Современная физика космических лучей: Нелинейные Волны и Бифуркации

(Аналитические и численные резултаты)

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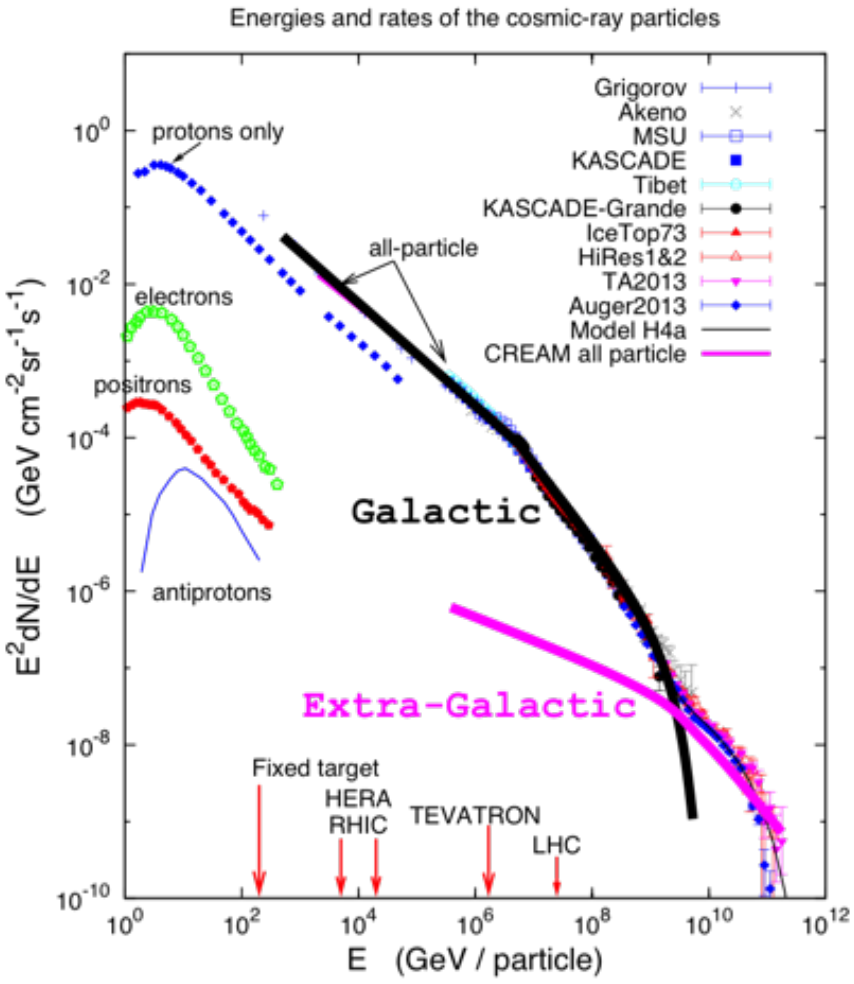
Contents

State of the Art CR energy spectrum

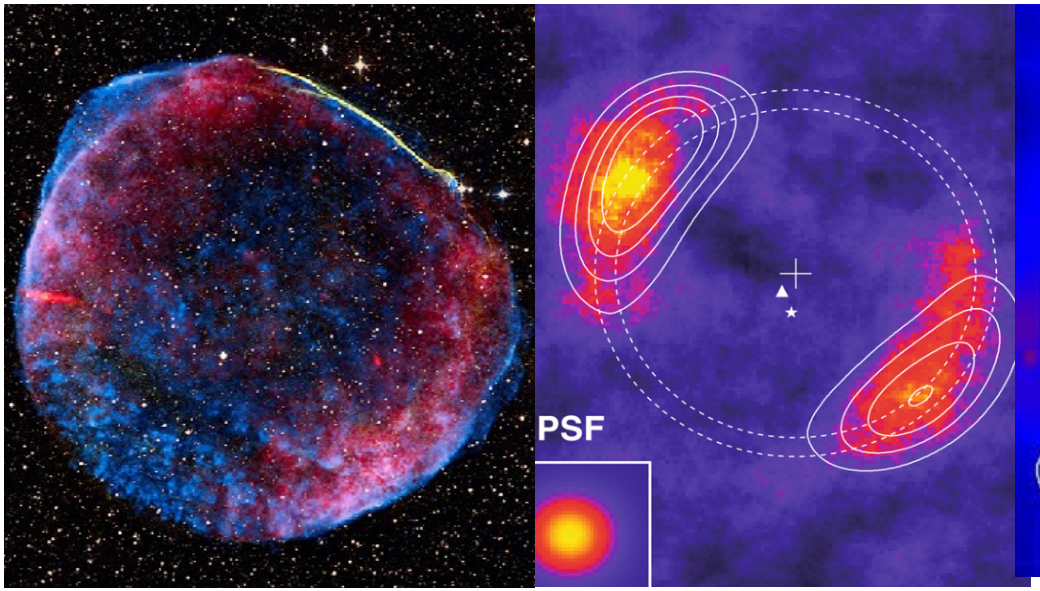


IceCube compilation of CR spectrum

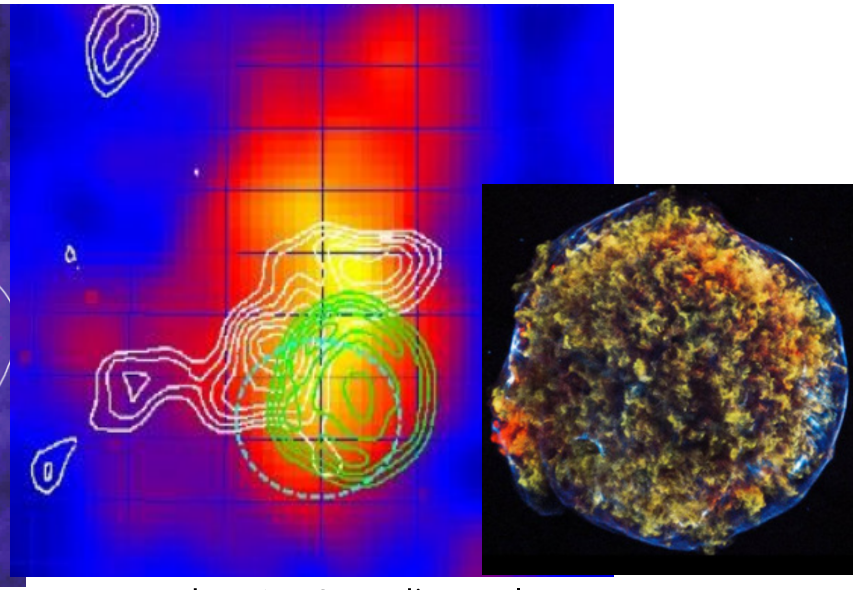
- CR energy spectrum long thought to be featureless (power law):
- consistent with popular acceleration mechanism:
  diffusive shock acceleration, DSA
- DSA rigidity (p/Z) spectra should be the same for all species
- propagation through the ISM may only change the PL-index
  - steepening by propagation losses (0.3-0.6 [!] in PL index)
- some predictions proved incorrect
  - difference in elemental rigidity spectra
  - breaks in individual spectra
- however, conclusion about PL holds up!

Goals and Issues

- Goal: where and how are CR accelerated?
- long-standing hypothesis for galactic CRs: Supernova Remnant (SNR) shocks
- proof can only be "beyond a *reasonable* doubt", by indirect reasoning. Why?
  - impossible to depropagate CR from Earth back to their putative sources (e.g., SNR)
  - difficult to disentangle hadronic and leptonic emission



SNR 1006: X, radio, optical, gamma

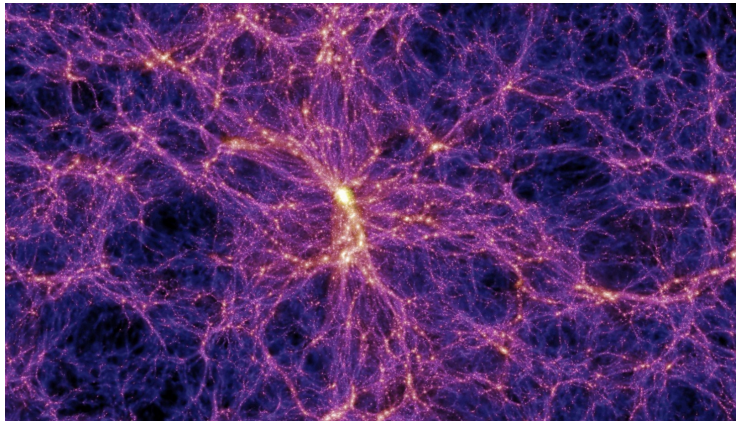


Tycho (1572): radio, mol. gas, gamma

Macroscopic Energy Sources for Cosmic Rays

Generic source: gravitational energy of

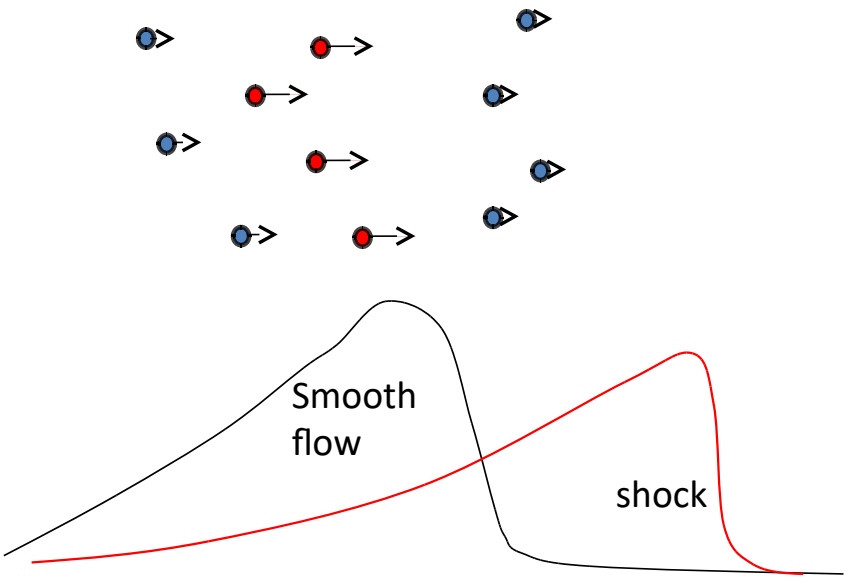
- stars, black holes
- clouds of dense molecular gases
- dark matter filaments and nodes of the "cosmic web" (galaxy clusters)
- exotic sources: strings (topological defects from BB), DM decay and annihilation



Energy extraction mechanisms:

- inhomogeneous flows of conducting gases (plasmas) usually terminated by **SHOCKS**
- accretion flows on galactic clusters, BHs, jets, ..
- stellar winds, colliding winds, galactic winds, SN explosions →SNR shocks

CR mechanism: Diffusive Shock Acceleration (DSA)



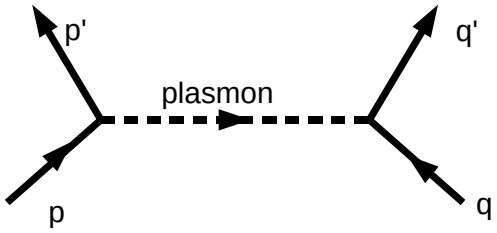
flow velocity

-Most shocks of interest are collisionless

-Big old field in plasma physics

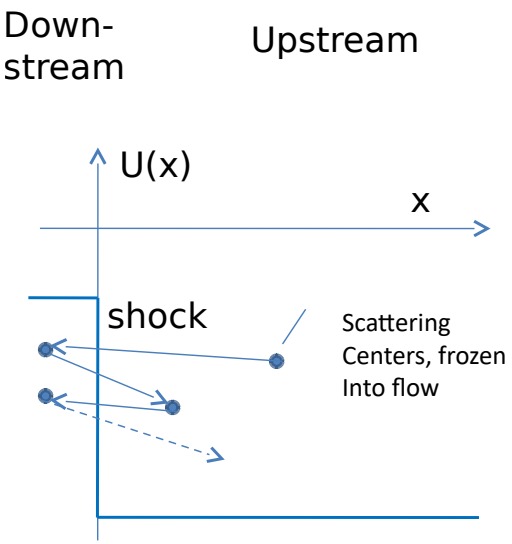
Problems:

- How to transfer momentum and energy from fast to slow gas envelopes if there are no binary collisions?
- waves...
- driven by particles whose distribution is almost certainly unstable...! collective mode



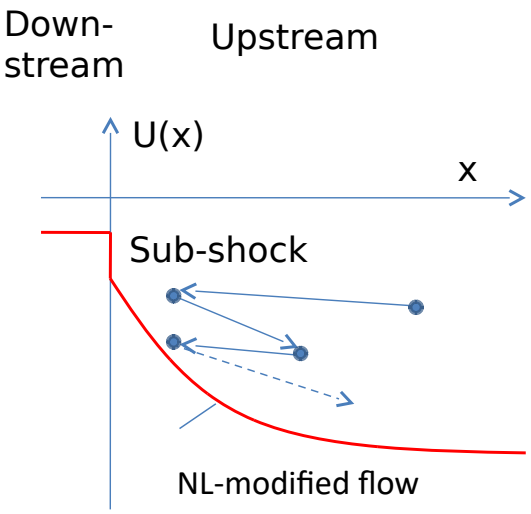
Essential DSA (aka Fermi-I process, E. Fermi, ~1950s)

Linear (TP) phase of acceleration



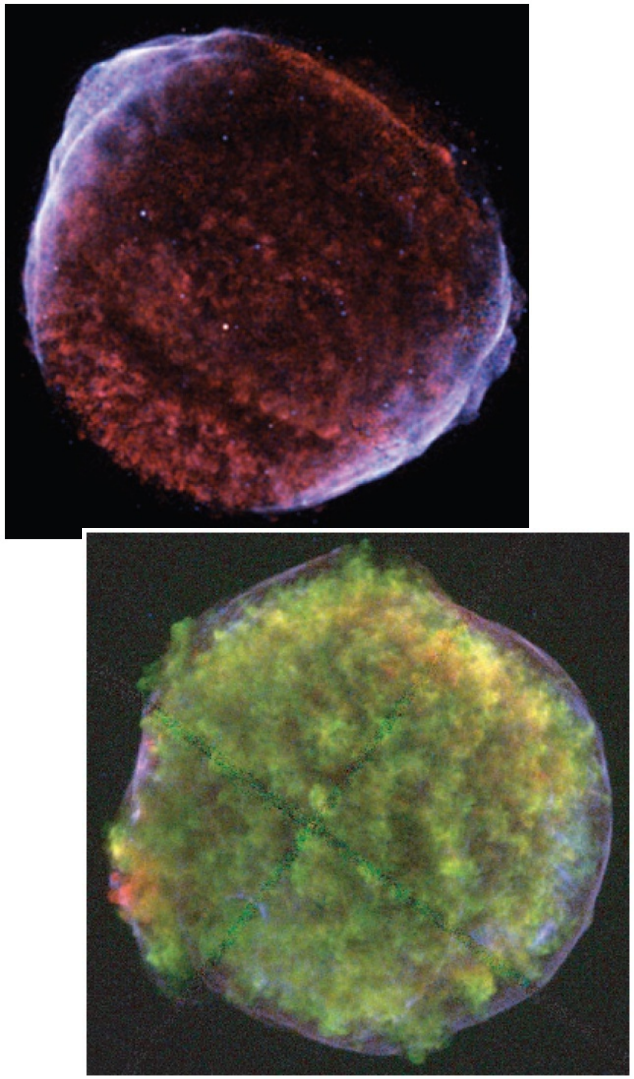
- CR trapped between converging mirrors: $p\Delta x \approx const$
- CR spectrum depends on shock compression, r: $f \sim p^{-q}, \quad q = 3r/(r-1)$, $r = q = 4$, Mach $M \to \infty$

NL, with CR back-reaction



- Ind $q \to q(p)$: soft at low p:
  - $q = 3r_s/(r_s - 1) \sim 5$
- hard at high p : $q \to 3.5$
- for $M > 10$, $E_{max} \gtrsim 1$ TeV (MM'97) acceleration must go nonlinear (confirmed by other analyses and numerics in 2000s)

CR acceleration in SNRs



SN 1006 and SN 1572 (Tycho), Reynolds 2008 and Warren et al 2005

- At least some of the galactic SNR are expected to produce CR up to $10^{15} eV$ (knee energy)
- "Direct" detection is possible only as secondary emission
  - observed from radio to gamma
  - electron acceleration up to $\sim 10^{14} eV$ is considered well established, synchrotron emission in x-ray band (Koyama et al 1995)
  - tentative evidence of proton acceleration from nearby molecular clouds:

$$pp \rightarrow \gamma$$

Fermi-LAT, HESS, Agile,..

Convection-Diffusion Equation: shock solution

- energetic particles, pitch-angle scattered by MHD waves frozen into a plasma flow of the speed $u(x)$

$$\frac{\partial f}{\partial t} + u(x)\frac{\partial f}{\partial x} - \frac{\partial}{\partial x}\kappa(p)\frac{\partial f}{\partial x} = \frac{1}{3}\frac{du}{dx}\frac{\partial f}{\partial p}$$

- at a simple shock, u is a step function $u = u_1, u_2$ for $x > 0$, $x < 0$

$$f(x,p) = f_0(p)\exp\left[-\frac{u_1}{\kappa}x\right], \quad x > 0; \qquad f(x,p) = f_0(p), \quad x \leq 0$$

- matching at $x = 0$ (shock position) leads to the particle spectrum

$$f_0(p) \propto p^{-q}, \quad q = 3r/(r-1), \quad r = u_1/u_2$$

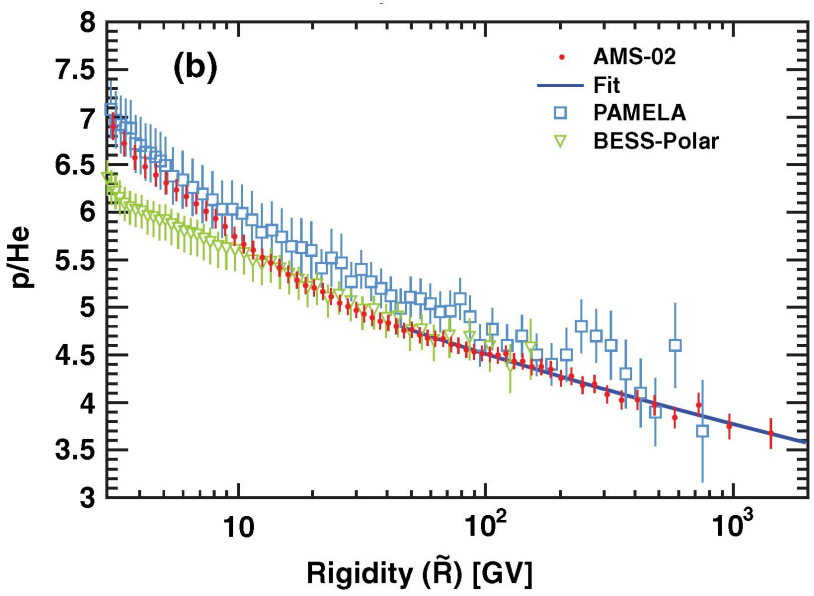
Krymsky 77, Blandford & Ostriker 78, Bell 78, Axford et al 78

Problems with simple test particle solution

1. Does not determine the normalization
   1. number of accelerated particles N_{CR} remains unknown
      1. So-called "injection problem": how and in what number are particles extracted from the thermal pool
   2. the level of turbulence driven by them remains unknown
      1. since DSA is a bootstrap process, acceleration rate, i.e. $p_{max}(t)$ depends on the scattering rate, that is on turbulence level
   3. particle backreaction on the shock structure is unknown
2. for high Mach numbers, typical for young SNRs ($M \sim 100$), $r = 4$, $q = 4$ →CR pressure diverges with p_{max}
3. high pressure of CR may totally change the shock structure, drive instabilities near the shock, change the CR confinement condition and the shock compression rate
   - in fact, it does!

- Bottom line: even the PL index is no longer determinate

New instruments make injection models testable



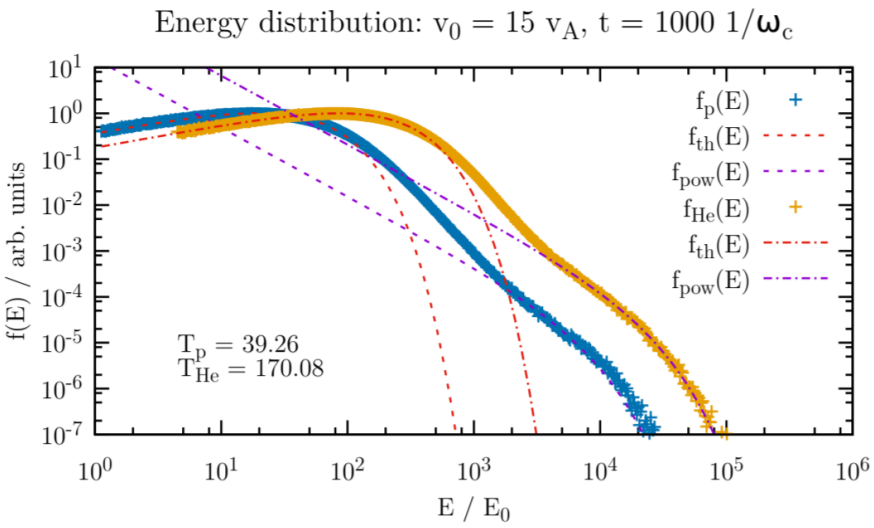
AMS-02 (2015) results along with earlier data

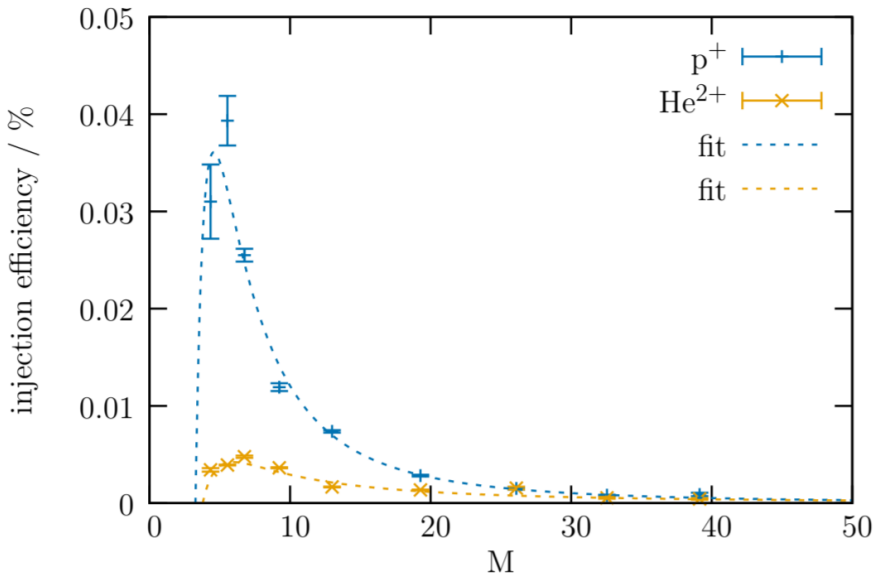
- strange result, at the first glimpse
- all elements with the same rigidity must have the same spectra under a steady state acceleration conditions

Key Observations and Disagreements with theory:

- Several instruments revealed deviation ≈ 0.1 in spectral index between He and p's (claimed inconsistent with DSA (e.g., Adriani et al 2011)
- DSA predicts a flat spectrum for the He/p ratio
- points to initial phase of acceleration where elemental similarity (rigidity dependence only) does not apply
- A/Z is the same for He and C

Validating Physical ideas by hybrid Simulations

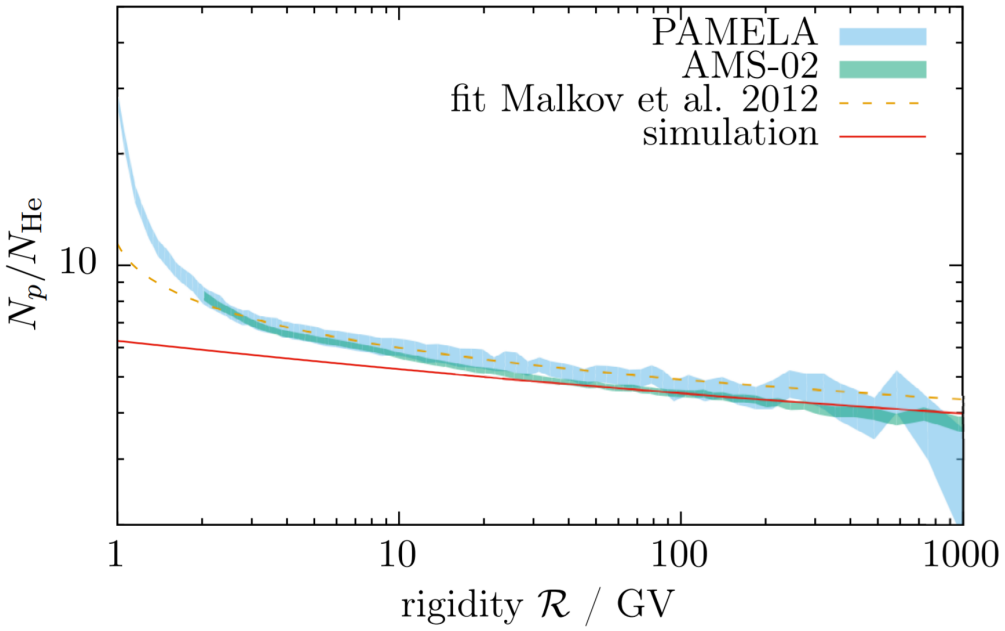




- 1D in configuration space, full velocity space simulations
  - shock propagates into ionized homogeneous plasma
- p and He are thermalized downstream according to Rankine-Hugoniot relations
- preferential injection of He into DSA for higher Mach numbers is evident
- injection dependence on Mach is close to theoretically predicted $\eta \sim M^{-1} \ln M$ (MM'98)

plots from A. Hanusch, T. Liseykina, MM, 2017

p/He ratio integrated over SNR life



p/He from A. Hanusch, T. Liseykina, MM, 2017

- p/He result automatically predicts the p/C ratio since the rest rigidity (A/Z) is the same for C and He

Some Conclusions

- the p/He ratio at $\mathcal{R} \gg 1$, is not affected by CR propagation, regardless the individual spectra
- telltale signs, intrinsic to the particle acceleration mechanism
- reproducible theoretically with no free parameters
- PIC and hybrid simulations confirm p and He injection scalings with Mach number Hanusch et al, ICRC 2017

Backreaction of accelerated CRs on shock

- DC equation with $u(x)$ to be determined self-consistently with $f(x,p)$:

$$u\frac{\partial f}{\partial x} + \kappa(p)\frac{\partial^2 f}{\partial x^2} = \frac{1}{3}\frac{du}{dx}p\frac{\partial f}{\partial p}, \tag{1}$$

- $f(x,p) = \langle f(x,\boldsymbol{p})\rangle$
- BC: $f \to 0$, $x \to \infty$; $f < \infty$, $f < \infty$, $x \to -\infty$
- CR diffusivity $\kappa(p)$ is of the Bohm type, $\kappa(p) = Kp^2/\sqrt{1+p^2}$ (p is normalized to mc, $\kappa \sim r_{\rm g}(p)$
- K depends on $\delta B/B$ of MHD turbulence that scatters the particles in pitch angle
- $K \sim mc^3/eB$ if $\delta B \sim B$.
- $x < 0$ $\quad f(x,p) = f_0(p) \equiv f(0,p), \;\; u \equiv u_2$
- $x > 0$: need to solve eq.(1) coupled with eq. for $u(x)$

Solving DC self-consistently with backreaction of CR

Introduce : $$P_{\rm CR}(x) = \frac{4\pi}{3} mc^2 \int_{p_0}^{p_1} \frac{p^4\,dp}{\sqrt{p^2+1}} f(p,x)$$

$$P_{\rm CR} + \rho u^2 = \rho_1 u_1^2, \qquad \rho u = \rho_1 u_1, \quad x > 0$$

$$u\frac{\partial f}{\partial x} + \kappa(p)\frac{\partial^2 f}{\partial x^2} = \frac{1}{3}\frac{du}{dx} p \frac{\partial f}{\partial p}, \qquad u\,(x) = u_1 - P_{CR}/\rho_1 u_1$$

$$r_{\rm s} \equiv \frac{u_0}{u_2} = \frac{\gamma+1}{\gamma - 1 + 2R^{\gamma+1}M^{-2}}, \qquad u_2 < u_0 \equiv u\,(x+0) < u_1 \tag{2}$$

precursor compression $R \equiv u_1/u_0$ and γ -the adiabatic index

- key substitution

$$f(x,p) = f_0(p)\exp\left[-\frac{q}{3\kappa}\Psi\right]$$

$$q(p) = -d\ln f_0/d\ln p, \qquad \Psi = \int_0^x u(x')dx'$$

Self-consistent solution of DC equation

- one-parameter (λ) family (MM '99)

$$f_0 = f_0\left(p_0\right)\left[1 + \frac{q\left(p_0\right)}{\lambda\kappa\left(p_0\right)} p_0^{-3/\lambda} \int_{p_0}^{p} \kappa\left(p'\right) p'^{3/\lambda-1} dp'\right]^{-\lambda}$$

- flow potential

$$\Psi\left(x\right) = \Psi_0^{-\lambda/(\lambda-1)}\left[\left(1-\lambda\right) u_0 x + \Psi_0\right]^{1/(1-\lambda)}$$

- λ=1/2 comes from the condition of pressure balance in the shock precursor $P_{CR} + \rho u^2 = const$.
- solution implicitly (trough Ψ_0) depends on p_1- maximum momentum (cut-off)
- tends to exact solution in the limit $p_1 \to \infty$ ($M = \infty$, as no thermal pressure), zeroth order term in $1/p_1$
- for this solution to exist $p_1 > 10^3$ (SNRs $p_1 > 10^6$)
- current hybrid simulations $p_1 \sim 1$ (e.g., Caprioli & Spitkovsky, 2017)

Integral Transform of DC: $u\frac{\partial f}{\partial x} + \kappa(p)\frac{\partial^2 f}{\partial x^2} = \frac{1}{3}\frac{du}{dx}p\frac{\partial f}{\partial p}$

- use self-similar solution to build a kernel of integral transform of DC

$$f(x,p) = f_0(p)\exp\left[-\frac{q}{3\kappa}\Psi\right], \quad \Psi = \int_0^x u(x')dx', \quad q(p) = -d\ln f_0/d\ln p$$

The integral transform is as follows (MM '97)

$$U(p) = \frac{1}{u_1}\int_{0-}^{\infty}\exp\left[-\frac{q(p)}{3\kappa(p)}\Psi\right]du(\Psi) \tag{3}$$

and it is related to $q(p)$ through

$$q(p) = \frac{d\ln U}{d\ln p} + \frac{3}{r_s R U(p)} + 3 \tag{4}$$

$U(p)$ yields both the flow profile and particle distribution. Using the linearity of equation $P_{\rm CR}(x) + \rho u^2 = \rho_1 u_1^2$ ($\rho u = const$),

- obtain an integral equation for U by applying transform (3)

$$U(t) = \frac{r_s - 1}{R r_s} + \frac{\nu}{K p_0}\int_{t_0}^{t_1} dt'\left[\frac{1}{\kappa(t')} + \frac{q(t')}{\kappa(t)q(t)}\right]^{-1}\frac{U(t_0)}{U(t')}\exp\left[-\frac{3}{R r_s}\int_{t_0}^{t'}\frac{dt''}{U(t'')}\right] \tag{5}$$

where $t = \ln p$, $t_{0,1} = \ln p_{0,1}$. Injection parameter ν

$$\nu = \frac{4\pi}{3}\frac{mc^2}{\rho_1 u_1^2}p_0^4 f_0(p_0) = K p_0\left(1 - R^{-1}\right)\left\{\int_{t_0}^{t_1}\kappa(t)dt\frac{U(t_0)}{U(t)}\exp\left[-\frac{3}{R r_s}\int_{t_0}^{t}\frac{dt'}{U(t')}\right]\right\}^{-1} \tag{6}$$

Approximate solution of Integral Equation

- re-scaling, remapping, simplifications..., obtain $U \to F, \quad p \to \tau$, $p_0 \to \epsilon \ll 1$, $p_1 \to \epsilon^{-1}$

$$F(\tau) = \int_{\epsilon}^{1/\epsilon} \frac{d\tau'}{\tau + \tau'} \frac{1}{\tau' F(\tau')} + A(R, r_s, \nu) \tag{7}$$

- expand in $\sqrt{\epsilon} \ll 1$: $F = F_0(\tau) + \ldots, \quad \tau + \epsilon \equiv y$:

$$F_0(y) = \int_0^{\infty} \frac{dy'}{y + y'} \frac{1}{y' F_0(y')}$$

- solution

$$F_0 = \sqrt{\pi / y}$$

- using the symmetry of eq.(7) $\tau \mapsto 1/\tau$, $F \mapsto \tau F$ $(A = 0)$
- using the branch points $\tau = \epsilon, \epsilon^{-1}$, restore the full solution

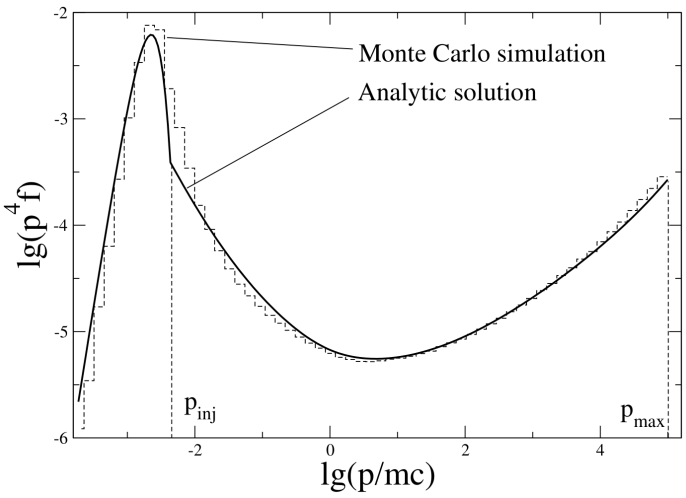
$$F = \sqrt{\frac{\pi}{(\tau + \epsilon)(1 + \epsilon\tau)}}$$

Nonlinear Spectrum

- returning to physical variables, $p \gg p_0$ (simplified spectrum)

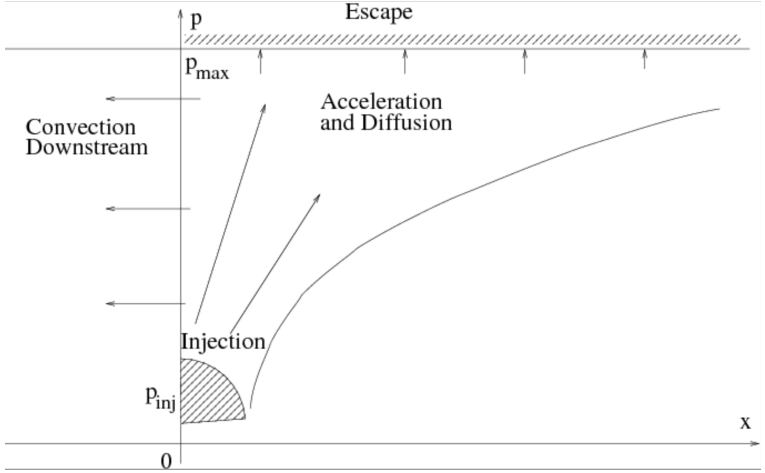
$$f_0(p) = \frac{C}{p^{7/2}} \sqrt{3q(p) + p/p_1}$$

- more accurately, from int. eq.:

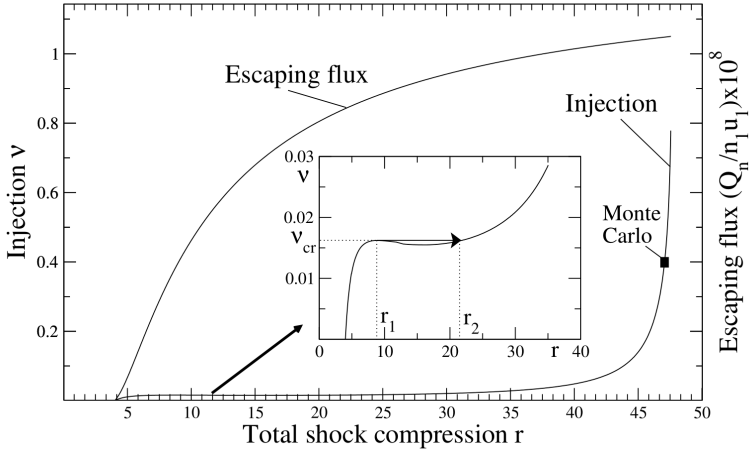


MC: Ellison & Berezhko '99
Anal. Sol.: MM '97

Phase space of acceleration

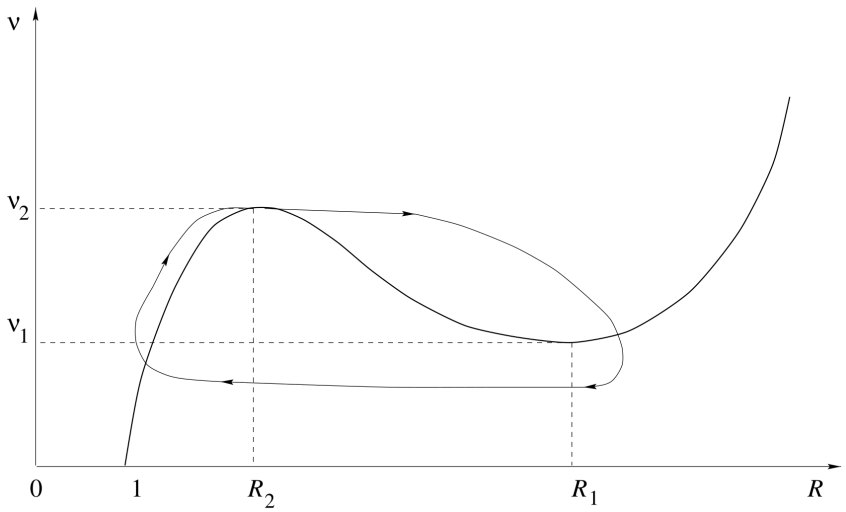


Escape flux and bifurcation of acceleration regime

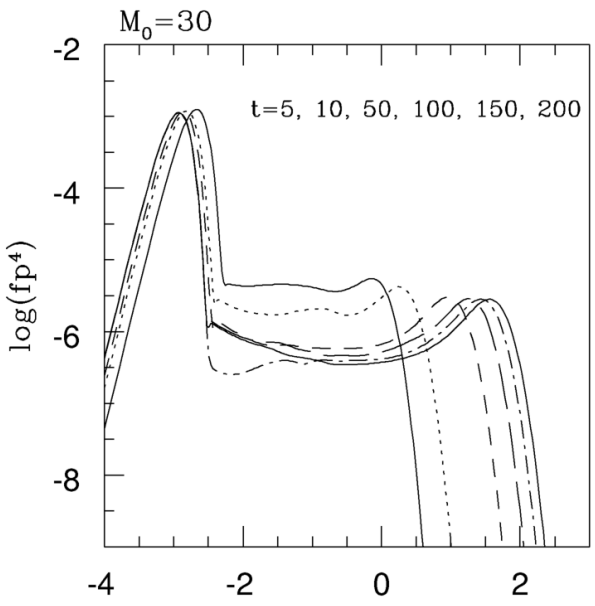


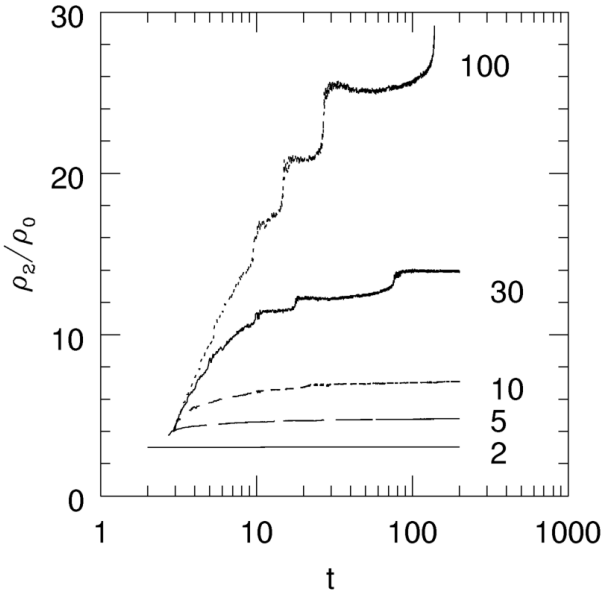
Limit Cycle Oscillations in CR Acc'n

- calculated bifurcation diagram
  (schematically from MM&L.Drury 2001)



- undergoes adiabatic deformation when $p_{max}(t)$ grows
- suggests hysteresis and limit-cycle oscillations in course of acceleration
- such LCO's have indeed been observed in numerical modeling of acceleration by Kang and Jones 2002

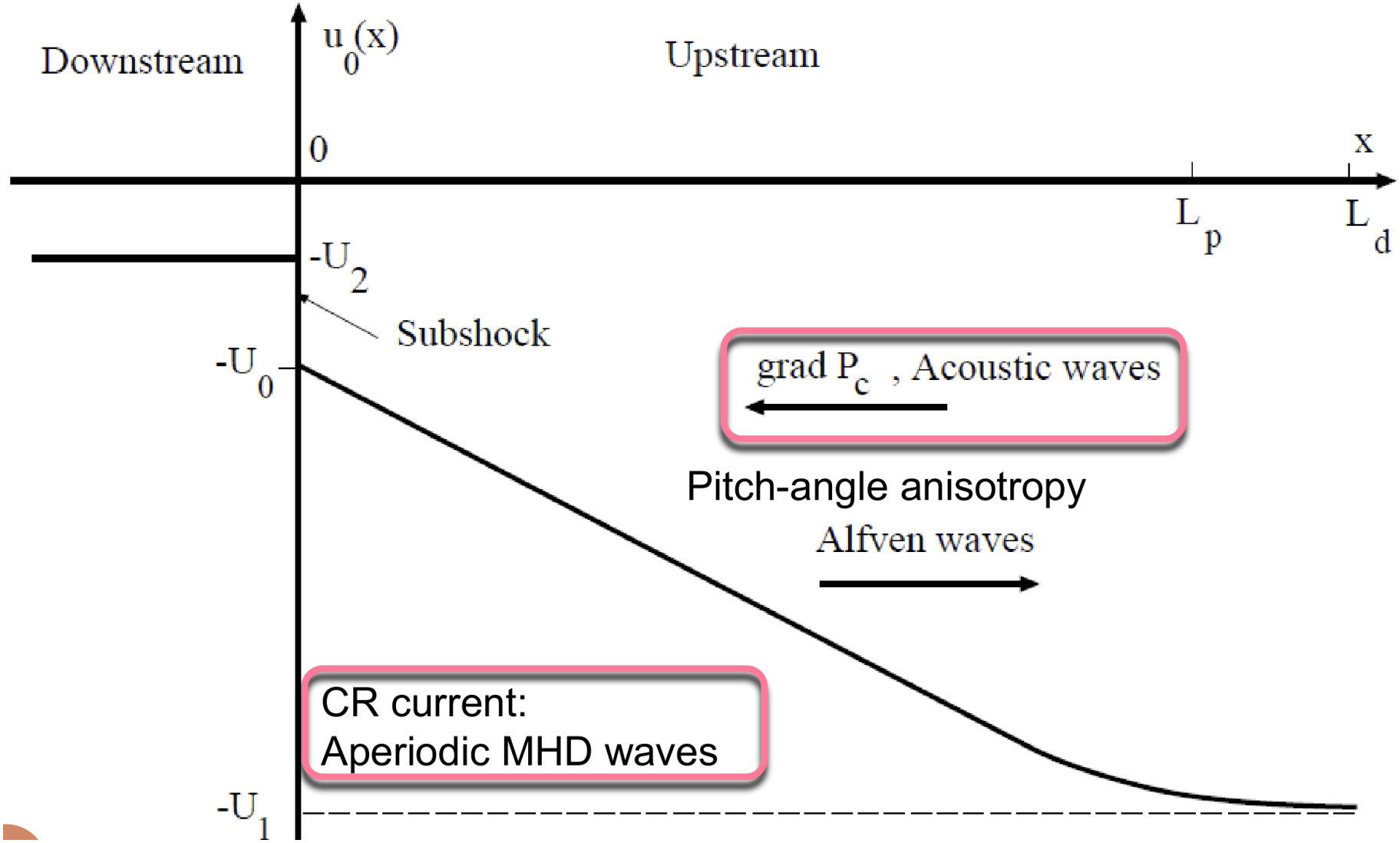




- long, $\kappa(p_{max})/U_{shock}$ CR precursor with velocity and CR pressure gradients
- CR modified shock constitutes an NL front propagating into weakly turbulent ISM
- the background ISM turbulence does not provide enough CR scattering to accelerate them to appreciable energy
- particles need to create waves by themselves
  - Bootstrap acceleration
- most relevant instabilities
  - resonant ion-cyclotron instability of CRs in shock precursor
  - nonresonant aperiodic (Bell's) instability driven by the return current of CRs
  - acoustic (Drury's) instability driven by the CR pressure gradient
- large scale magnetic field generation
- plasma heating in CR precursor (last two items poorly understood)

Microphysics of CR shock precursor

Instabilities, important for particle transport in CRP



Nonlinear waves in CR shock precursor

MHD Equations

$$\frac{d\rho}{dt} + \rho \frac{\partial}{\partial x} U_x = 0$$

$$\frac{d\mathbf{B}_\perp}{dt} = B_0 \frac{\partial \mathbf{U}_\perp}{\partial x} - \mathbf{B}_\perp \frac{\partial U_x}{\partial x}$$

$$\frac{dU_x}{dt} = -\frac{1}{\rho}\frac{\partial}{\partial x}\left(P_c + P_g + \frac{B_\perp^2}{8\pi}\right)$$

$$\frac{d\mathbf{U}_\perp}{dt} = \frac{B_0}{4\pi\rho}\frac{\partial \mathbf{B}_\perp}{\partial x} + \frac{J_c}{c\rho}\mathbf{n} \times \mathbf{B}_\perp$$

$$d/dt \equiv \partial/\partial t + U_x \partial/\partial x$$

Lagrangian variable

$$d\xi = \frac{\rho}{\rho_0}(dx - U_x dt)$$

MHD reduction

$$\frac{\partial^2 b}{\partial t^2} - C_A^2 \frac{\partial^2}{\partial \xi^2} \frac{\rho}{\rho_0} b = \frac{i}{c\rho_0} B_0 J_c \frac{\partial b}{\partial \xi}$$

$$\frac{\partial^2}{\partial t^2} \frac{\rho_0^2}{\rho} + \frac{\partial^2}{\partial \xi^2} \left(P_g + \frac{|b|^2}{8\pi} \rho^2 \right) = -\frac{\partial^2}{\partial \xi^2} P_c$$

Instability

$$b = \frac{1}{\rho} (B_y + iB_z) \qquad C_A^2 = \frac{B_0^2}{4\pi\rho_0}$$

$$P_c = \frac{4\pi}{3} mc^2 \int \frac{p^4}{\sqrt{1+p^2}} f\,dp \qquad J_c = eun_c = 4\pi eu \int p^2 f\,dp$$

Linear vs. nonlinear regime (sh. precursor)

Two-way balance

$$f = f_0(p)\exp\left[\frac{1}{\kappa(p)}\phi(x)\right], \quad x \geq 0$$

$$\boxed{\frac{\partial f}{\partial t} + u\frac{\partial f}{\partial x} - \kappa(p)\frac{\partial^2 f}{\partial x^2} = \frac{p}{3}\frac{\partial u}{\partial x}\frac{\partial f}{\partial p}}$$

Three-way balance

$$f = f_0(p)\exp\left[\frac{q(p)}{3\kappa(p)}\phi(x)\right], \quad x \geq 0$$

$$u = \partial\phi/\partial x \qquad q(p) = -\partial \ln f_0/\partial \ln p$$

9

q is well constrained: 3.5 < q < 4 (< q_sub, close to the sub-shock)

Instabilities (summary)

- Resonant, anisotropy-driven Alfven waves $k \sim r_g^{-1}(p)$

(use as a seed for the next two)

→Bell '78

✓ Non-resonant, CR-current driven (Bell's) instability

→Achterberg '83, Shapiro and Quest '98,
Bell and Lucek '01, 04, Reville et al 08, Bykov et al 08

→ *Bell '04 particularly complete characterization*

(short-scale, needs an inverse cascade or extreme amplification to confine particles)

✓ Hydrodynamic, CR-pressure-gradient-driven (Drury's) instability

→Drury 84, Drury and Falle 86, Zank et al 90, Kang et al 92…

Advantages:

- drive all wave numbers, $\gamma(k) \approx const$
- insensitive to CR distribution function
- staibilizes only nonlinearly (not quasi-linearly)
- long scales, much needed for particle confinement are naturally produced → Diamond and MM 07

Linear Theory: density vs. magnetic NR perturbations

$$\boxed{\begin{aligned} \omega^2 &= k^2 C_A^2 - 2\gamma_B k C_A \\ \omega^2 &= k^2 C_s^2 - 2i\gamma_D k C_S \end{aligned}}$$

$$\gamma_B = \sqrt{\frac{\pi}{\rho_0}} J_c / c$$

$$\gamma_D = -\frac{1}{2\rho_0 C_S} \frac{\partial \overline{P}_c}{\partial x}$$

Compare the growthrates:

$$\frac{\gamma_D}{\gamma_B} = \frac{C_A}{C_s} \frac{c^2}{3\omega_{ci}} \left\langle \frac{p^2}{\sqrt{1+p^2}} \frac{q}{3\kappa} \right\rangle$$

$$q/3 \to 1$$

In TP acceleration regime

$$\langle \cdot \rangle \equiv \frac{\int (\cdot) f_0(p) p^2 \exp(q\phi/3\kappa) dp}{\int f_0(p) p^2 \exp(q\phi/3\kappa) dp}$$

Comparing density (Drury) unstable mode with B-field (Bell) mode

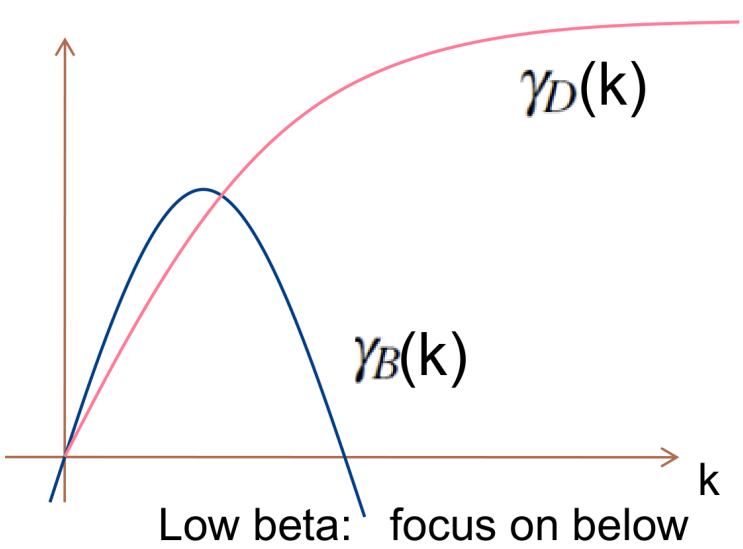
Bohm diffusion

$$\kappa = r_g(p)\, v/3$$

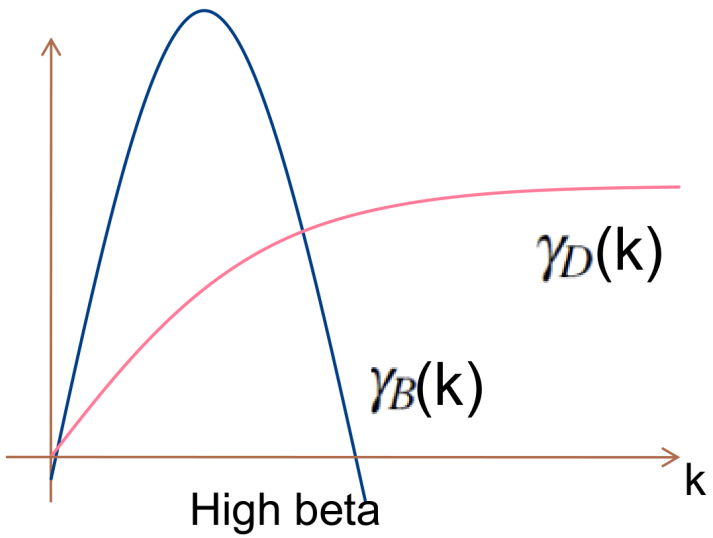
$$\boxed{\frac{\gamma_D}{\gamma_B} = \left\langle \frac{q}{3} \right\rangle \frac{C_A}{C_s}}$$

$$q/3 \to 1$$

In TP acceleration regime



Low beta: focus on below



High beta

Weakly nonlinear theory (acoustic)

Instability driver

CR linear response

$$\left(\frac{\partial}{\partial t} - c_s \frac{\partial}{\partial \xi}\right)\left(\frac{\partial}{\partial t} + c_s \frac{\partial}{\partial \xi}\right)\tilde{\rho} = -\frac{1}{\rho_0}\frac{\partial \bar{P}_c}{\partial \xi}\frac{\partial \tilde{\rho}}{\partial \xi} + \frac{\partial^2 \tilde{P}_c}{\partial \xi^2} + c_s^2 \frac{\gamma_g - 2}{2\rho_0}\frac{\partial^2 \tilde{\rho}^2}{\partial \xi^2} - \frac{\partial}{\partial \xi}\left(2\tilde{u}\frac{\partial \tilde{\rho}}{\partial t} + \tilde{\rho}\frac{\partial \tilde{u}}{\partial t}\right) - 2\mu\rho_0 \frac{\partial^3 \tilde{u}}{\partial \xi^3}$$

Use Lagrangian coordinate: $d\xi = dx - u_0(x)\,dt$

→ Burgers eq.

$$\frac{\partial \tilde{\rho}}{\partial t} - c_s \frac{\partial \tilde{\rho}}{\partial \xi} - \frac{\gamma_g + 1}{2\rho_0} c_s \tilde{\rho}\frac{\partial \tilde{\rho}}{\partial \xi} - \mu \frac{\partial^2 \tilde{\rho}}{\partial \xi^2} = \gamma \tilde{\rho}$$

where the acoustic instability growth rate is

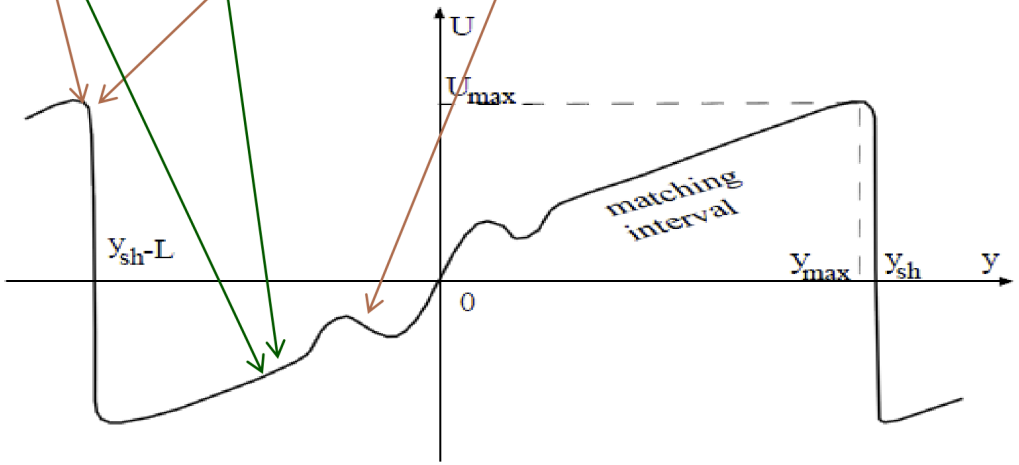
$$\gamma = -\frac{1}{2\rho_0 c_s}\frac{\partial \bar{P}_c}{\partial \xi} - \frac{\bar{q}}{18}\frac{mc^2 p_*}{\kappa_*}\frac{n_c(x)}{\rho_0}$$

Traveling wave solution driven by acoustic and cyclotron instabilities

$$\frac{\partial \hat{\rho}}{\partial t} + \hat{\rho}\frac{\partial \hat{\rho}}{\partial \zeta} - \gamma\hat{\rho} - \mu\frac{\partial^2 \hat{\rho}}{\partial \zeta^2} = Q\left(\zeta - vt\right)$$

Resonant Ion-cyclotron Instability (seed) term

balance

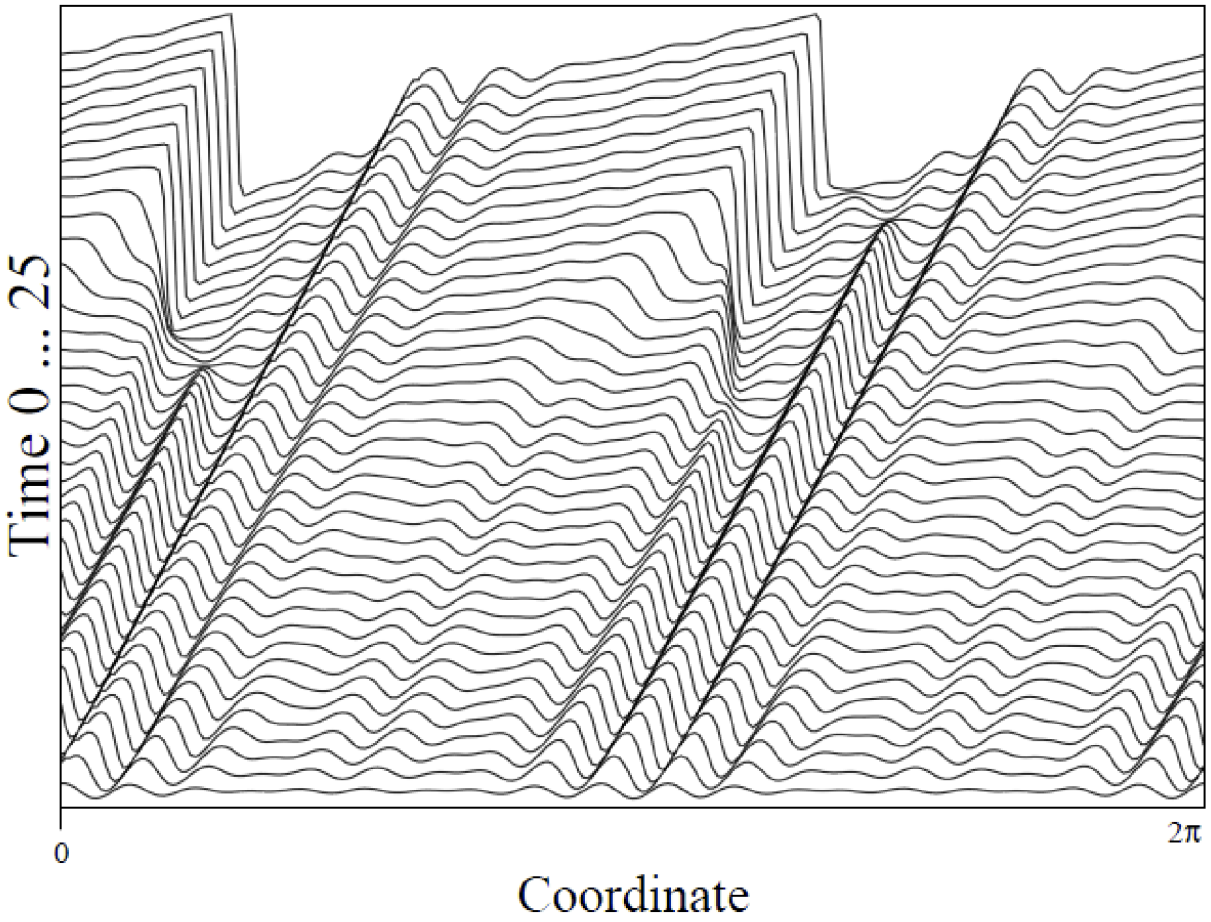


More general, 'magnetic' versions of this solution but with a cyclotron-unstable (no acoustic instability term) are also available

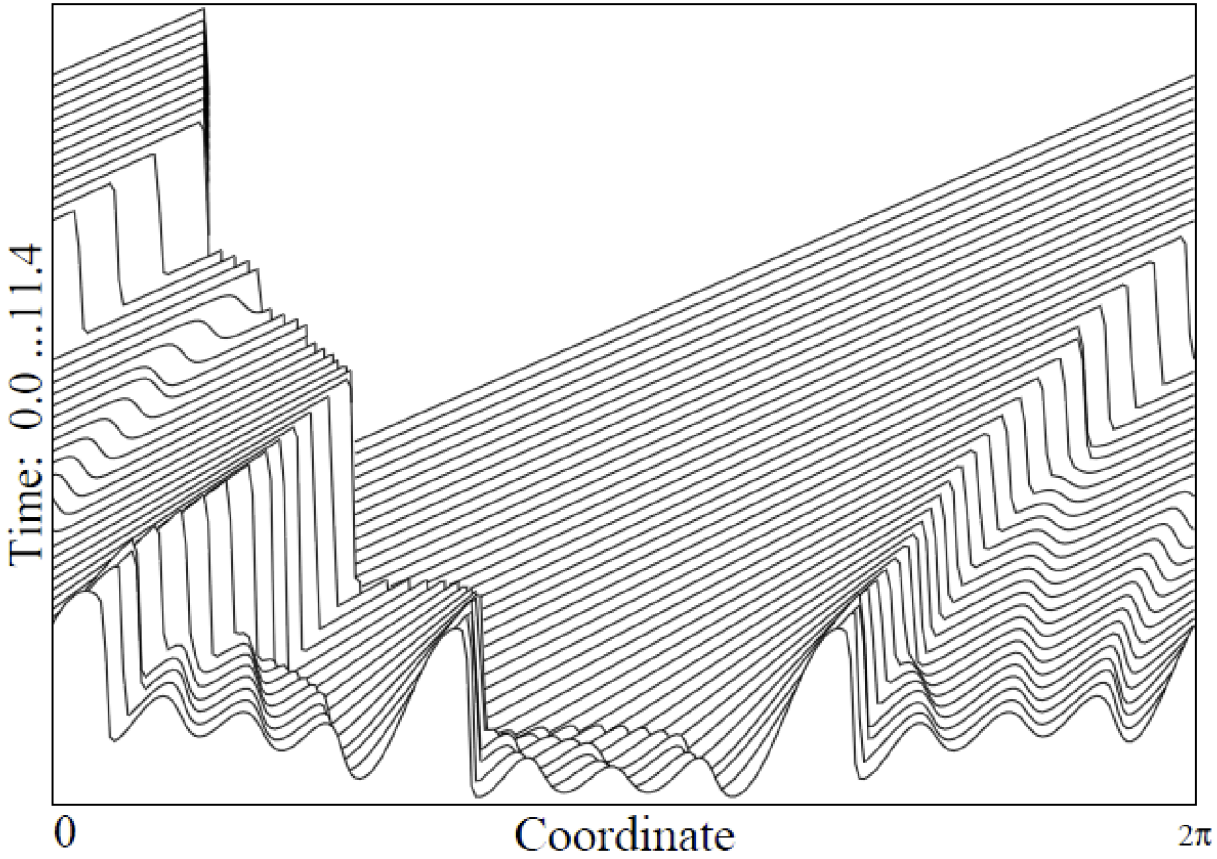
→ Kennel et al JETP Let. '88,

→ MM et al Phys. Fluids '90

Numerical verification of the traveling wave solution (acoustic instability +IC inst

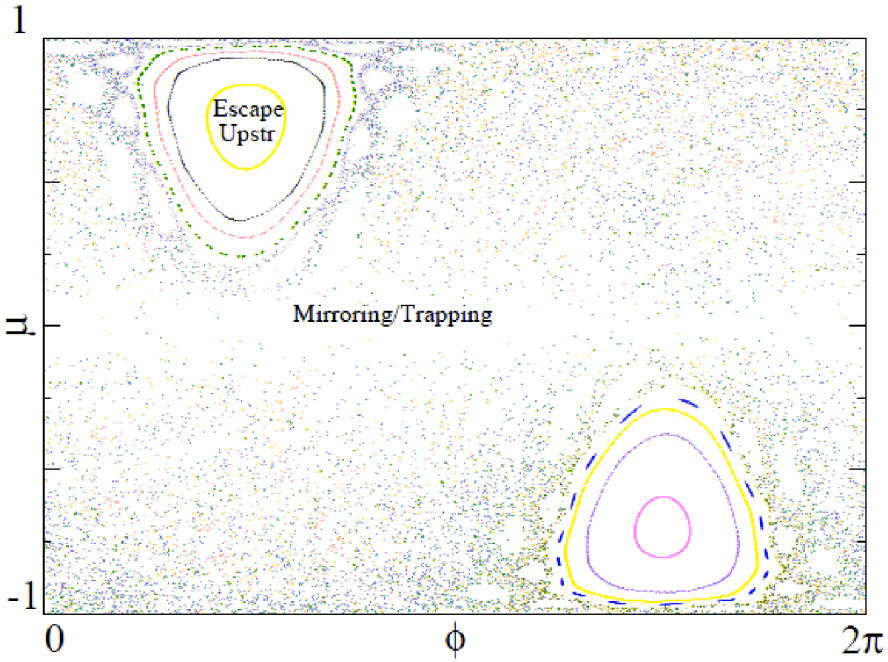


Numerical verification of the traveling wave solution (acoustic instability only)



Initial perturbation profile steepens into 3 relatively weak shocks
They merge to form one strong shock

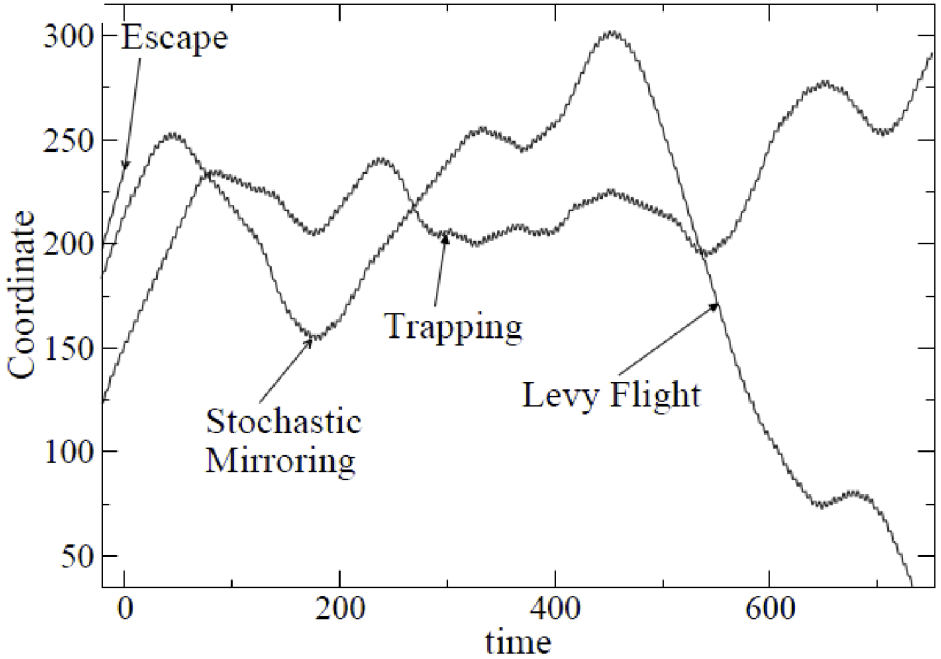
Particle dynamics in shock train



Pitch-angle/Gyro-phase
Poincare map
(Pitch-angle wrt shock normal, 45 deg here)

$r_g(p)/L \simeq 3$

Particle trajectories



Particle spectrum: change of confinement Regime leads to formation of a spectral break

For particles with momentum below the break at p_* the spectrum should be determined from nonlinear self-consistent solution of kinetic and HD equations.

Above the break at $p=p_*$ --no significant contribution of those particles to the CR pressure

Fermi '49 general spectral index

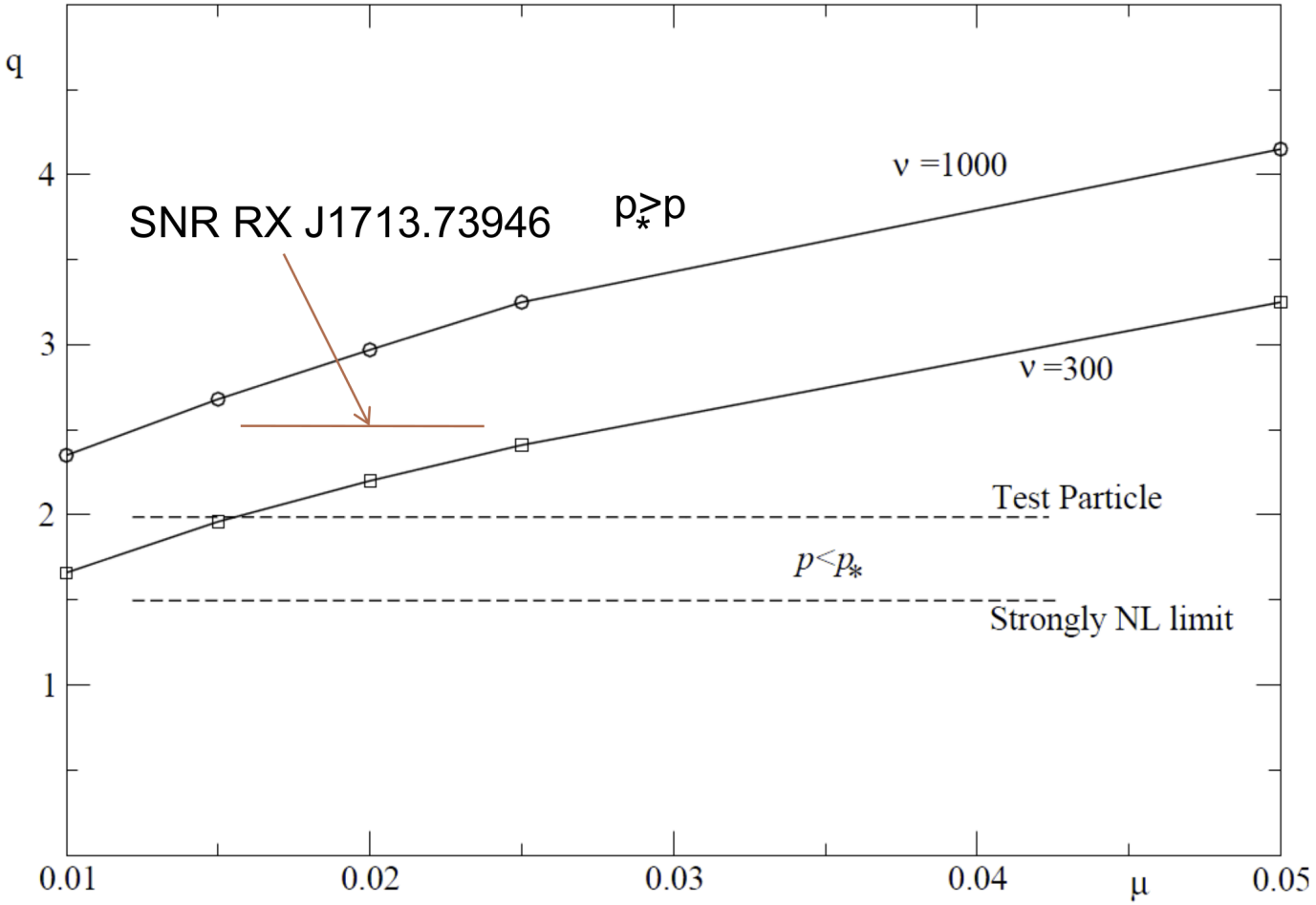
$$q = 3 + \tau_{acc}/\tau_{conf}$$

$$q_e = 3 + \frac{\ln\left[(U_+/U_0)\left(\mathscr{P}_L^2/\mathscr{P}_{tr}\right)\right]}{K(\vartheta)\ln(1/\mathscr{P}_L)}$$

$\mathscr{P}_{tr}$ Trapping probability

$\mathscr{P}_L$ Detrapping probability (Levy flight)

Softening of the spectrum



$$\nu = \frac{U_+\beta}{U_0\alpha_+} \simeq \frac{U_+}{U_0}\frac{\tau_L\tau_L^+}{\tau_{tr}^2} \sim \frac{c}{U_0} \gg 1$$

$$\mu \equiv \frac{\beta\kappa_0}{2\pi\eta^2 p_0 p_*(U_0 - U_2)^2}.$$

CR current driven Breathers

$$\frac{\partial^2}{\partial t^2}\frac{B}{\rho} - C_A^2\frac{\partial^2}{\partial \xi^2}\frac{B}{\rho_0} = \frac{i}{c\rho_0}B_0 J\frac{\partial}{\partial \xi}\frac{B}{\rho} \tag{8}$$

$$\frac{\partial^2}{\partial t^2}\frac{\rho_0^2}{\rho} + \frac{\partial^2}{\partial \xi^2}\frac{|B|^2}{8\pi} = 0, \tag{9}$$

$$B = B_y + iB_z \text{ and } C_A^2 = \frac{B_0^2}{4\pi\rho_0}.$$

- traveling wave solution

$$B = B_{\max} v(\zeta) e^{-i\omega t}, \qquad \rho = \rho(\zeta) \tag{10}$$

where $\zeta = \xi - Ct$, C is the (constant) propagation speed

$$\frac{\rho_0}{\rho} = 1 - \frac{|B|^2}{B_{\max}^2} \tag{11}$$

where $B_{\max}^2 \equiv 8\pi\rho_0 C^2$

Solution

$$\frac{\partial^2}{\partial\zeta^2}\left(a-|v|^2\right)v - iK\frac{\partial}{\partial\zeta}\left(1-|v|^2\right)v - \frac{\omega^2}{C^2}\left(1-|v|^2\right)v = 0. \tag{12}$$

with notation

$$K = \frac{B_0 J}{c\rho_0 C^2} - 2\frac{\omega}{C}, \quad a = 1 - 2\frac{B_0^2}{B_{max}^2} = 1 - \frac{C_A^2}{C^2}, \tag{13}$$

$C_A^2 = B_0^2/4\pi\rho_0$ The linear dispersion $v(\zeta) \propto e^{ik\zeta}$, $v \to 0$ in eq.(12):

$$\omega = kC \pm \sqrt{k^2 C_A^2 + B_0 Jk/c\rho_0}. \tag{14}$$

- Substituting $v(\zeta) = \sqrt{w}e^{i\Theta}$, obtain 1-st integral ($s = K\zeta/2$)

$$\left(\frac{dw}{ds}\right)^2 - \frac{w^2}{(3w-a)^2(a-w)^2}\sum_{n=0}^{4} C_n w^n = 0 \tag{15}$$

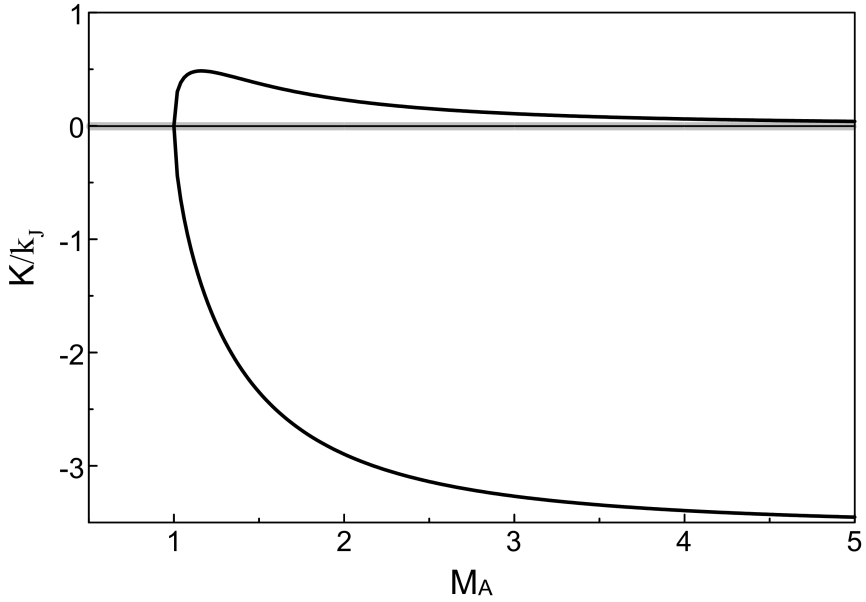
- passing through two singularities, obtain a closed-form solution (cumbersome)
- small-amplitude limit

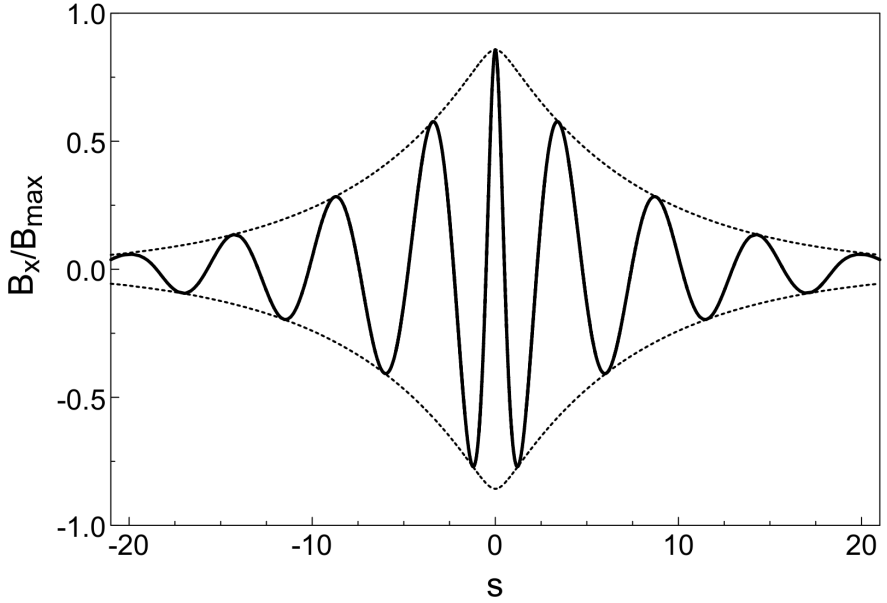
$$w(s) = w_0 \cosh^{-2}\left(\frac{\sqrt{C_0}}{2a^2}s\right)$$

NL dispersion relation

$$\omega = \frac{k_J C}{M_A^2 \left(1 \pm \sqrt{\left(1 - M_A^{-2}\right) / \left(1 + 1/8 M_A^2\right)}\right)},$$

- $k_J = 2\pi J / c B_0$
- Strong solitons with $M_A \equiv C / C_A \gg 1$





Conclusions and Outlook

- observational basis of the CR research is rapidly improving
- DSA theory accounts for most observations of the main (proton) CR energy spectrum
- however, some aspects need further studies
  - no consensus as to what maximum energy achievable in SNRs
    - estimates range from $10^{14} eV$ to $10^{16} eV$ and even higher (often backed off, though)
  - reason: lack of understanding of magnetic field generation of sufficiently large scale in CR shock precursor
- chemical composition remains partly controversial
  - observations are rapidly improving on e^+, He, C, N, O, Be, ...
  - largely by the new instrument AMS-02 on board ISS
  - theoretical work is ongoing
- CR shock precursor – excellent laboratory for NL waves and other NL phenomena

Спасибо за внимание!