Chimera States in Ensembles of Nonlocally Coupled Chaotic Systems. Control and Synchronization

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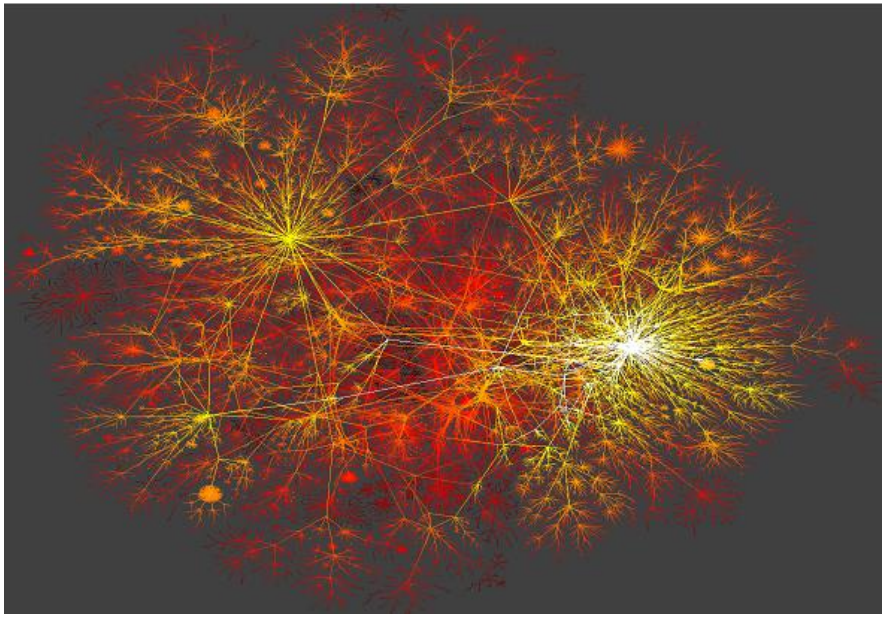
Saratov, Russia

Nizhny Novgorod,

February, 2018

CONTENTS

1. Introduction. Technological Complex Networks

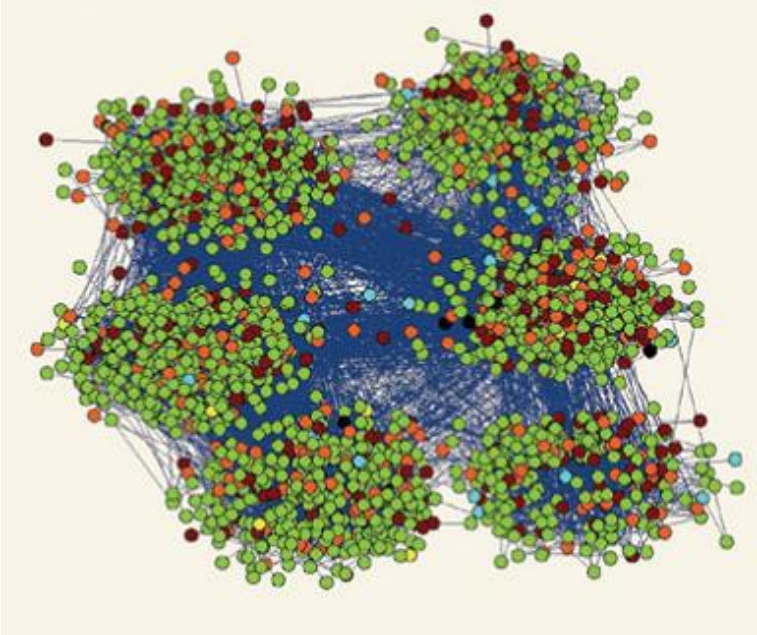


Traditionally the analysis of **the Internet structure** is made by means of ***traceroutes.*** That is to say, by exploring all the paths from a given point to all the possible destinations.

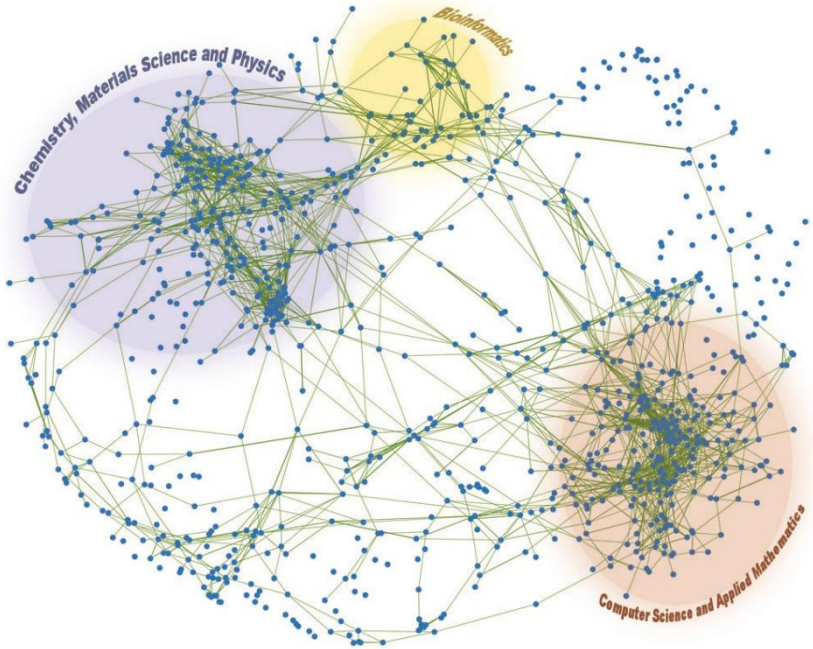
G. Caldarelli
CNR-INFM Centre SMC Dep. Physics University "Sapienza" Rome, Italy

Power grid network in the North America

Complex Social Networks

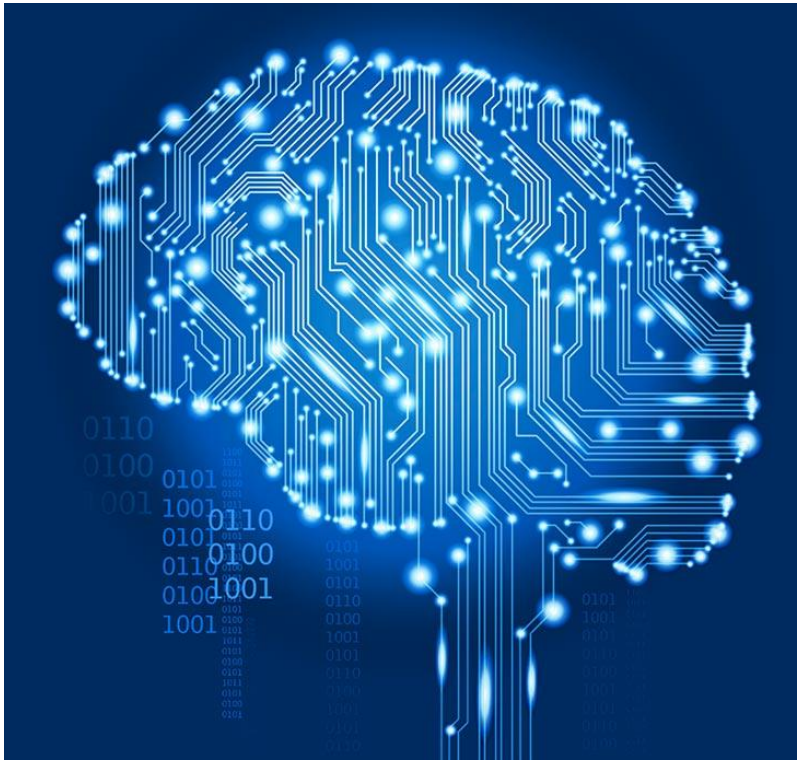


Friendship links in a school in the United States (from G. Caldarelli)



Knowledge Networks: networks to capture the collective knowledge of the communities of users of online resources, such as the scientific literature and digital libraries, Wikipedia, as well as social media such as Twitter and Instagram.

Complex Networks in Nature



Complex structure of neuron links in the human brain

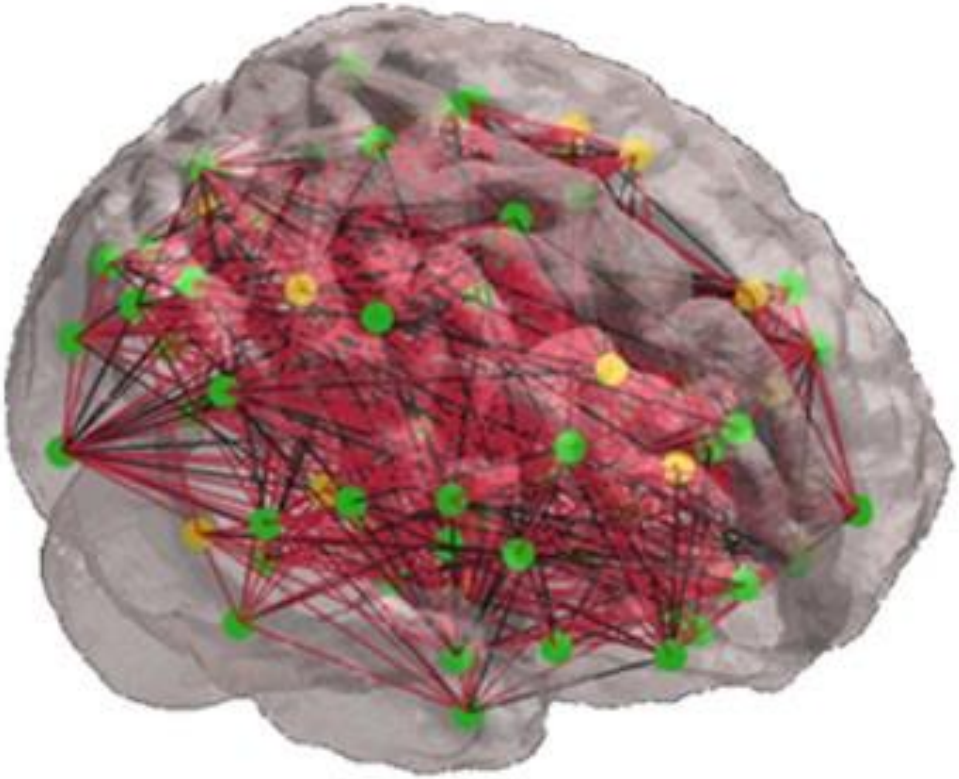
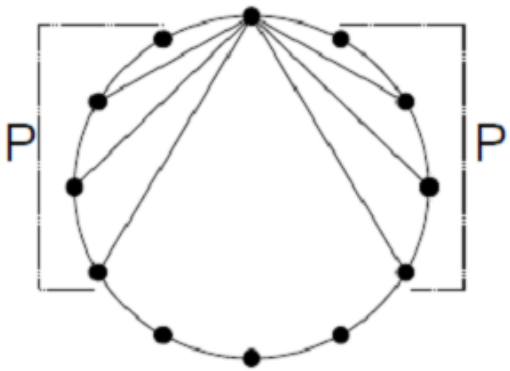


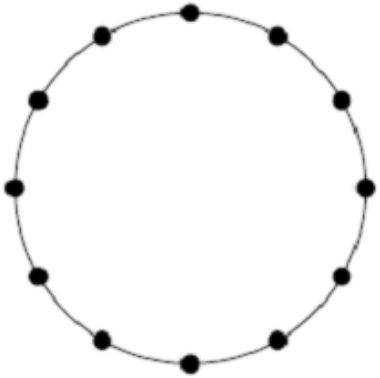
Figure 1. Agent based brain model. Each of 90 gray matter brain regions is represented by a node. Lines indicate functional connections, defined by correlated functional activity measured using fMRI. Nodes may either be on (green) or off (yellow) and connections indicate positive (red) or negative (black) correlations.

Numerical observation of chimera states:
Network of nonlocally coupled identical systems

$$\dot{x}_i(t) = f(x_i(t)) + \frac{\sigma}{2P}\sum_{j=i-P}^{i+P}[f(x_j(t)) - f(x_i(t))]$$

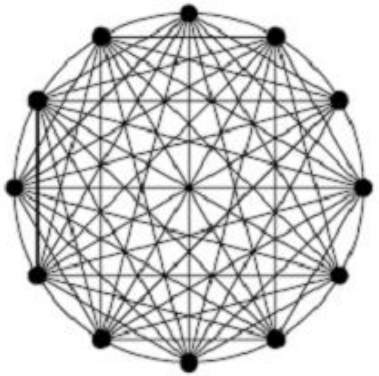


x_i are real dynamic variables ($i = 1, \ldots, N, N \gg 1$ and the index i is periodic $mod\ N$);
t denotes time;
σ is the coupling strength;
P specifies the number of neighbors in each direction coupled with the ith element;
r is the coupling radius (range), $r = P/N$;
$f(x)$ defines dynamics of individual element:



P=1

- Stuart-Landau harmonic self-sustained oscillators;
- discrete-time systems (maps);
- continuous-time chaotic models;
- FitzHugh-Nagumo neural systems;
- Van der Pol oscillators;
- quantum oscillator systems;
- …..



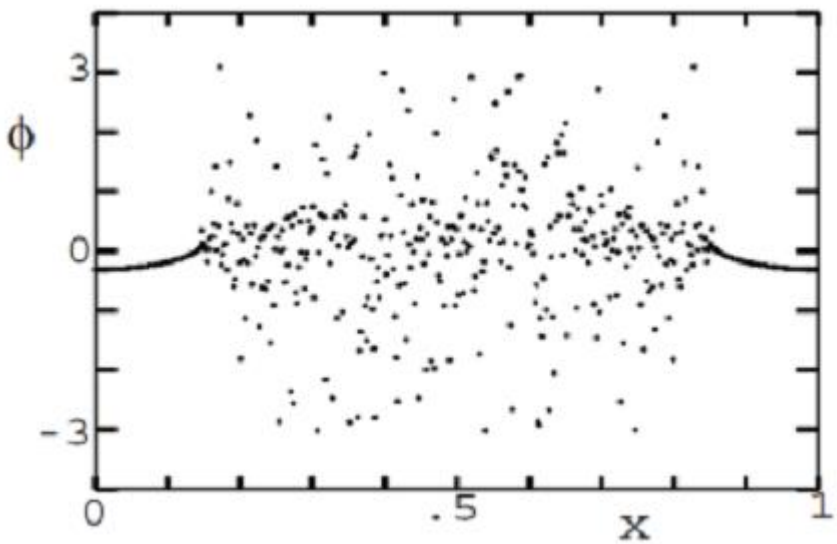
P=N/2

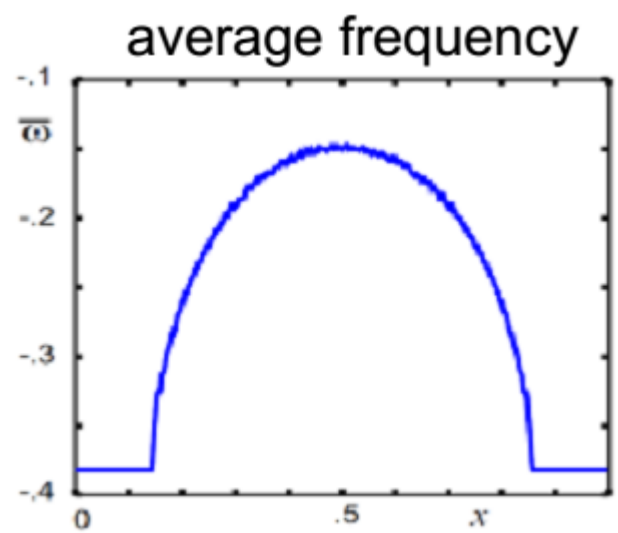
Chimera states in networks of nonlocally coupled identical oscillators

Kuramoto phase oscillator model:

$$\frac{\partial \psi}{\partial t} = \omega - \int_{-\pi}^{\pi} G(x-x')\sin[\psi(x,t)-\psi(x',t)+\alpha]dx' \quad \text{with} \quad \underbrace{G_{\exp}(x) = \frac{\kappa}{2}e^{-\kappa|x|}}_{\text{Exponential coupling function}}$$

Snapshot of chimera state





Coherent domains of periodic in-phase oscillations coexist with incoherent domains, characterized by a chaotic behavior in time and in space.

Y. Kuramoto and D. Battogtokh, Nonlinear Phenom. in Complex Syst. **5,** 380 (2002).

VOLUME 93, NUMBER 17 PHYSICAL REVIEW LETTERS week ending 22 OCTOBER 2004

Chimera States for Coupled Oscillators

Daniel M. Abrams* and Steven H. Strogatz†
Department of Theoretical and Applied Mechanics, Cornell University, 212 Kimball Hall, Ithaca, New York 14853-1503, USA

A chimera state was defined as a **spatio-temporal pattern** in which an array of identical oscillators is split into **coexisting regions** of **coherent** (phase and frequency locked) and **incoherent** (drifting) oscillations.

"In Greek mythology, ***the chimera*** was a fire-breathing monster having a lion's head, a goat's body, and a serpent's tail.
Today the word refers to anything composed of incongruent parts, or anything that seems fantastic."

Chimera of Arezzo

2. Basic models of chaotic systems with different types of chaotic attractors

It has been shown that chimera states can be obtained only in networks of chaotic systems with **non-hyperbolic attractors** and cannot be found in networks of chaotic systems with **hyperbolic** (**singular-hyperbolic**) attractors.

N. Semenova, A. Zakharova, E. Schöll, V. S. Anishchenko, Europhys. Lett. 112 (2015) 40002.

Discrete-time system

with a non-hyperbolic attractor:

Henon map

$$x_{n+1} = 1 - \alpha x_n^2 + y_n$$
$$y_{n+1} = \beta x_n$$

It describes the properties of chaotic attractors in the Poincare section for the spiral chaos systems: the Rössler oscillator, the Anishchenko-Astakhov oscillator etc.

Discrete-time system

with a singular-hyperbolic attractor:

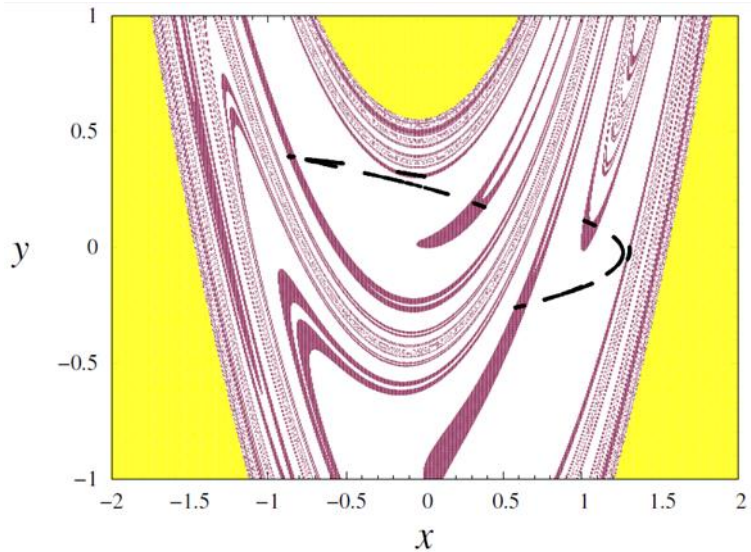
Lozi map

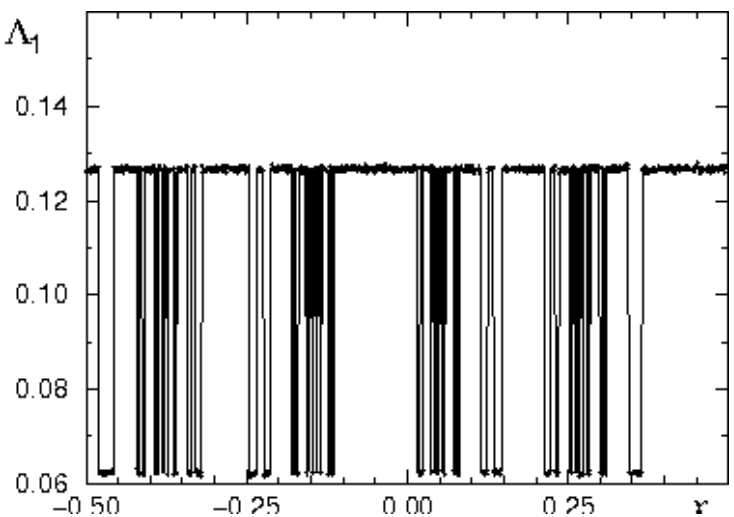
$$x_{n+1} = 1 - \alpha |x_n| + y_n$$
$$y_{n+1} = \beta x_n$$

It represents the properties of the Lorenz-type attractors in the Poincare section.

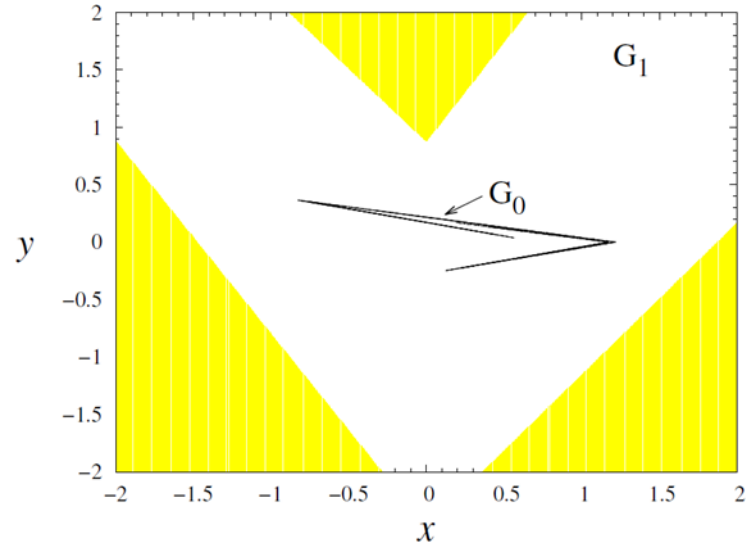
Peculiarities of chaotic attractors

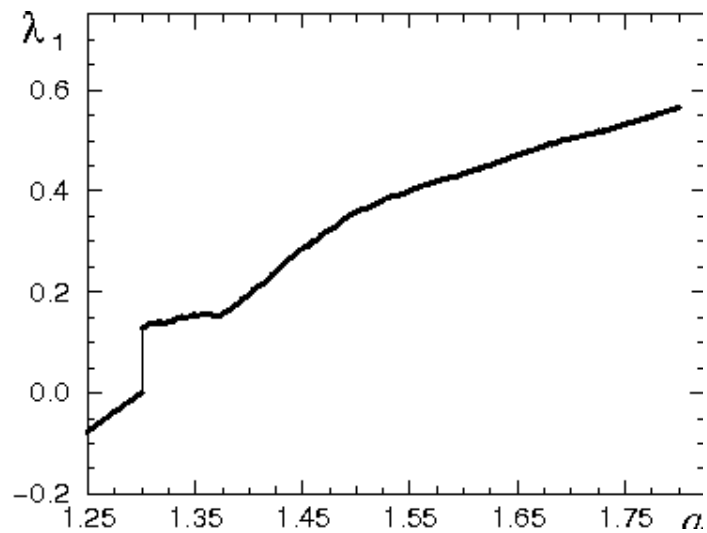
Henon map



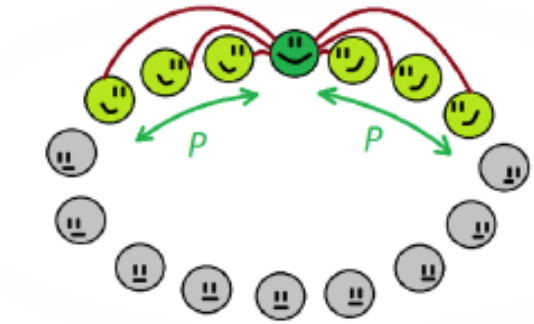


Lozi map





Ensembles of nonlocally coupled Henon and Lozi maps

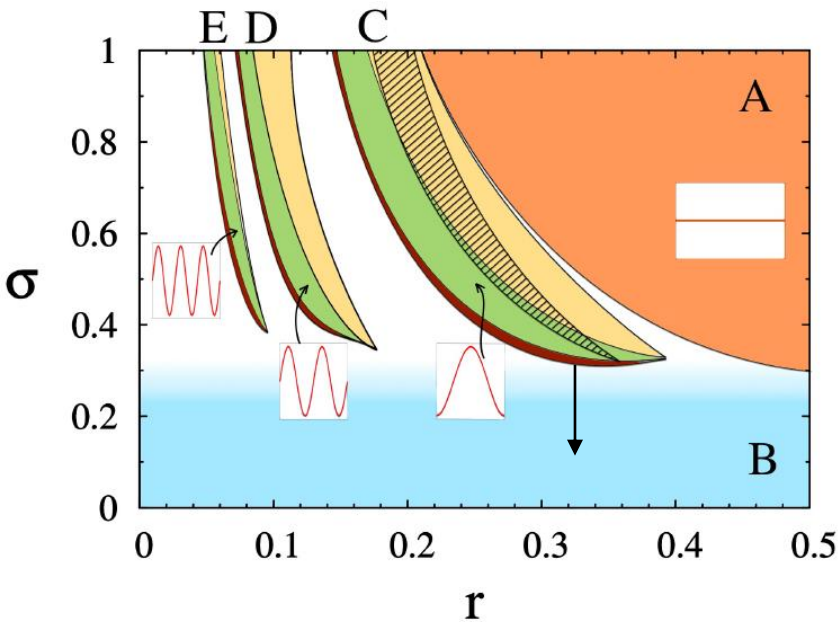


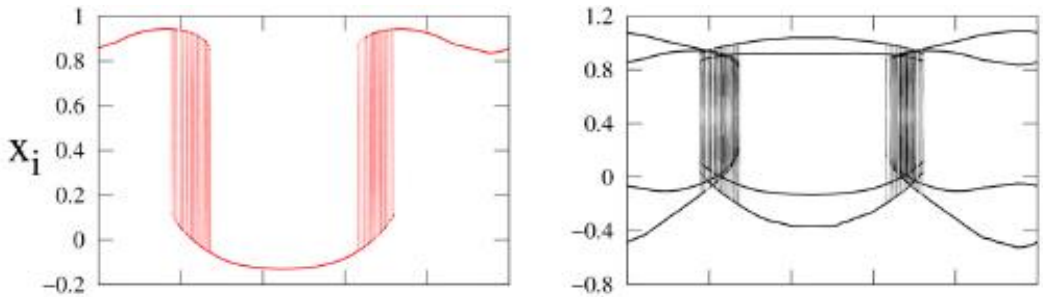
$$x_i^{t+1} = f(x_i^t, y_i^t) + \frac{\sigma}{2P} \sum_{j=i-P}^{i+P} \left[f(x_j^t, y_j^t) - f(x_i^t, y_i^t) \right],$$

$$y_i^{t+1} = \beta x_i^t .$$

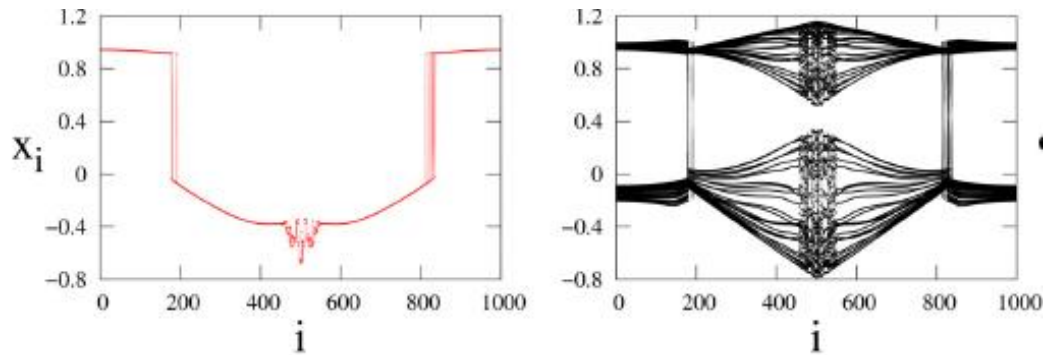
- x_i, y_i are real dynamical variables ($i = 1,2, \ldots, N$, $N = 1000$ is the number of the network elements, the index i is periodic *mod N),*
- t denotes the discrete time,
- $f(x_i , y_i) = 1 - \alpha(x_i)^2 + y_i$ (Henon map) and $f(x_i , y_i) = 1 - \alpha|x_i| + y_i$ (Lozi map),
- σ is the coupling strength,
- P specifies the number of neighbors on the left and right of the ith element,
- α =1.4, β = 0.3 are the parameters of the ensemble elements, which correspond to the chaotic dynamics in an uncoupled map,
- r is the coupling radius (range), $r = P/N$,
- Initial conditions are randomly distributed in the interval [-0.5; 0.5].

Phase and amplitude chimera states in the ensemble of nonlocally coupled Henon maps





Phase chimera for σ = 0.296

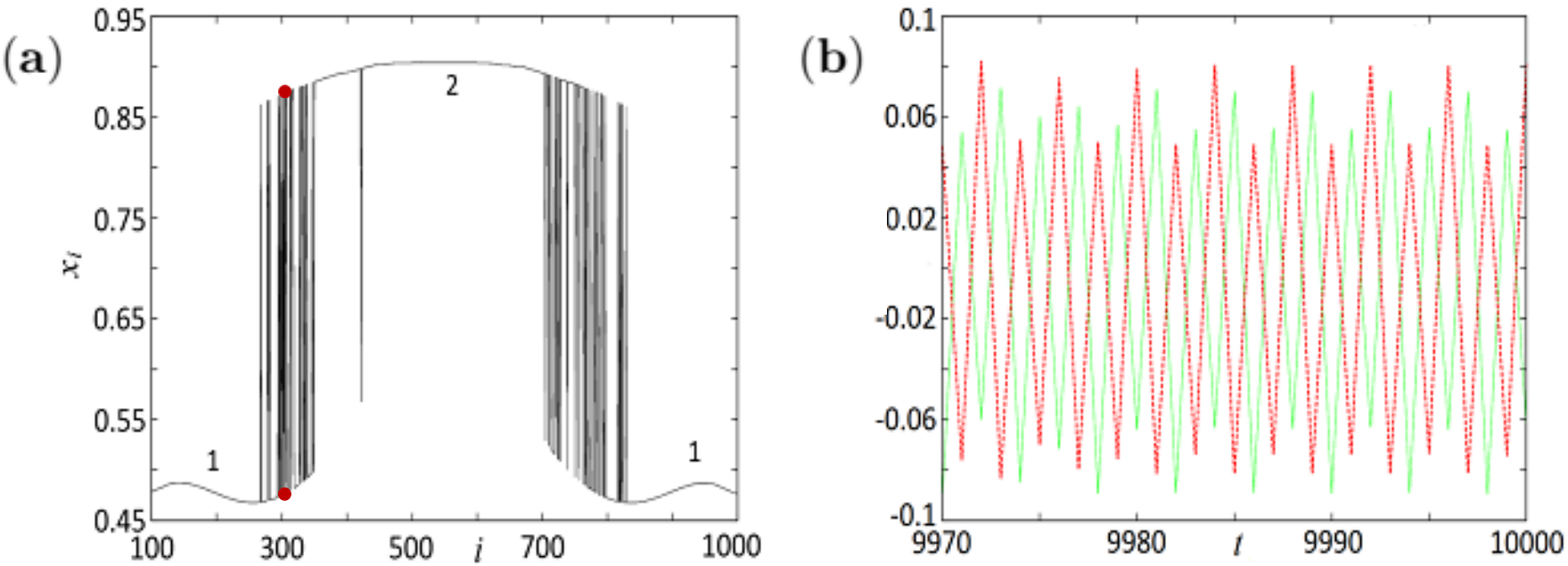


Phase and amplitude chimeras for σ = 0.26

I. Omelchenko, Y. Maistrenko, P. Hövel, and E. Schöll, Phys. Rev. Lett. **106**, 234102 (2011).

N.I. Semenova, E.V. Rybalova, G.I. Strelkova, V.S. Anishchenko, Regular and Chaotic Dynamics **22**(2), 148-162 (2017).

Phase chimera



Elements in regions 1 and 2 demonstrate **weakly chaotic oscillations which are close to the 2-cycle and shifted in phase by one iteration.**

Phase chimera : **random switchings** between the in-phase (region 1) and anti-phase (region 2) oscillations.

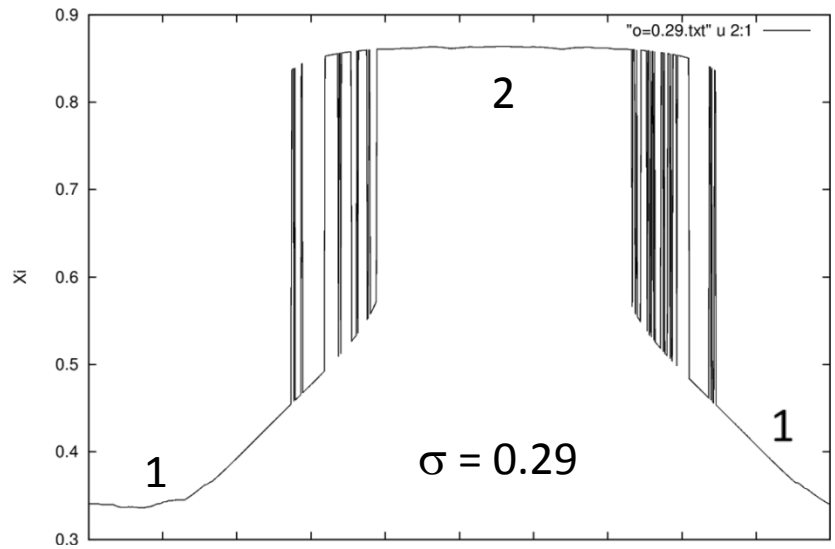
S.A. Bogomolov, A.V. Slepnev, G.I. Strelkova, E. Schöll, V.S. Anishchenko, Commun. Nonlinear Sci. Numer. Simul. **43,** 25 (2017).

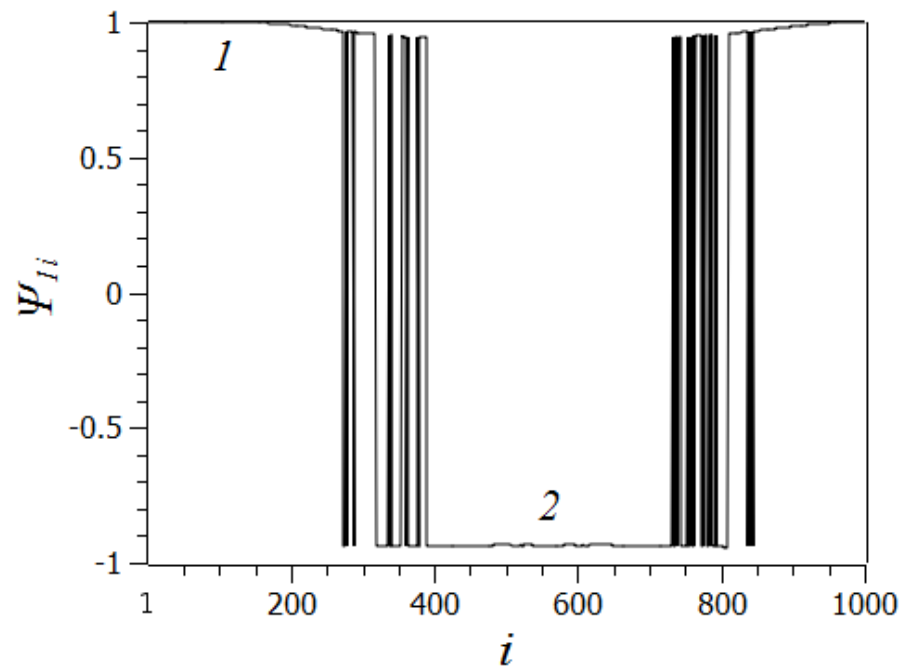
Cross-correlation coefficient in the phase chimera state

$$\Psi_{1,i} = \frac{\langle \tilde{x}_1(t)\tilde{x}_i(t)\rangle}{\sqrt{\langle \tilde{x}_1^2(t)\rangle\langle \tilde{x}_i^2(t)\rangle}}, \quad i = 2, 3, \ldots, N$$

$$\tilde{x}(t) = x(t) - \langle x(t)\rangle$$

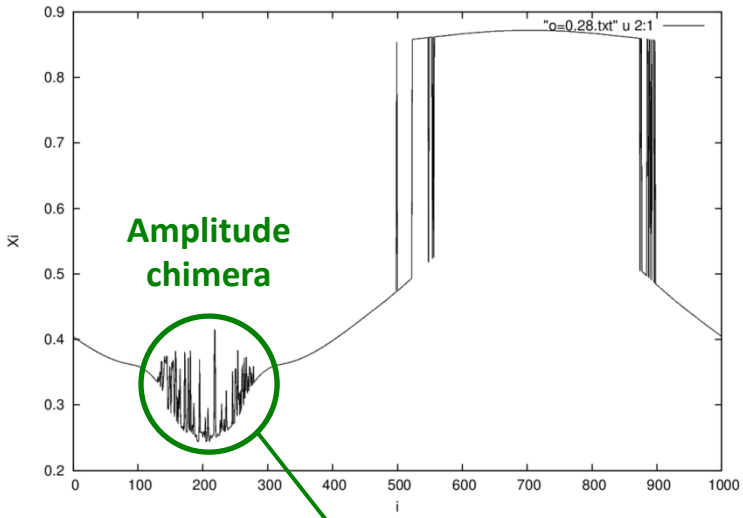
is a fluctuation around the average value.
The brackets mean time averaging.



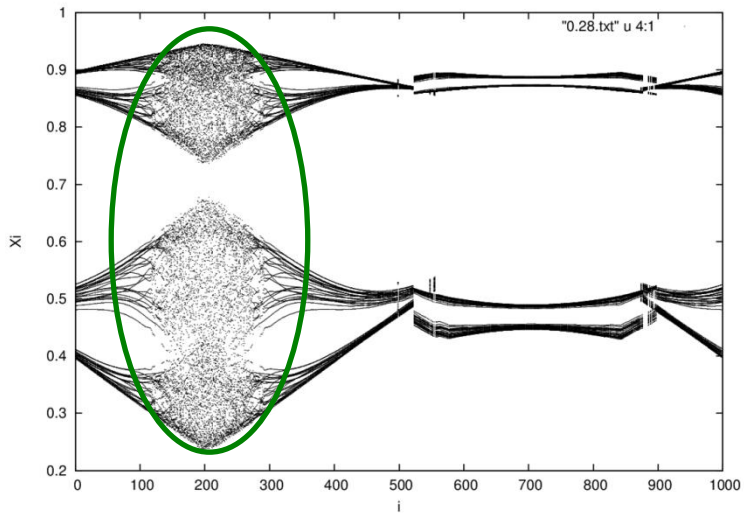


The CCC in the regime of phase chimera changes between either +1 or -1 and characterizes random switchings between the in-phase (+1) and anti-phase (-1) oscillations.

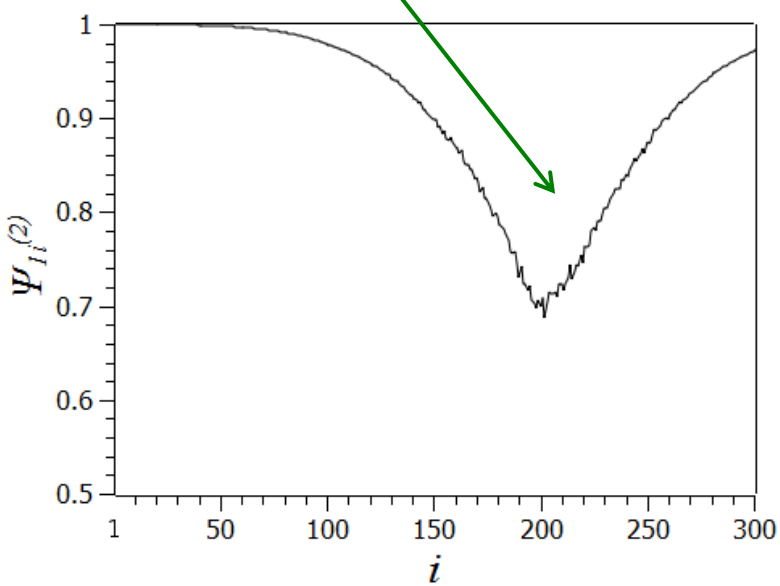
Amplitude chimera



Snapshot



Space-time profile

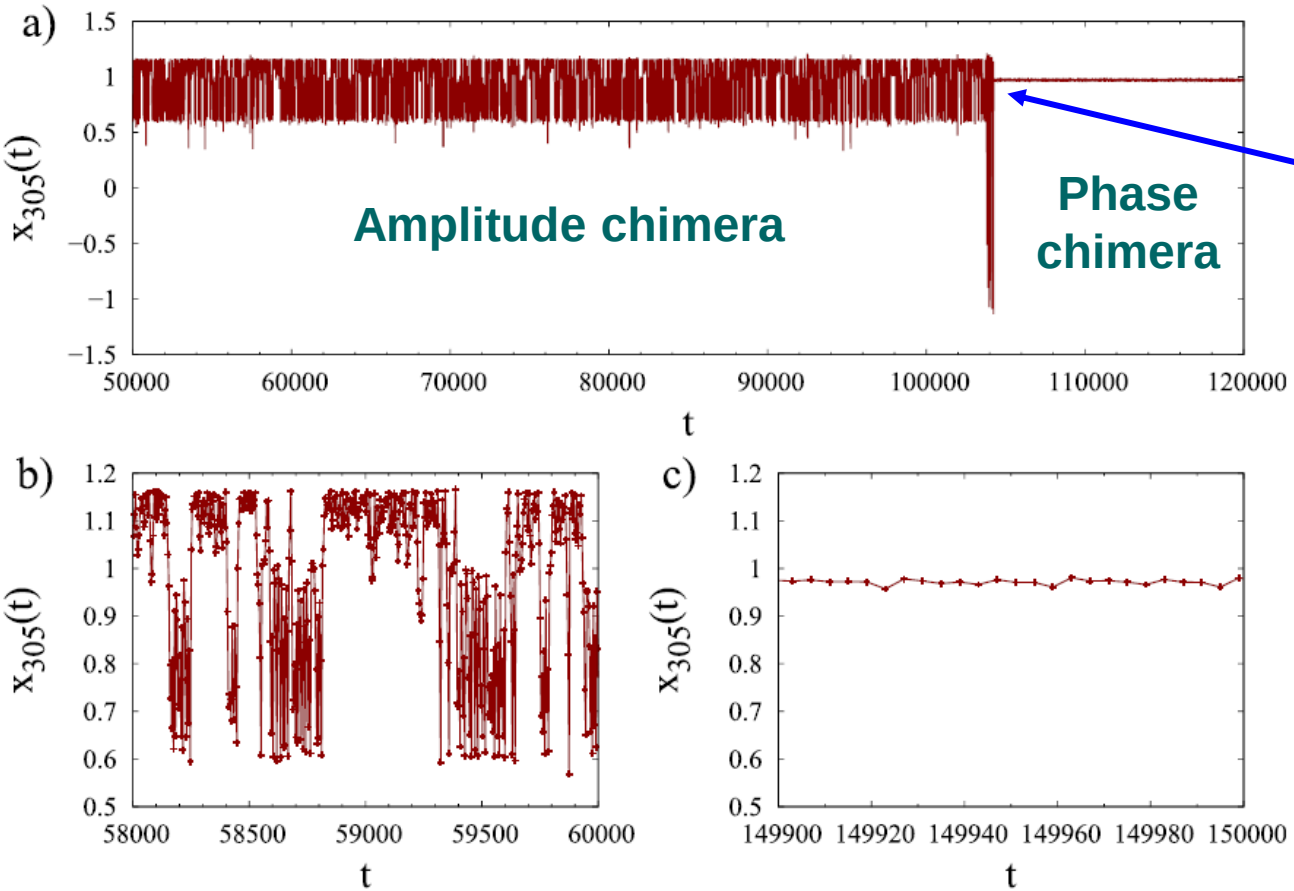


Cross-correlation

Amplitude chimera state appears when the cluster of elements ($120 < i < 290$) demonstrates developed chaotic behavior.

S.A. Bogomolov, A.V. Slepnev, G.I. Strelkova, E. Schöll, V.S. Anishchenko, Commun. Nonlinear Sci. Numer. Simul. **43,** 25 (2017).

Temporal switchings of oscillations of the network elements in an amplitude chimera. Finite lifetime of the amplitude chimera

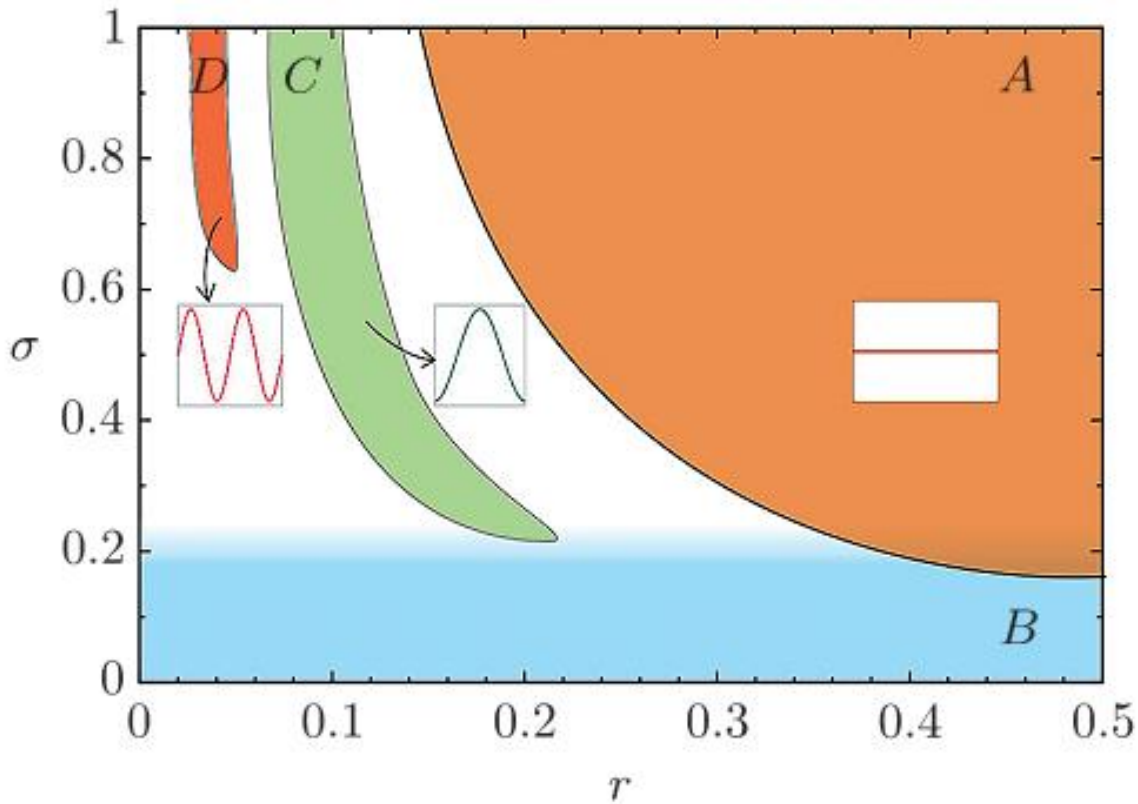


$\sigma = 0.304$,
$r = 0.32$

Amplitude chimera **Phase chimera**

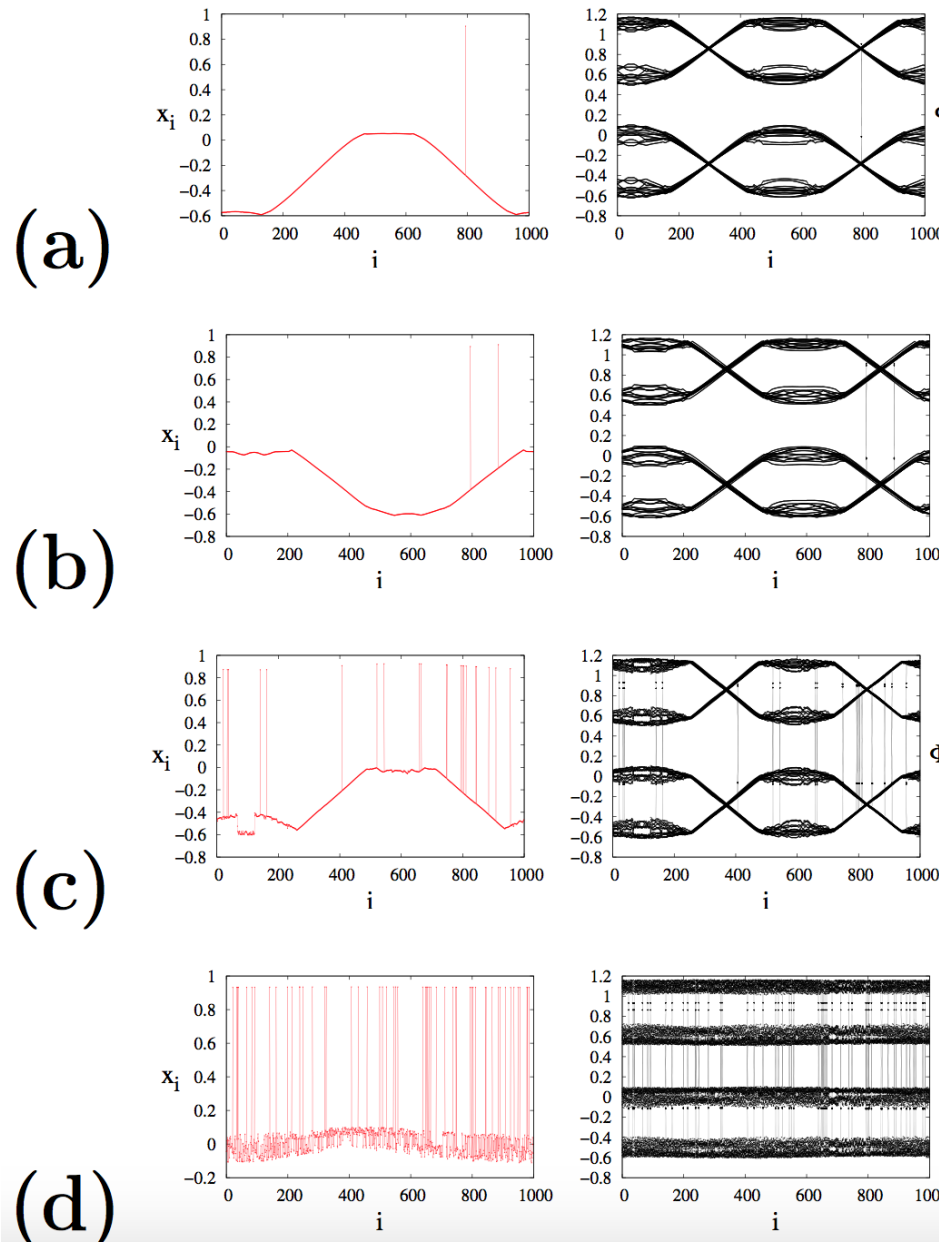
N.I. Semenova, G.I. Strelkova, V.S. Anishchenko, and A. Zakharova, Temporal intermittency and the lifetime of chimera states in ensembles of nonlocally coupled chaotic oscillators. CHAOS **27**, 061102 (2017).

Dynamics of the ensemble of nonlocally coupled Lozi maps



N.I. Semenova, E.V. Rybalova, G.I. Strelkova, V.S. Anishchenko, The European Phys. J. Special Topics **226**, 1857 (2017).

Transition to incoherence through solitary states in the network of Lozi maps for *r* = 0.193



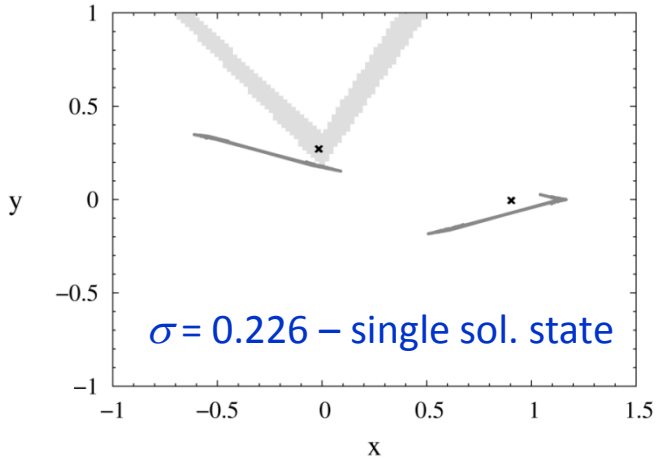
σ = 0.226

σ = 0.219

σ = 0.193

σ = 0.165

Mechanism of solitary state emergence in the ring of Lozi maps

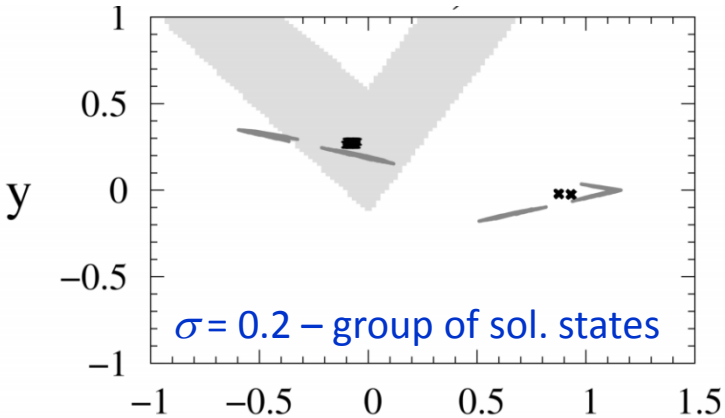


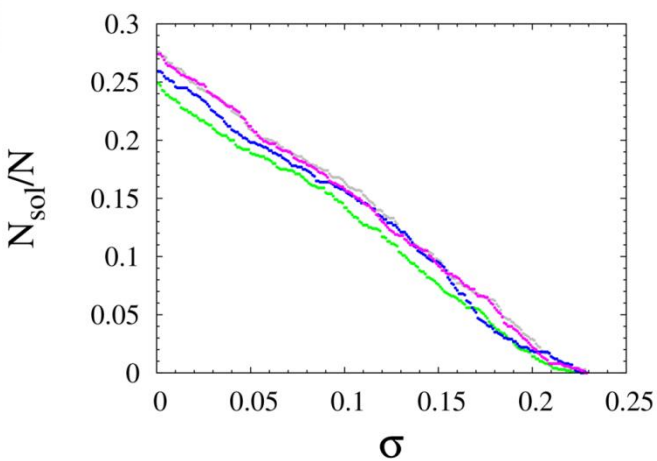
$$x_i^{t+1} = f(x_i^t, y_i^t) + \frac{\sigma}{2P} \sum_{j=i-P}^{i+P} [f(x_j^t, y_j^t) - f(x_i^t, y_i^t)],$$

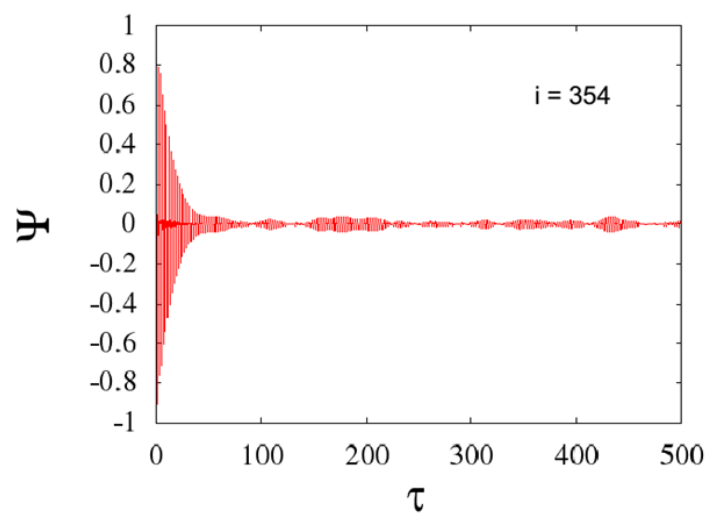
$$y_i^{t+1} = \beta x_i^t.$$

$$x_i^{t+1} = (1-\sigma) f(x_i^t, y_i^t) + \frac{\sigma}{2P} \sum_{j=i-P}^{i+P} f(x_j^t, y_j^t),$$

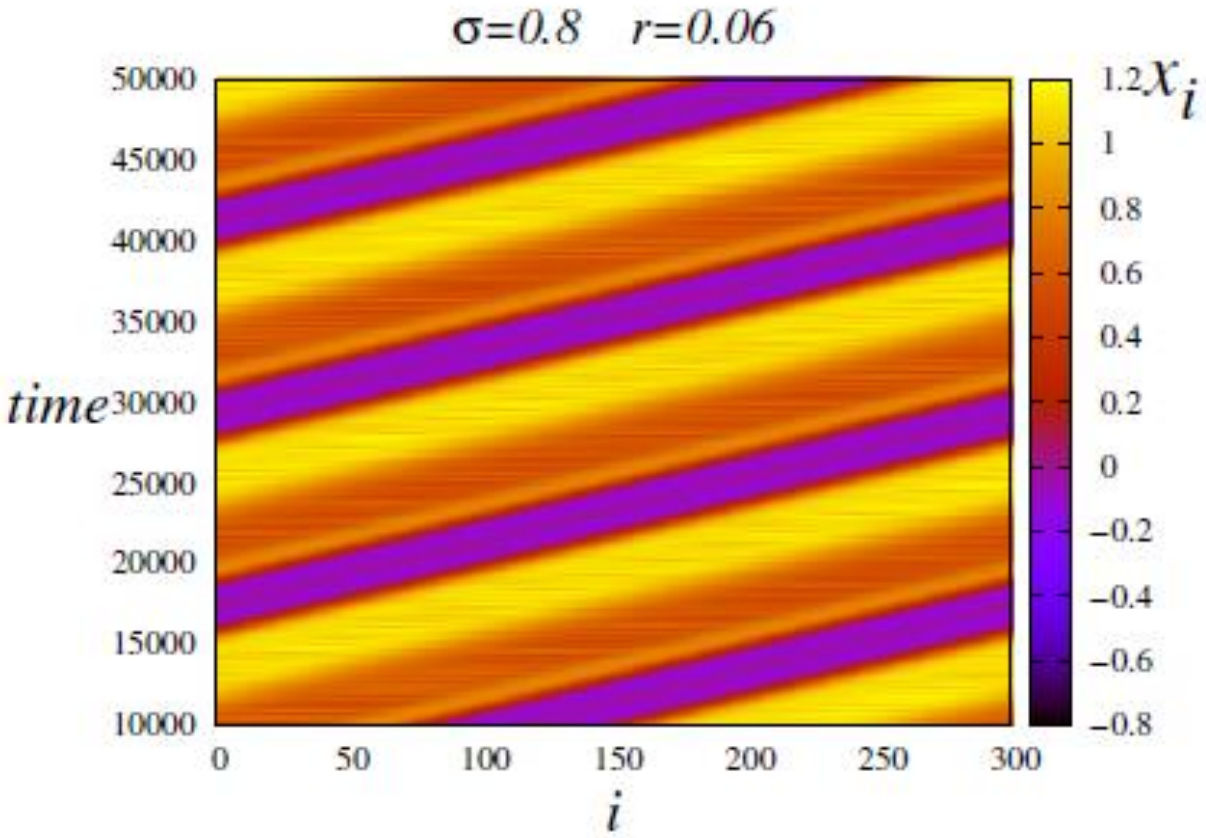
$$y_i^{t+1} = \beta x_i^t.$$



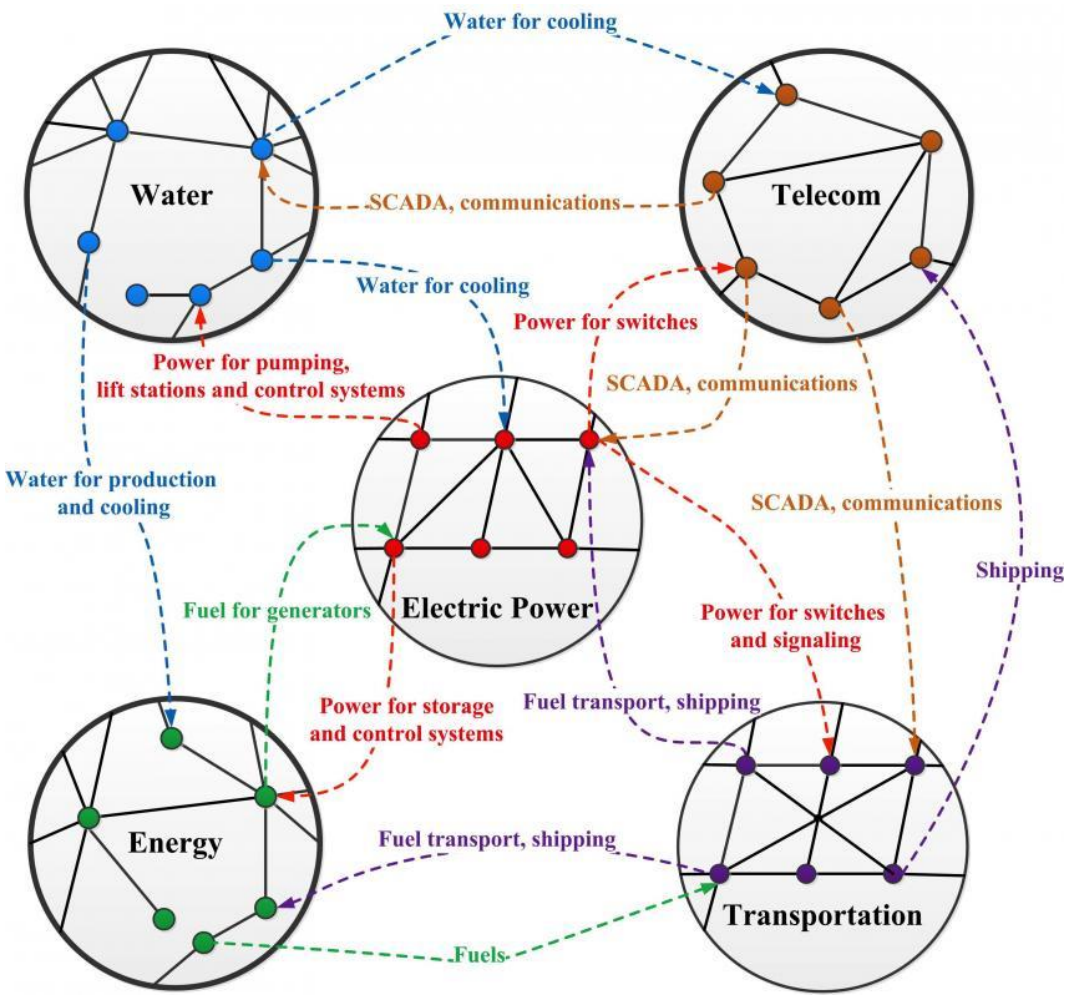




Travelling waves in the ensemble of Lozi maps



Network of networks



In nature and technology, it is typical the connection (and thus, the creation of a network) between different networks with different inner and external couplings.

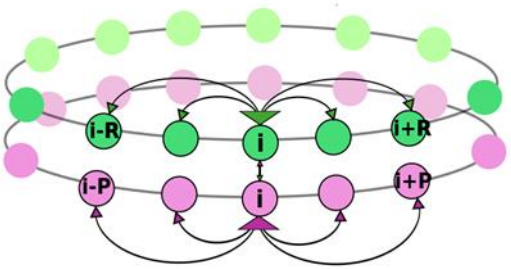
Two coupled rings of Henon and Lozi maps: Simple model of a network of networks

$$x_i^{t+1} = f_i^t + \frac{\sigma_1}{2P} \sum_{j=i-P}^{i+P} \left[f_j^t - f_i^t \right] + \gamma F_i^t,$$

$$y_i^{t+1} = \beta x_i^t, \qquad f_i^t = 1 - \alpha (x_i)^2 + y_i$$

$$u_i^{t+1} = g_i^t + \frac{\sigma_2}{2R} \sum_{j=i-R}^{i+R} \left[g_j^t - g_i^t \right] + \gamma G_i^t,$$

$$v_i^{t+1} = \beta u_i^t. \qquad g_i^t = 1 - \alpha |x_i| + y_i$$



$i = 1, 2, \ldots, N = 1000$ and γ is the coupling coefficient between the two rings.

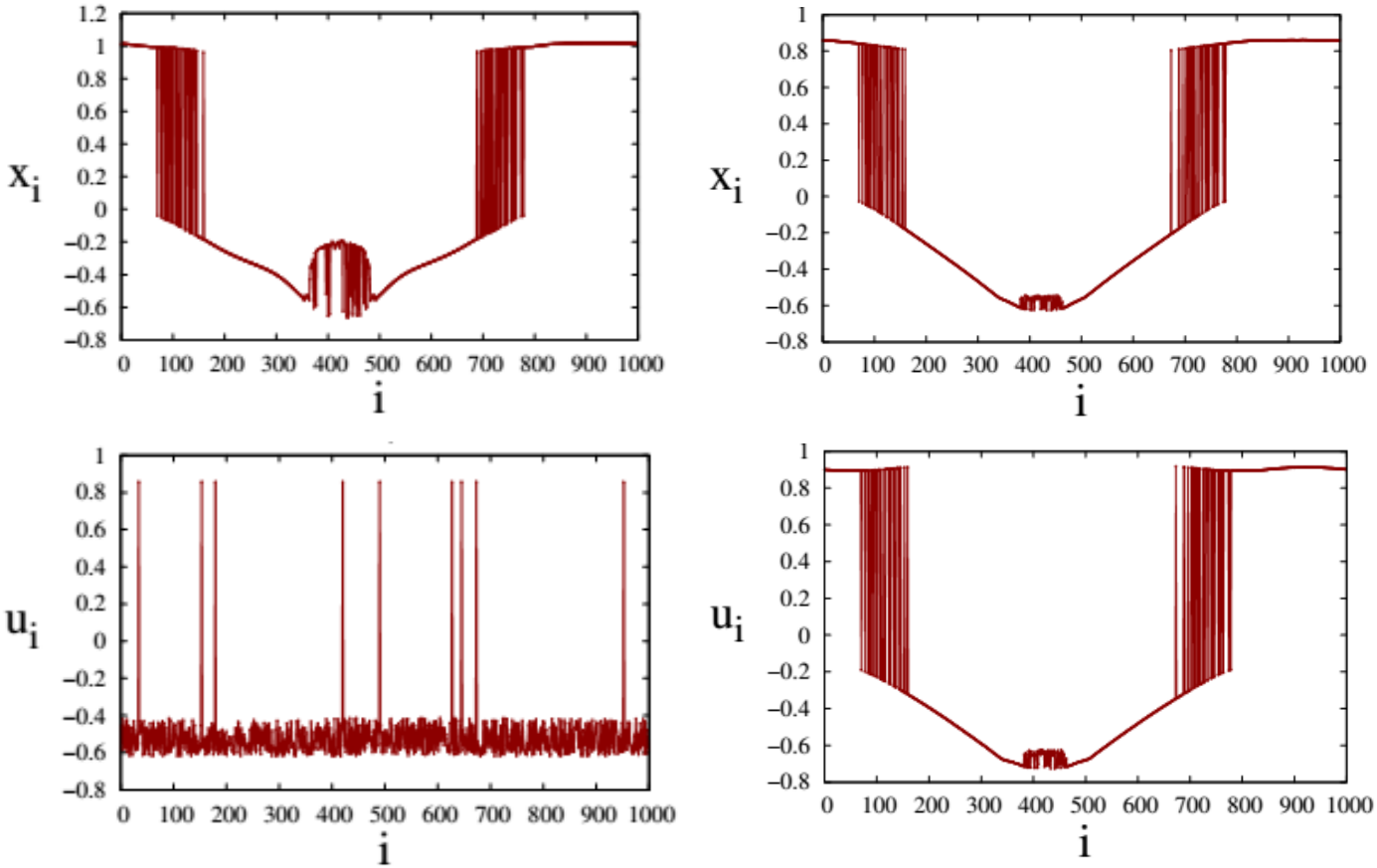
$$F_i^t = \bar{F}_i^t(x_i^t, y_i^t, u_i^t, v_i^t) = g(u_i^t, v_i^t) - f(x_i^t, y_i^t),$$
$$G_i^t = \bar{G}_i^t(x_i^t, y_i^t, u_i^t, v_i^t) = f(x_i^t, y_i^t) - g(u_i^t, v_i^t),$$

→ **Dissipative coupling**

$$F_i^t = \bar{\bar{F}}_i^t(x_i^t, y_i^t, u_i^t, v_i^t) = u_i^t - x_i^t,$$
$$G_i^t = \bar{\bar{G}}_i^t(x_i^t, y_i^t, u_i^t, v_i^t) = x_i^t - u_i^t.$$

→ **Inertial coupling**

Snapshots of the dynamics of the Henon and Lozi maps rings



Henon maps

Phase and amplitude chimeras are observed in both rings.

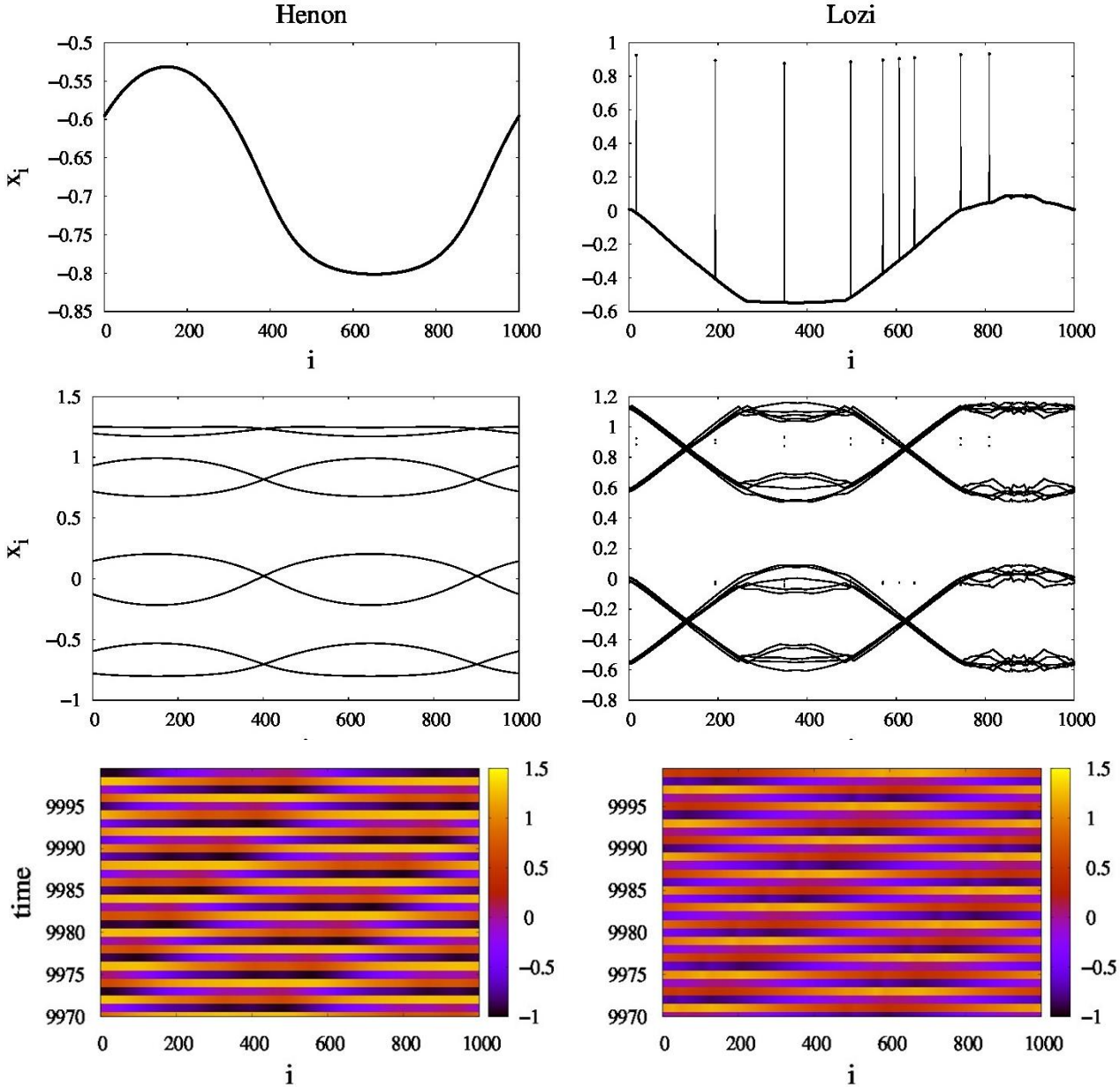
Lozi maps

$\gamma = 0$ $\gamma = 0.375$

$\sigma_1 = 0.32$, $\sigma_2 = 0.15$, $r_1 = 0.32$, $r_2 = 0.19$, $\alpha = 1.4$, $\beta = 0.3$

Traveling waves in the Henon and Lozi maps rings

$\sigma_1 = 0.344$,
$\sigma_2 = 0.22$



Snapshots

$\gamma = 0.0$

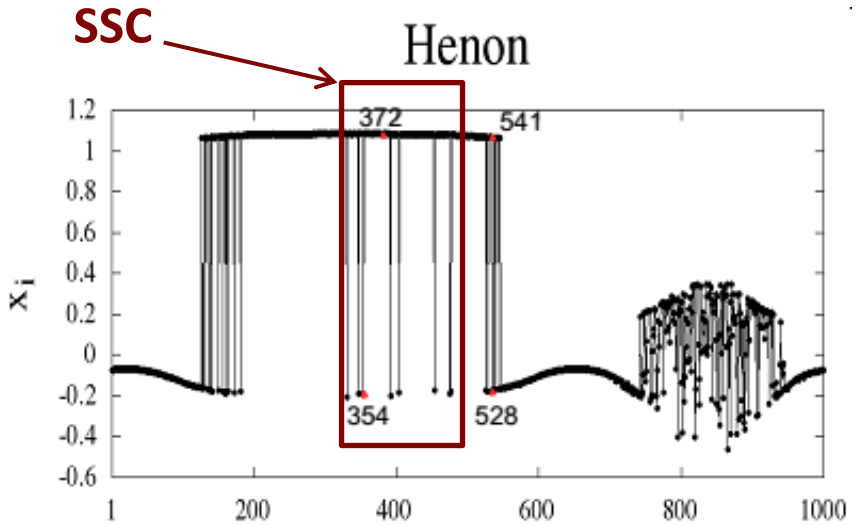
Spatio-temporal profiles

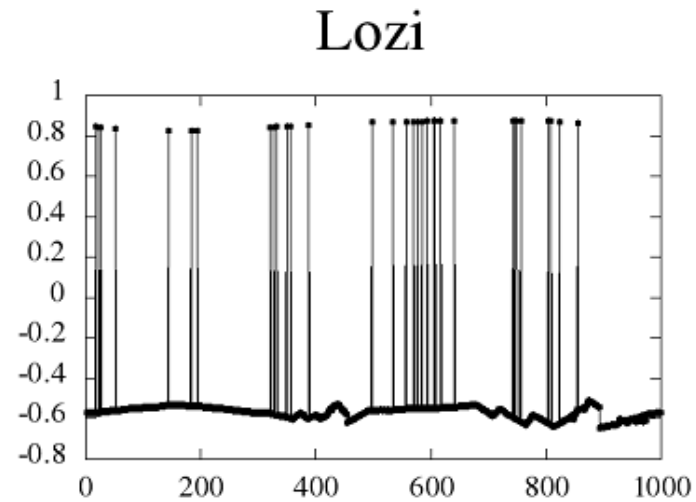
$\gamma = 0.0925$

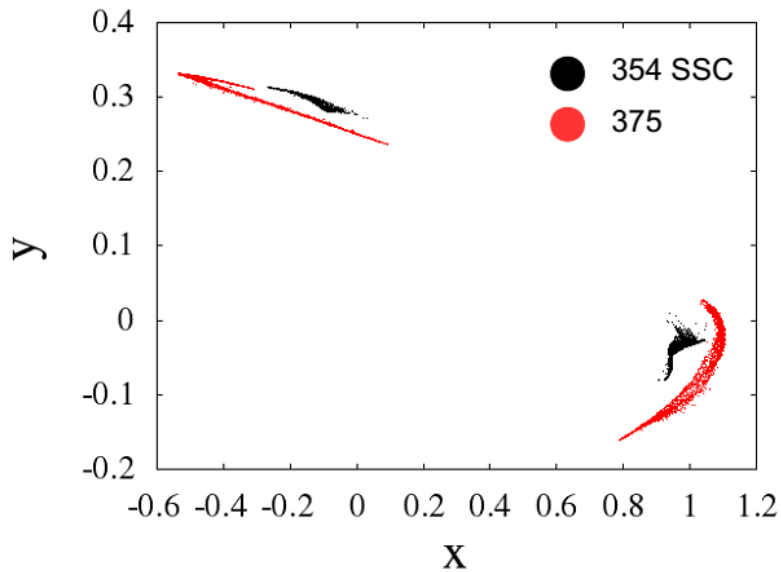
Traveling waves in both rings

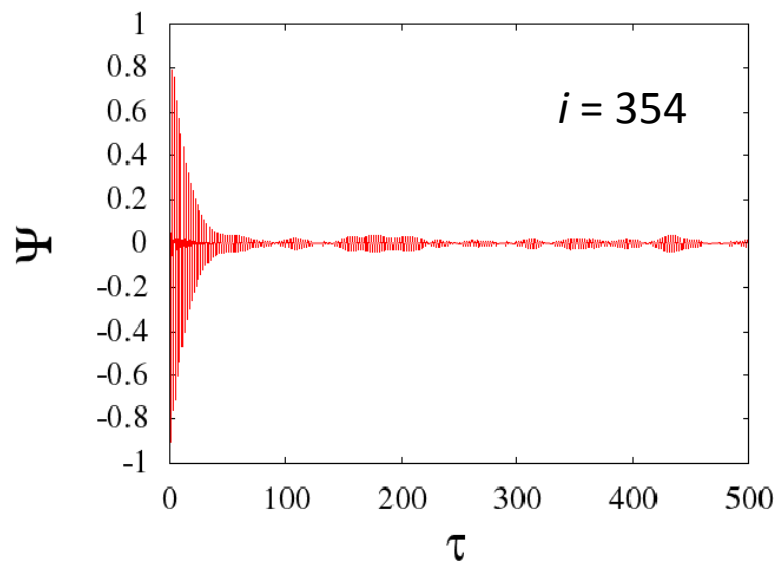
A new type of chimera - **solitary state chimera (SSC)**

(inertial coupling γ = 0.020)

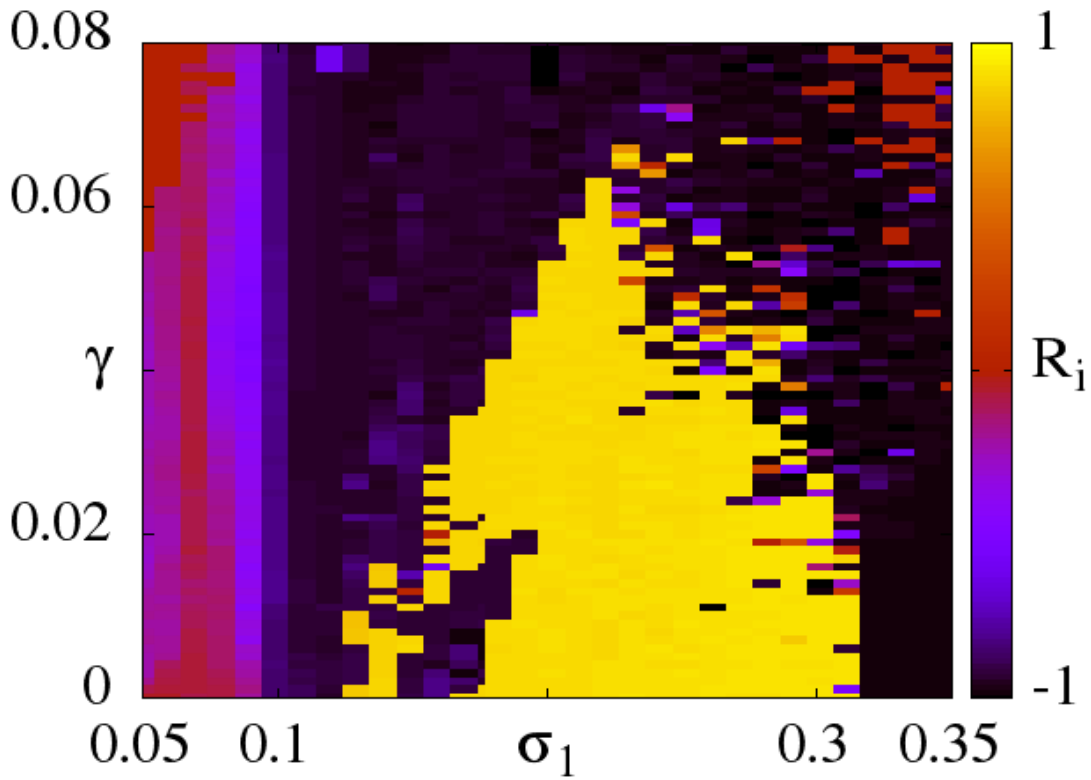








Region of existence of the solitary state chimera in the ring of Henon maps

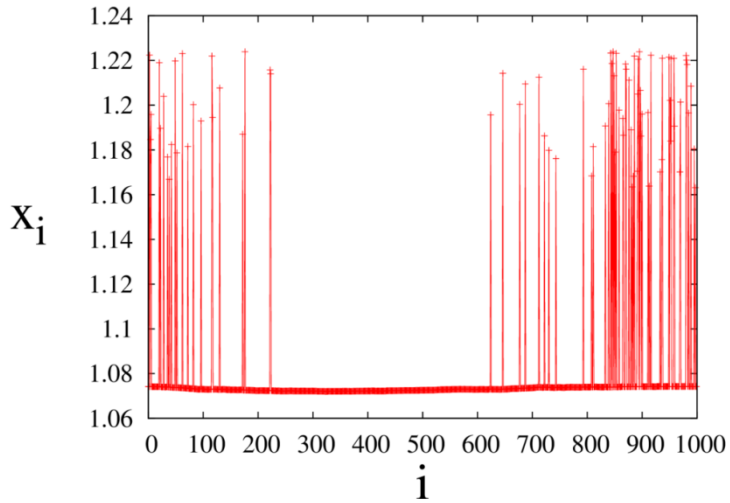


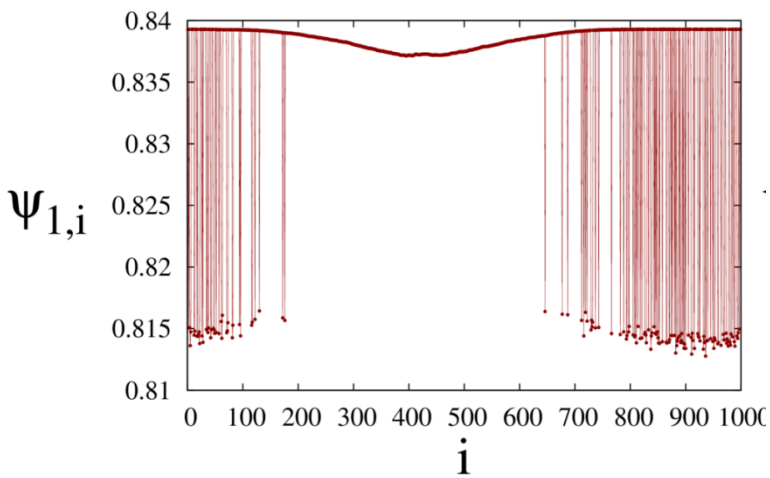
Solitary state chimera:

- All the elements - solitary states;
- Asynchronous chaotic behavior;
- Chaotic synchronization inside the coexisting coherence domains.

Solitary state chimera in the Henon map ensemble

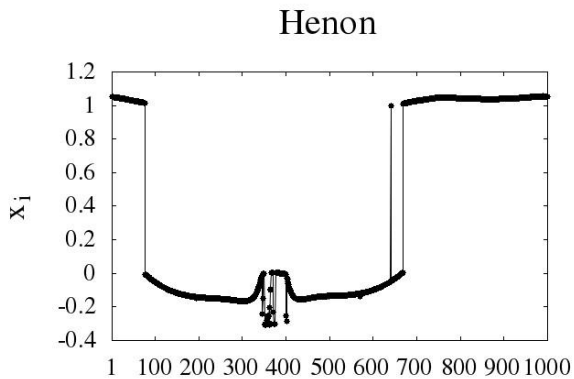
(dissipative coupling γ = 0.105)

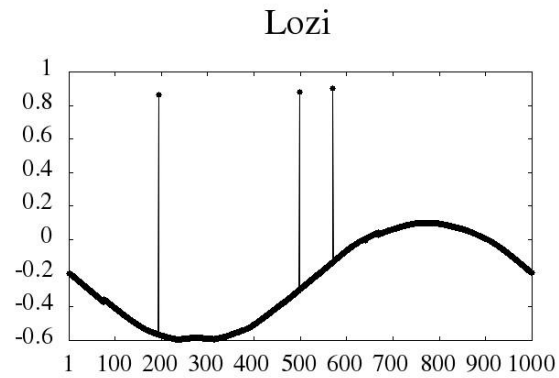




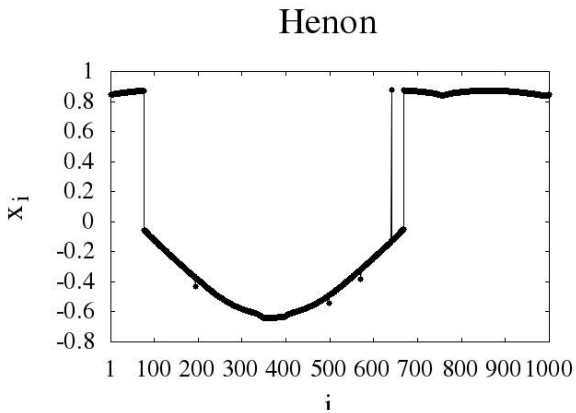
Temporal intermittency in the network of coupled ensembles of Henon and Lozi maps

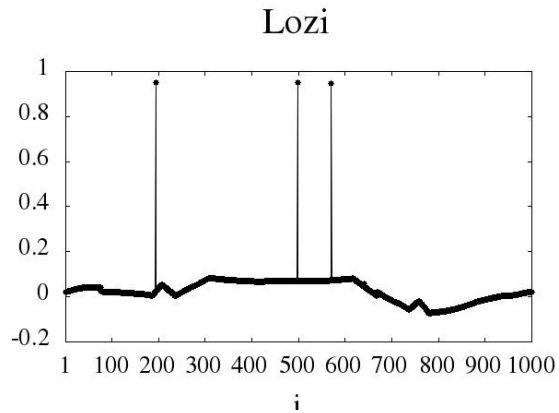
t_1



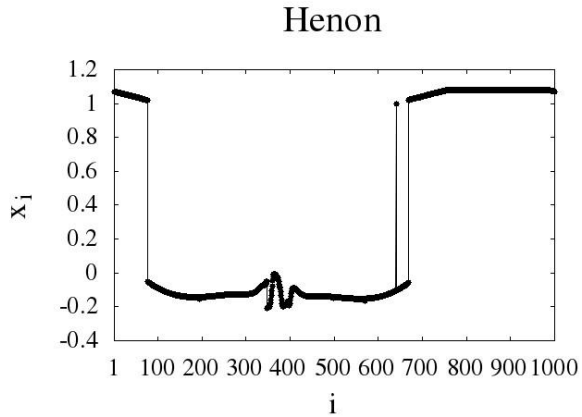


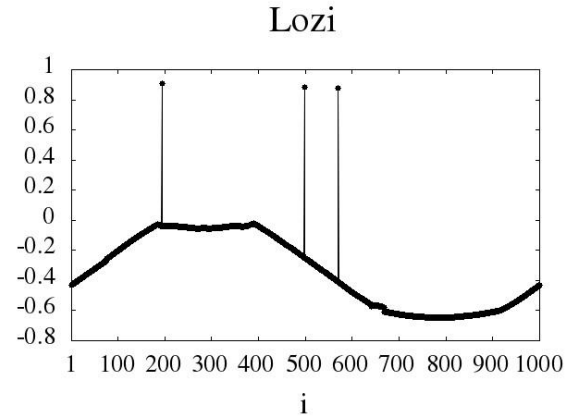
t_2





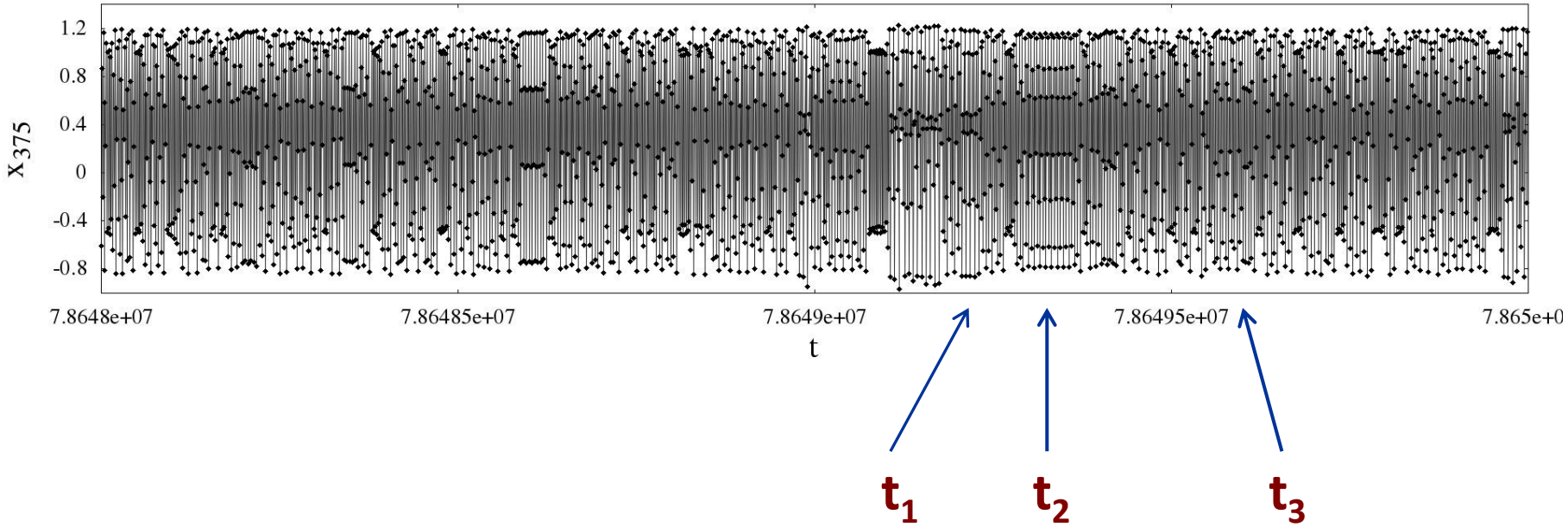
t_3



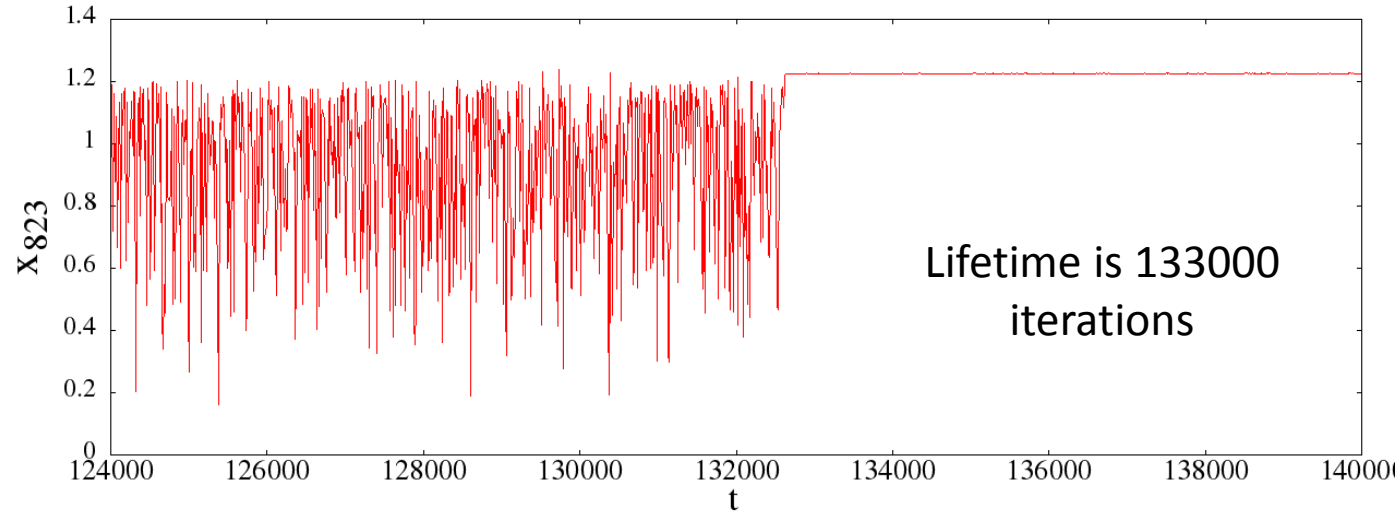


$\gamma = 0.019$,
$\sigma_1 = 0.344$,
$\sigma_2 = 0.23$,
$r_1 = 0.32$,
$r_2 = 0.193$,
$\alpha = 1.4$,
$\beta = 0.3$

Time series for the 375th element of the amplitude chimera

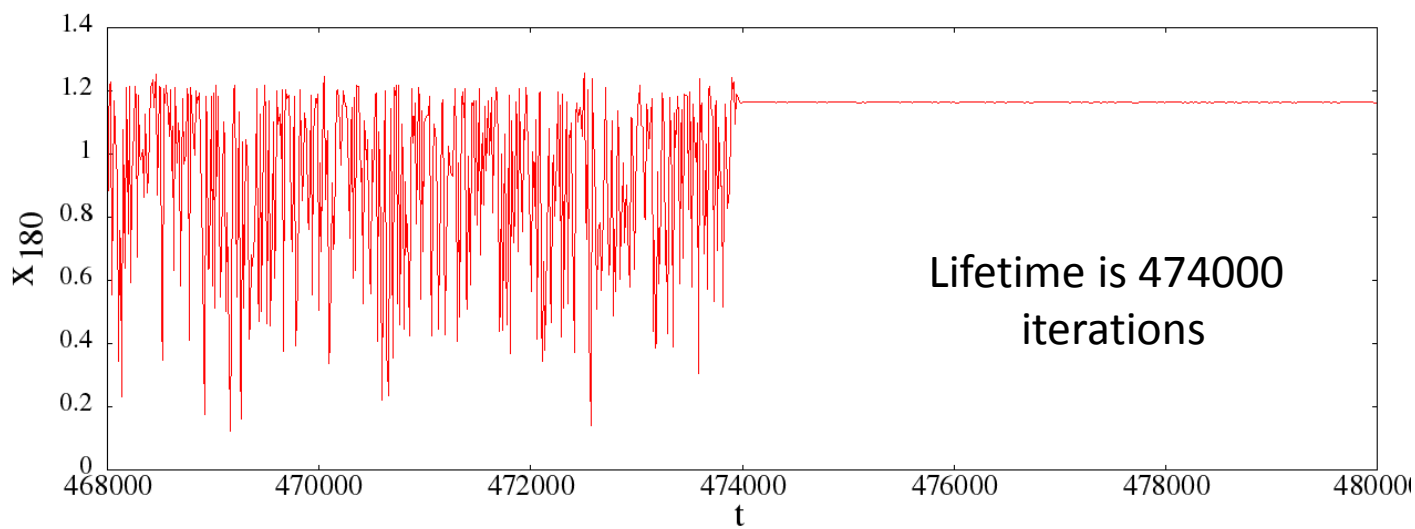


Lifetime of switching process and its control in two coupled ensembles

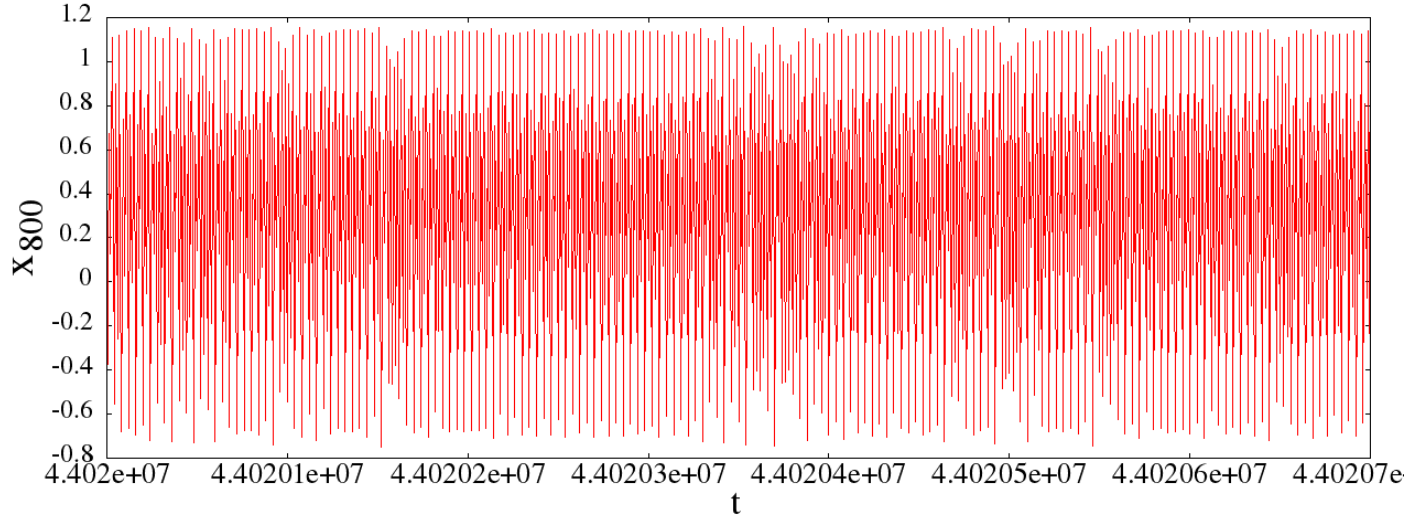


$\gamma = 0.015$

$\sigma_1 = 0.344$,
$\sigma_2 = 0.23$,
$r_1 = 0.32$,
$r_2 = 0.193$,
$\alpha = 1.4$,
$\beta = 0.3$

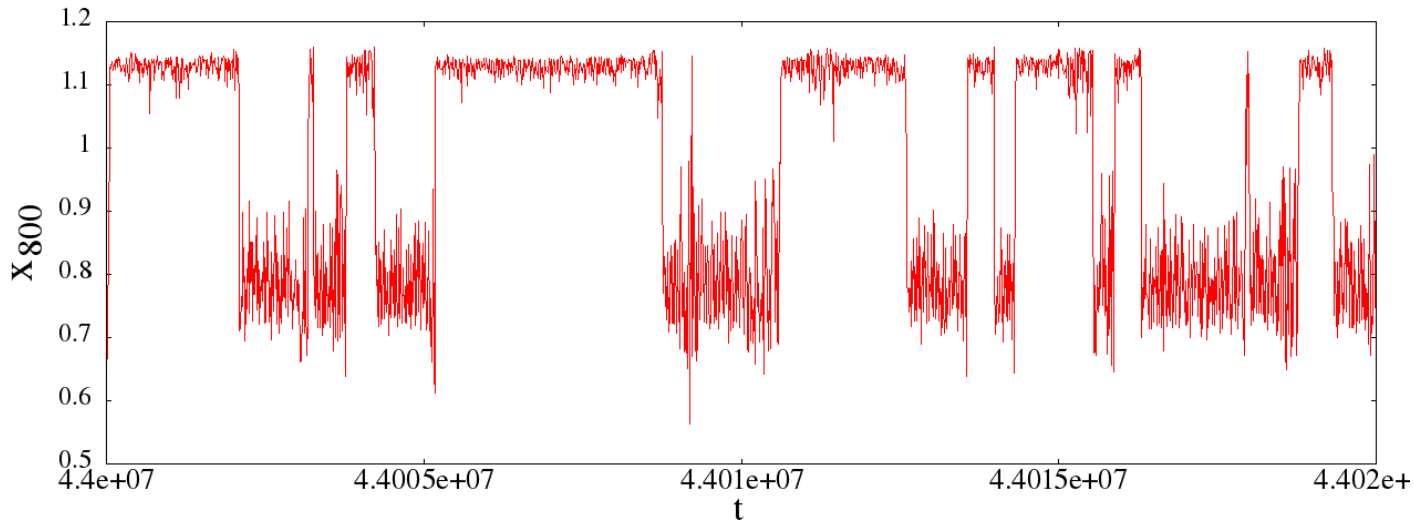


$\gamma = 0.003$



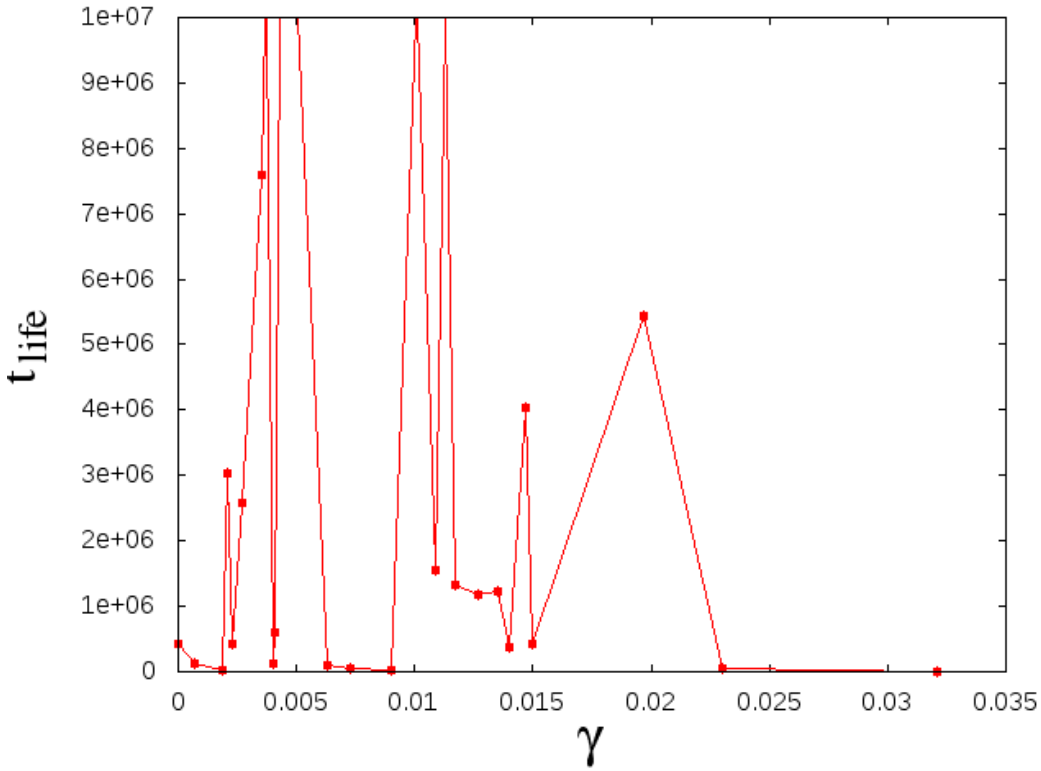
$\gamma = 0.031$

Lifetime > 10^8 and goes to infinity



Switchings between amplitude and phase chimeras

Lifetime of the amplitude chimera vs. the coupling γ



The amplitude chimera lifetime (or the intermittent process duration) depends nonlinearly on the coupling coefficient γ.

Mutual synchronization of spatio-temporal structures in coupled ensembles

When the parameters are varied in the system of coupled ensembles (1) , identical spatial structures appear simultaneously in both ensembles.

The question is whether these structures can be synchronized and what is the region of their synchronization?

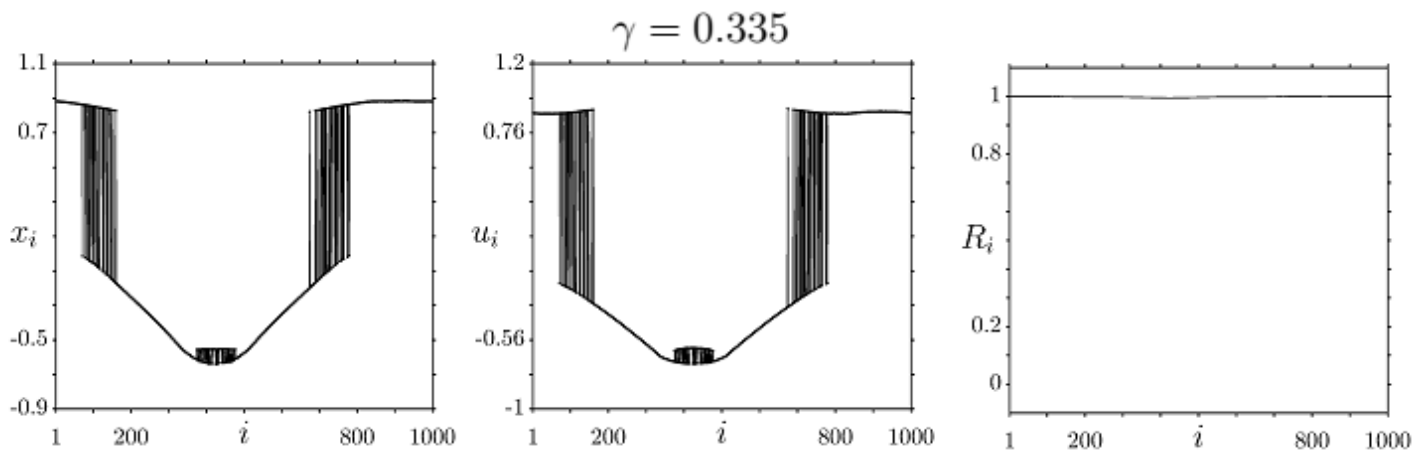
The identity of the observed structures is diagnosed by using **the mutual correlation coefficient:**

$$R_i = \frac{\langle \tilde{x}_i(t)\tilde{u}_i(t)\rangle}{\sqrt{\langle \tilde{x}_i^2(t)\rangle\langle \tilde{u}_i^2(t)\rangle}},$$

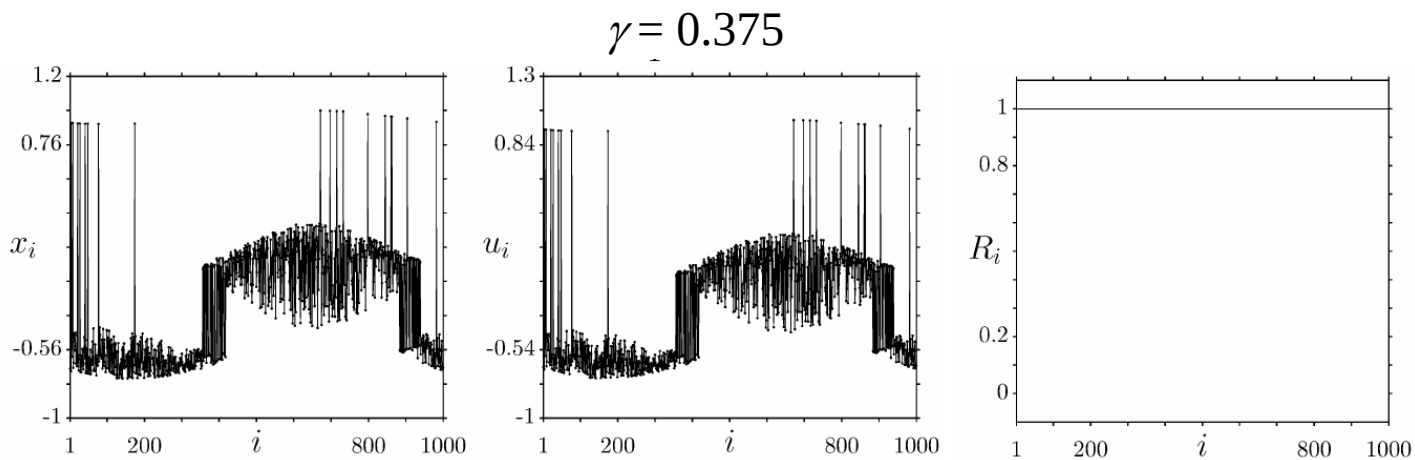
$\tilde{x}(t) = x(t) - \langle x(t)\rangle$ is a fluctuation around the average value. The brackets mean time averaging.

$i = 1, 2, \ldots, N$

$\sigma_{Henon} = 0.32$
$\sigma_{Lozi} = 0.15$

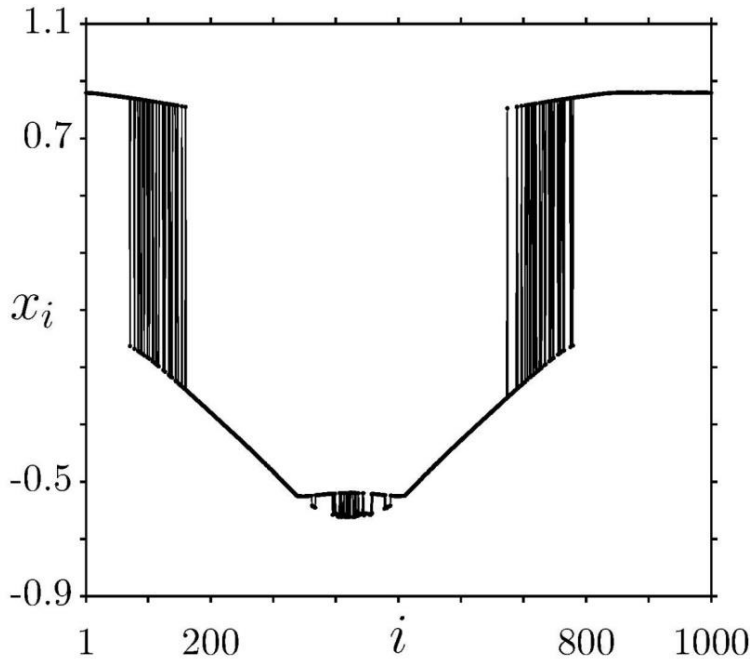


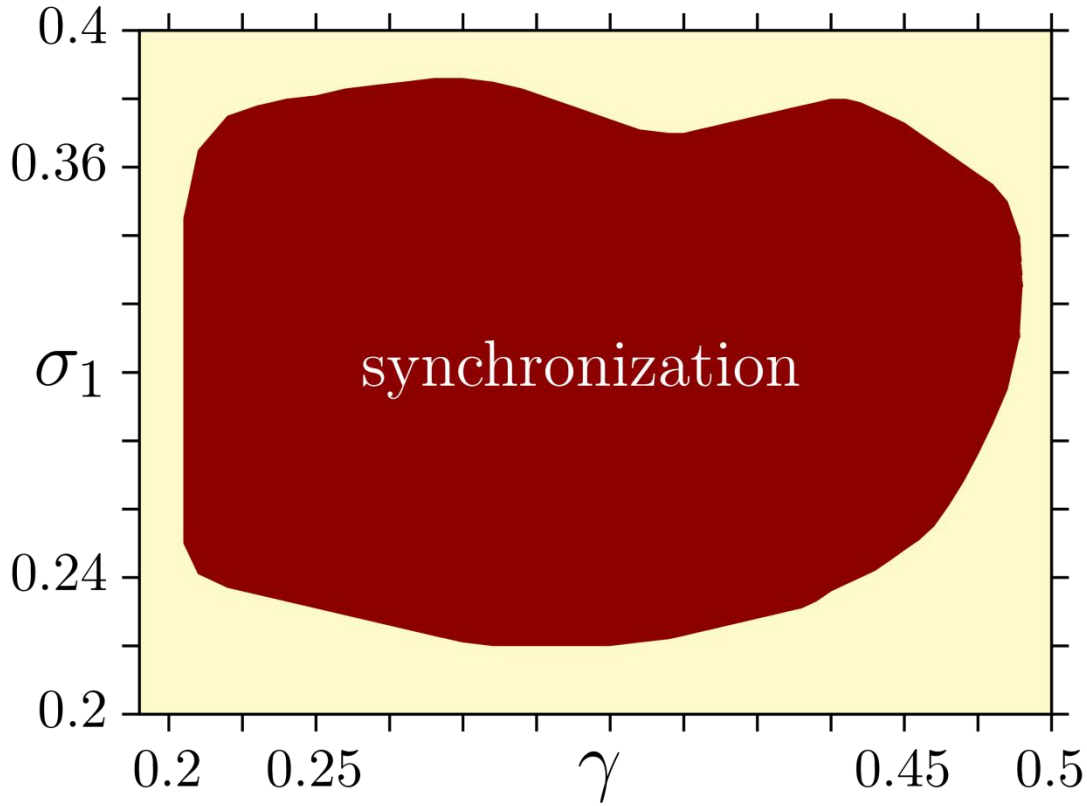
$\sigma_{Henon} = 0.18$
$\sigma_{Lozi} = 0.15$



Spatio-temporal structures in the Henon maps (left), the Lozi maps (middle) ensembles and the mutual correlation coefficient (right)

Synchronization region

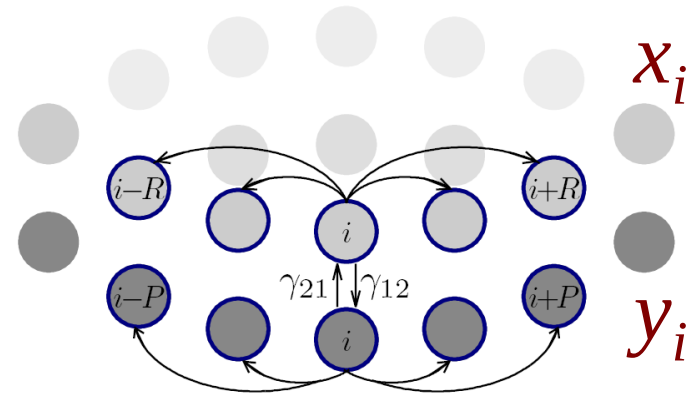




External and mutual synchronization in a two layer network of nonlocally coupled logistic maps

$$x_i^{t+1} = f_i^t + \frac{\sigma_1}{2R}\sum_{j=i-P}^{i+P}\left[f_j^t - f_i^t\right] + \gamma_{21}F_i^t,$$

$$y_i^{t+1} = g_i^t + \frac{\sigma_2}{2P}\sum_{j=i-R}^{i+R}\left[g_j^t - g_i^t\right] + \gamma_{12}G_i^t.$$



$$f_i^t = \alpha_1 x_i^t(1-x_i^t), \quad g_i^t = \alpha_2 y_i^t(1-y_i^t), \quad i = 1,2,\ldots,N = 1000$$

α_1 and α_2 are the control parameters which define the local dynamics;

σ_1 and σ_2 are the nonlocal coupling strengths (intra-couplings);

$P = R = 320$ denote the number of neighbors of the ith element from each side;

$F_i^t = (g_i^t - f_i^t), \quad G_i^t = (f_i^t - g_i^t)$ are the coupling functions;

γ_{21} and γ_{12} are the coupling strengths between the sub-networks (inter-coupling);

Initial conditions are randomly distributed in the interval [0; 1] for both rings.

Uncoupled case

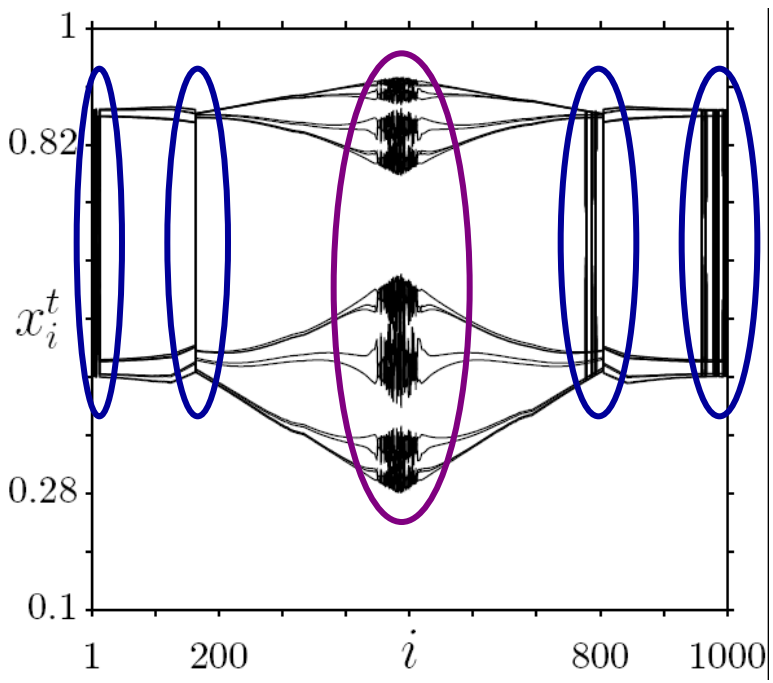
Parameter mismatch:

$\alpha_1 = 3.7,\ \alpha_2 = 3.85$

Different intra-couplings:

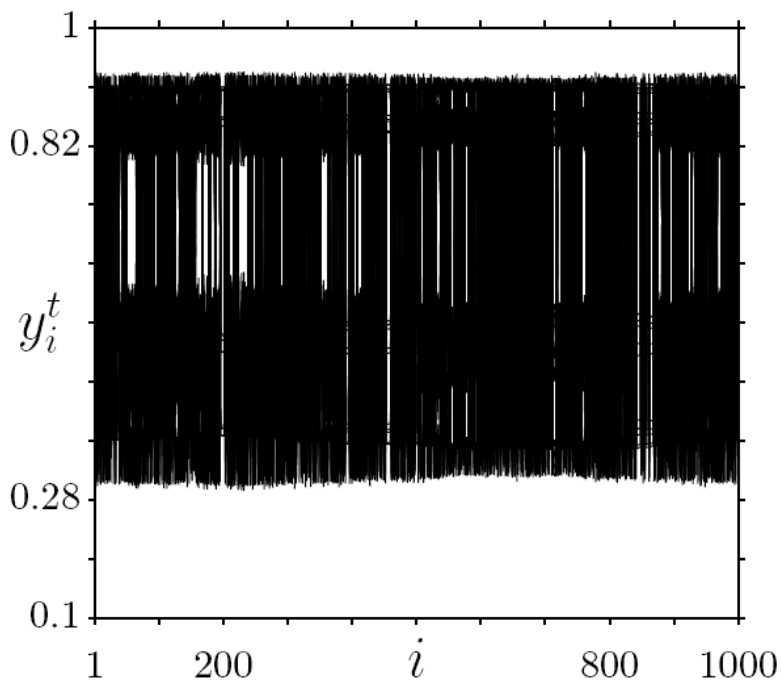
$\sigma_1 = 0.23,\ \sigma_2 = 0.15$

First ring



Phase and amplitude chimeras

Second ring

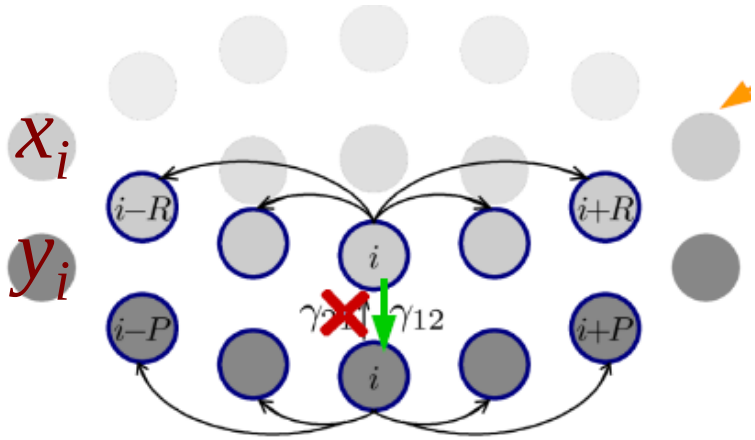


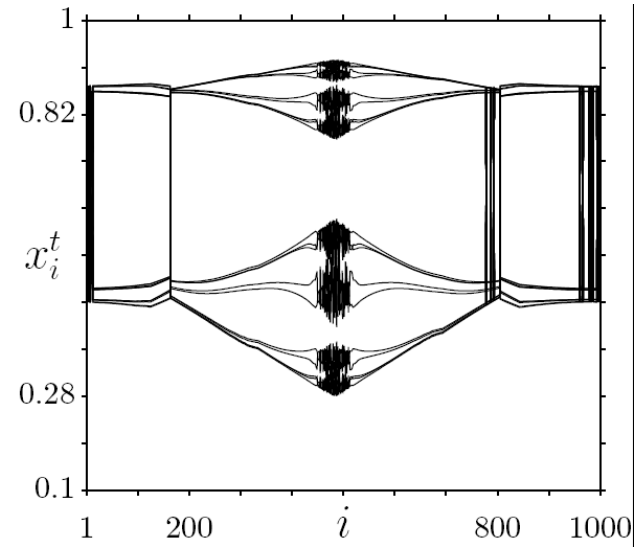
Spatiotemporal chaos

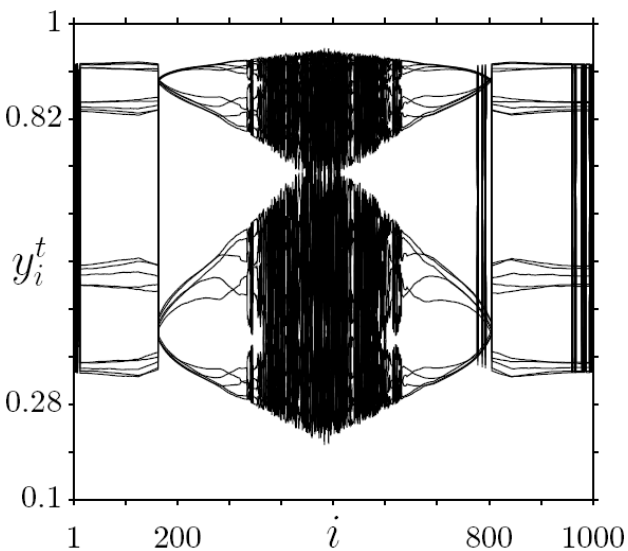
External synchronization of chimera states

driver x_i

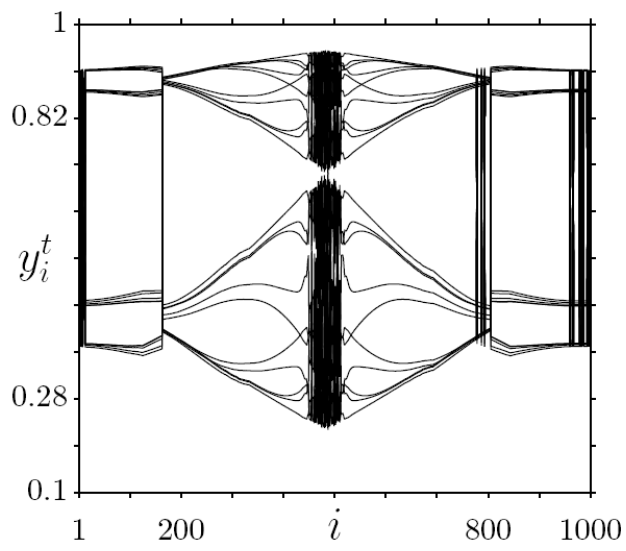
response y_i



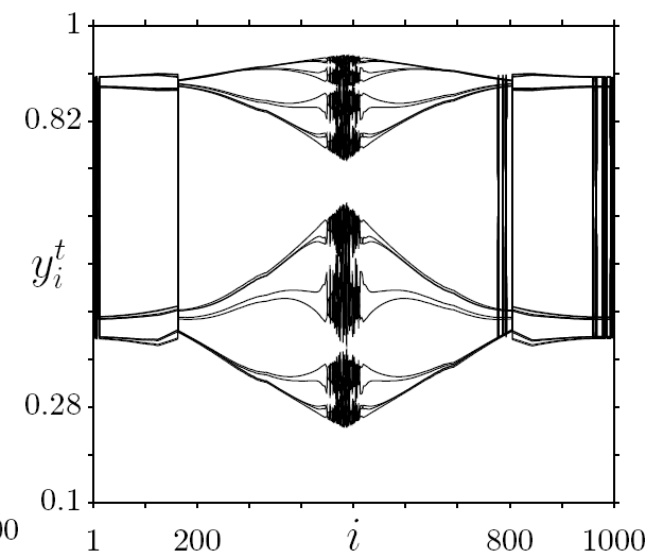




$\gamma_{12} = 0.15$

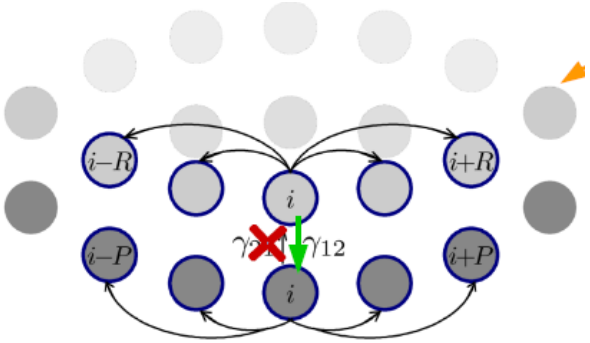


$\gamma_{12} = 0.3$



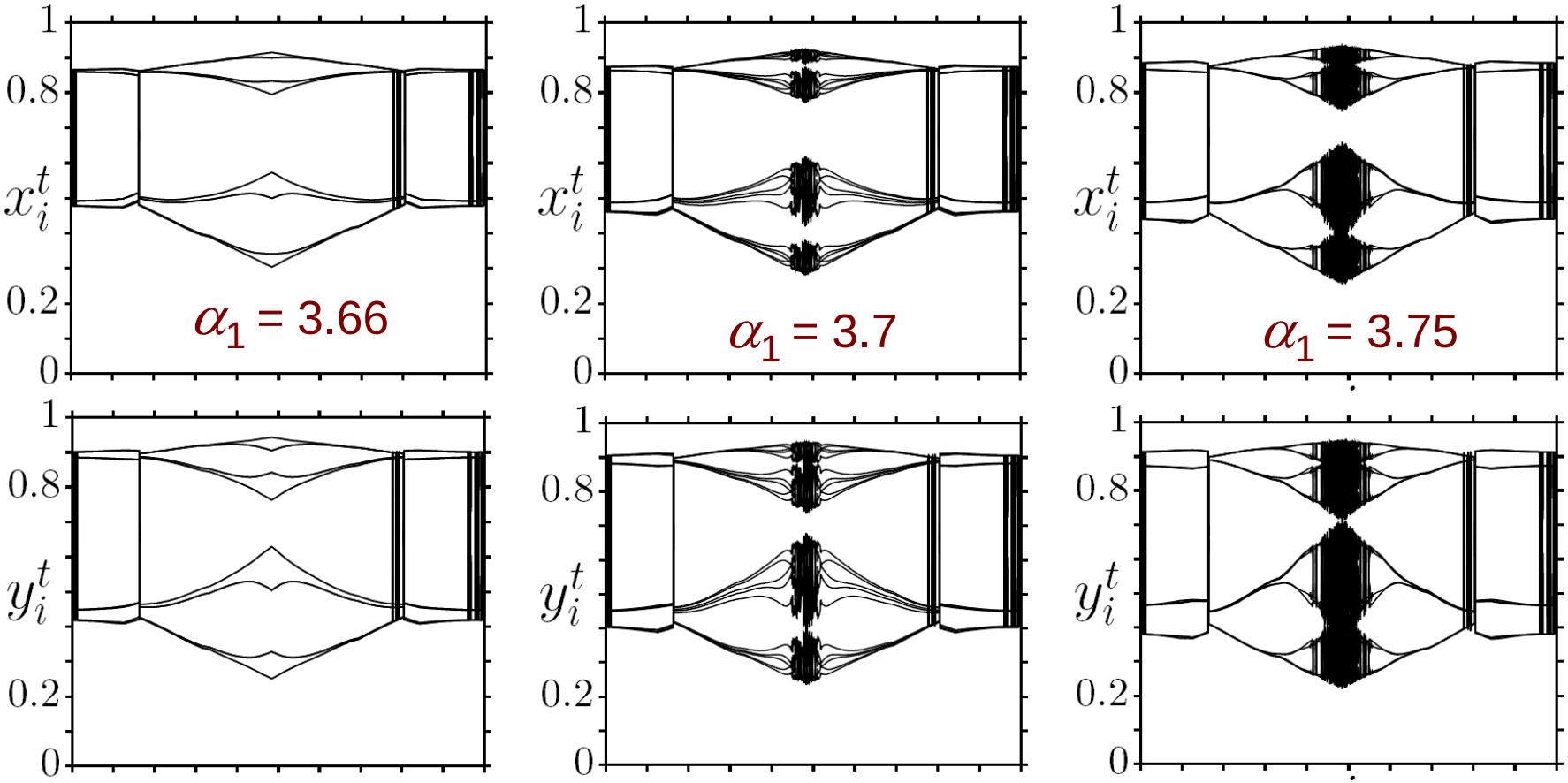
$\gamma_{12} = 0.45$

External synchronization of spatiotemporal patterns

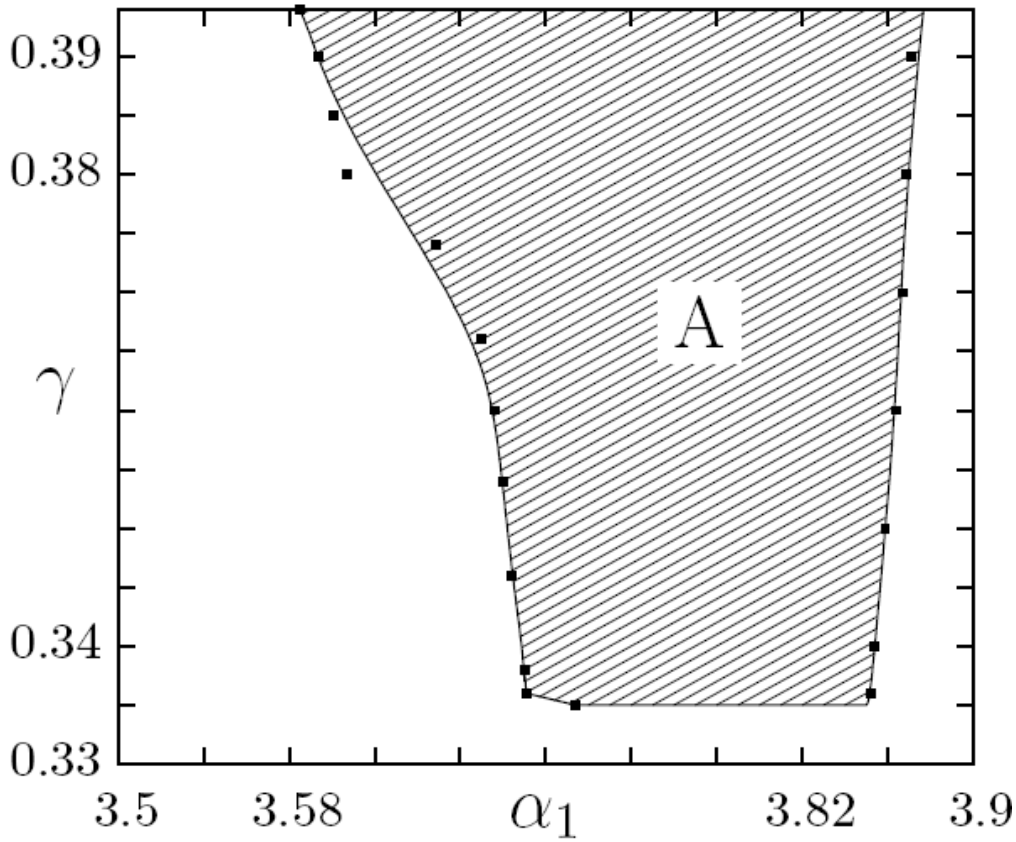


α_1 is varied in [3.5, 3.95],

$\alpha_2 = 3.85$, $\gamma_{12} = 0.4$

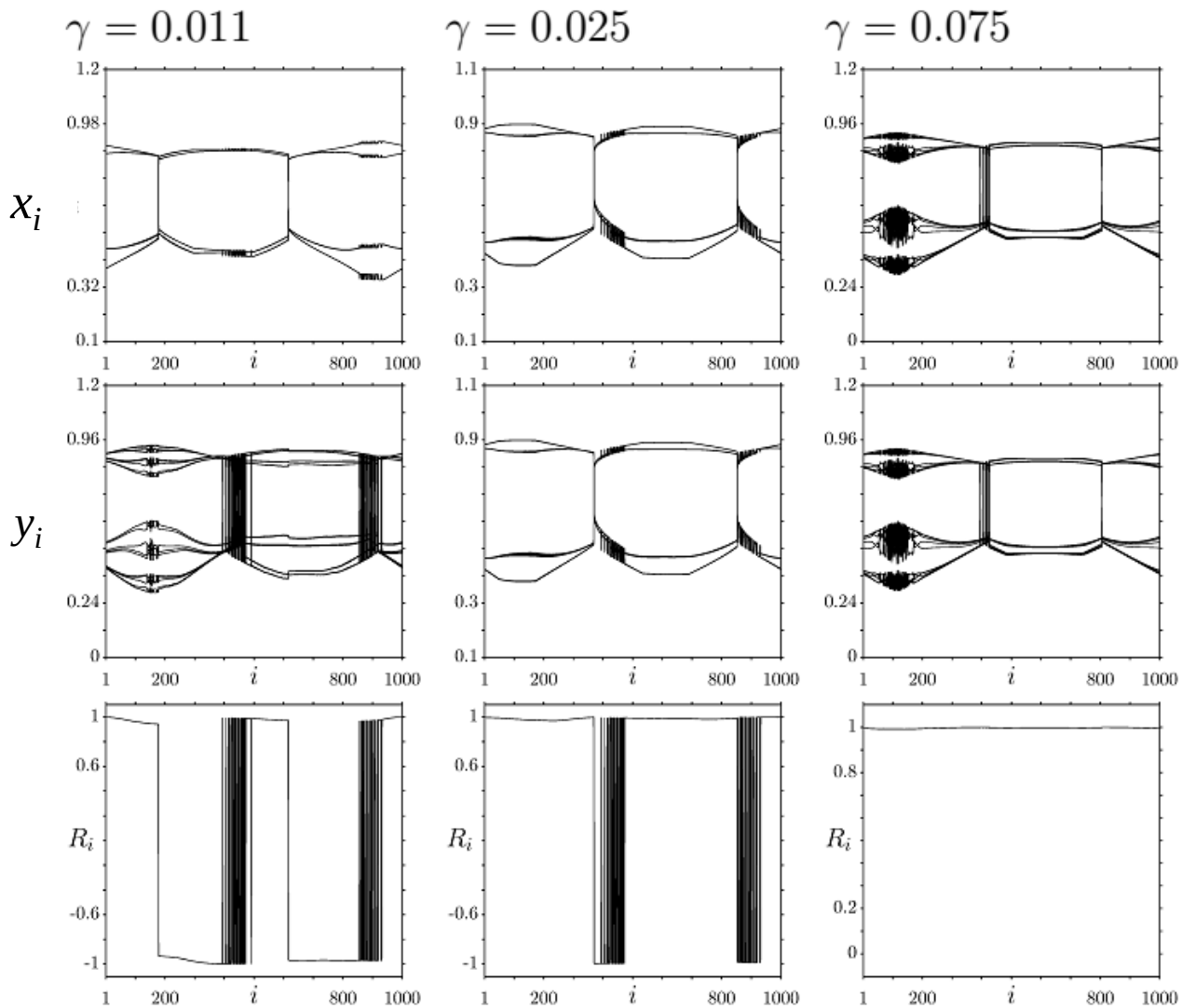


Region of external synchronization in the (α_1,γ) parameter plane



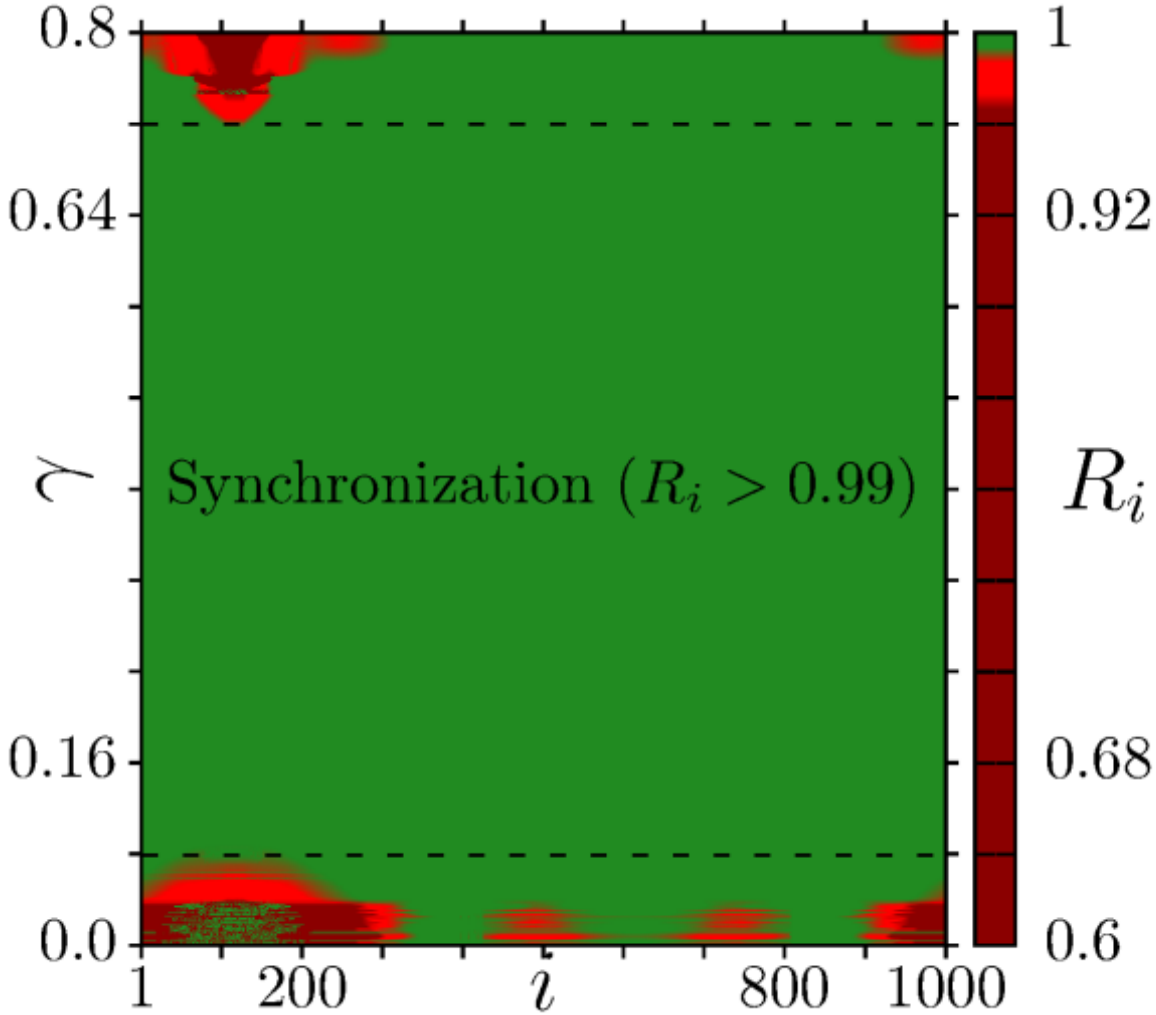
- Inside the synchronization regime, the state of the driver network **fully determines** the state in the response network.
- The structure in the response network **"is locked"** by the pattern in the driver network.

Mutual synchronization of chimera structures



The chimera structures are synchronized at $\gamma = \mathbf{0.075}$

Region of Synchronization



Conclusions

1. The basic models of discrete-time chaotic systems (the Henon and Lozi maps) have been selected to describe the spatiotemporal dynamics in ensembles of nonlocally coupled chaotic oscillators.
2. Possible spatiotemporal structures realized in those models have been analyzed in detail.
3. A novel type of chimera states, a solitary state chimera, has been revealed.
4. It has been shown that the amplitude chimeras are non-stationary and are characterized by a finite lifetime.
5. Effects of external and mutual synchronization of chimera states have been established and analyzed.

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Our papers

1. N. Semenova, A. Zakharova, E. Schöll, V. S. Anishchenko, Europhys. Lett. **112** (2015) 40002.
2. N. Semenova, A. Zakharova, E. Schöll, and V. Anishchenko, AIP Conference Proceedings 1738, 210014 (2016); doi: 10.1063/1.4951997.
3. S.A. Bogomolov, G.I. Strelkova, E. Schöll, and V.S. Anishchenko, Amplitude and phase chimeras in an ensemble of chaotic oscillators. Tech. Phys. Lett. **42**(7), 763-766 (2016).
4. S.A. Bogomolov, A.V. Slepnev, G.I. Strelkova, E. Schöll, and V.S. Anishchenko, Mechanisms of Appearance of Amplitude and Phase Chimera States in Ensembles of Nonlocally Coupled Chaotic Systems. Commun. Nonlinear Sci. Numer. Simulat. **43**, 25-36 (2017).
5. T.E. Vadivasova, G.I. Strelkova, S.A. Bogomolov, and V.S. Anishchenko, Correlation analysis of the coherence-incoherence transition in a ring of nonlocally coupled logistic maps. CHAOS, **26** (2016).
6. N.I. Semenova, E.V. Rybalova, G.I. Strelkova, and V.S. Anishchenko, “Coherence-incoherence” Transition in Ensembles of Nonlocally Coupled Chaotic Oscillators with Nonhyperbolic and Hyperbolic Attractors. Regular and Chaotic Dynamics, **22**(2), 148-162 (2017).
7. N.I. Semenova, G.I. Strelkova, V.S. Anishchenko, and A. Zakharova, Temporal intermittency and the lifetime of chimera states in ensembles of nonlocally coupled chaotic oscillators. CHAOS, **27**, 061102 (2017).
8. A. Bukh, E. Rybalova, N. Semenova, G. Strelkova, and V. Anishchenko, New type of chimera and mutual synchronization of spatiotemporal structures in two coupled ensembles of nonlocally interacting chaotic maps. CHAOS, **27**(11), 111102 (2017).

Thank you very much for your attention!