* Focus on the transient eddy forcing;

(2) *Transient impulses:* Eddy forcing is approximated by simple "plungers";

(3) *Dynamical consistency:* Effects of "plungers" are found by explicitly solving eddy-resolving linear dynamics.

• Other important points:

(4) Parameterization is *indirect*, and this is serious advantage;

(5) Physical focus is shifted from eddy fluxes to their divergences;

(6) Mathematical language is shifted from linear normal modes to Green's functions;

(7) Linear-dynamics problems can be solved *once* and used as the lookup table;

(8) Framework not only provides a *systematic strategy*, but also allows for many extensions, refinements and optimizations.

Systematic Strategy for Improving the Parameterization

(1) To use a lot more of the large-scale flow information;

(2) To upgrade the time dependence (e.g., make it stochastic) and spatial structure (e.g., make it anisotropic) of the plungers;

(3) To improve boundary conditions for the plunger problems and coarse-graining of footprints;

(4) To upgrade the plunger dynamics from the quasilinear to fully nonlinear;

(5) To extend the approach from the quasigeostrophic to primitive equations;

(6) To extend the approach to parameterizing transport and mixing of passive tracers.