Investigation of the history of our Universe before the Big Bang

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Brief history of the Universe

Inflation: background and generation of perturbations

Visualizing small differences in the duration of inflation

Model reconstruction from observational data

For future work: GW from inflation and features in the spectrum

Conclusions

Main epochs of the Universe evolution – before 1979

$H \equiv \frac{\dot{a}}{a}$ where $a(t)$ is a scale factor of an isotropic homogeneous spatially flat universe (a Friedmann-Lemaître-Robertson-Walker background):

$$ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2) + small\ perturbations$$

The history of the Universe in one line: two main epochs

$$? \longrightarrow FLRWRD \Longrightarrow FLRWMD \longrightarrow ?$$

Geometry

$$H = \frac{1}{2t} \Longrightarrow H = \frac{2}{3t}$$

Physics

$$p = \rho/3 \Longrightarrow p \ll \rho$$

Main epochs of the Universe evolution – now

The history of the Universe in one line: four main epochs

$$? \longrightarrow DS \Longrightarrow FLRWRD \Longrightarrow FLRWMD \Longrightarrow \overline{DS} \longrightarrow ?$$

Geometry

$$|\dot{H}| << H^2 \Longrightarrow H = \frac{1}{2t} \Longrightarrow H = \frac{2}{3t} \Longrightarrow |\dot{H}| << H^2$$

Physics

$$p \approx -\rho \Longrightarrow p = \rho/3 \Longrightarrow p \ll \rho \Longrightarrow p \approx -\rho$$

Duration in terms of the number of e-folds $\ln(a_{fin}/a_{in})$

$$> 60 \qquad \sim 55 \qquad 7.5 \qquad 0.5$$

Inflation

The inflationary scenario is based on the two main cornerstone ideas (hypothesis):

1. Existence of inflation (or, quasi-de Sitter stage) – a stage of accelerated, close to exponential expansion of our Universe in the past preceding the hot Big Bang with decelerated, power-law expansion.

2. The origin of all inhomogeneities in the present Universe is the effect of gravitational creation of particles and field fluctuations during inflation from a "vacuum" (no-particle) state.

First conjectures of the former:
1. E. B. Gliner (1965): equation of state of matter approaches $p = -\rho$ at $\rho \to \infty$.
Wrong if taken literally.

2. E. B. Gliner (1970): such equation of state leads to the de Sitter stage preceding the hot Big Bang.
Much closer to the modern paradigm. However: 1) no workable model; 2) no ideas of observational tests.

First field-theoretical models:
A. A. Starobinsky (1980); D. Kazanas (1980); K. Sato (1981); A. H. Guth (1981); A. D. Linde (1982); A. Albrecht and P. J. Steinhardt (1982); A. D. Linde (1983), etc.

First predictions for observations based on calculations of quantum-gravitational generation of perturbations during inflation:
A. A. Starobinsky (1979) - tensor ones in GR, V. F. Mukhanov and G. V. Chibisov (1981) - scalar ones in $f(R)$ gravity, S. W. Hawking (1982), A. A. Starobinsky (1982), A. H. Guth and S.-Y. Pi (1982) - scalar ones in GR, etc.

Main advantages of inflation

1. Aesthetic elegance
Inflation – hypothesis about an almost maximally symmetric (quasi-de Sitter) stage of the evolution of our Universe in the past, before the hot Big Bang. If so, preferred initial conditions for (quantum) inhomogeneities with sufficiently short wavelengths exist – the adiabatic in-vacuum ones. In addition, these initial conditions represent an attractor for a much larger compact open set of initial conditions having a non-zero measure in the space of all initial conditions.

2. Predictability, proof and/or falsification
Given equations, this gives a possibility to calculate all subsequent evolution of the Universe up to the present time and even further to the future. Thus, any concrete inflationary model can be proved or disproved by observational data.

3. Naturalness of the hypothesis

Remarkable qualitative similarity between primordial and present dark energy.

4. Relates quantum gravity and quantum cosmology to astronomical observations

Makes quantum gravity effects observable at the present time and at very large – cosmological – scales.

5. Produces (non-universal) arrow of time for our Universe

Origin – initial quasi-vacuum fluctuation with a fantastically large correlation radius.

Present status of inflation

Now we have numbers.

P. A. R. Ade et al., arXiv:1502.01589

The primordial spectrum of scalar perturbations has been measured and its deviation from the flat spectrum $n_s = 1$ in the first order in $|n_s - 1| \sim N^{-1}$ has been established (using the multipole range $\ell > 40$):

$$< \zeta^2(\mathbf{r}) >= \int \frac{P_\zeta(k)}{k}\, dk, \quad P_\zeta(k) = \left(2.21^{+0.07}_{-0.08}\right) 10^{-9} \left(\frac{k}{k_0}\right)^{n_s - 1}$$

$$k_0 = 0.05 \mathrm{Mpc}^{-1}, \quad n_s - 1 = -0.035 \pm 0.005$$

Two fundamental observational constants of cosmology in addition to the three known ones (baryon-to-photon ratio, baryon-to-matter density and the cosmological constant). Existing inflationary models can predict (and predicted, in fact) one of them, namely $n_s - 1$, relating it finally to $\ln \frac{k_B T_\gamma}{\hbar H_0} \approx 67.2$.

From "proving" inflation to using it as a tool

Present status of inflation: transition from "proving" it in general and testing some of its simplest models to applying the inflationary paradigm to investigate particle physics at super-high energies and the actual history of the Universe in the remote past using real observational data on $n_s(k) - 1$ and $r(k)$.

Simple (one-parameter, in particular) models may be good in the first approximation (indeed so), but it is difficult to expect them to be absolutely exact, small corrections due to new physics should exist (indeed so).

The reconstruction approach – determining curvature and inflaton potential from observational data.

The most important quantities:
1) for classical gravity – $H, \dot{H}$
2) for super-high energy particle physics – m^2_{infl}.

Physical scales related to inflation

"Naive" estimate where I use the reduced Planck mass $\tilde{M}_{Pl} = (8\pi G)^{-1}$.

I. $H \sim \sqrt{P_\zeta}\tilde{M}_{Pl} \sim 10^{14}\mathrm{GeV}$

In the simplest inflationary models, H gives the curvature (the Hubble function) value around ~ 55 e-folds before the end of inflation.

II. $m \sim H\sqrt{|1 - n_s|} \sim 10^{13}\mathrm{GeV}$

In the simplest models, m gives the inflaton mass after the end of inflation.

New range of mass-energy scales significantly less than the GUT scale. However, this mass occurs in curved, not flat, space-time.

Generation of scalar and tensor perturbations during inflation

A genuine quantum-gravitational effect: a particular case of the effect of particle-antiparticle creation by an external gravitational field. Requires quantization of a space-time metric. Similar to electron-positron creation by an electric field. From the diagrammatic point of view: an imaginary part of a one-loop correction to the propagator of a gravitational field from all quantum matter fields including the gravitational field itself, too.

The effect can be understood from the behaviour of a light scalar field in the de Sitter space-time.

Early papers on particle creation in an external electric field: Nikishov (ZhETF 1970), Narozhnyi and Nikishov (Yad. Fiz. 1970).

Early papers on particle creation in FRLW cosmology: Chernikov and Tagirov (1968), Parker (1968), Grib and Mamaev (1969), Zeldovich (1970).

First calculation of a renormalized average value of the energy-momentum tensor of a quantum field: Zeldovich and Starobinsky (1971) (made for a more general case of anisotropic homogeneous cosmology).

However, it appeared finally that quantum field fluctuations themselves, i.e. first order quantities, are the most interesting and observable ones.

De Sitter space-time

Constant curvature space-time.

$$R_{\alpha\beta\gamma\delta} = H_0^2(g_{\alpha\gamma}g_{\beta\delta} - g_{\alpha\delta}g_{\beta_\gamma})$$

4 most popular forms of its space-time metric (only the first metric covers the whole space-time):

$$ds^2 = dt_c^2 - H_0^{-2}\cosh^2(H_0 t_c)\left(d\chi_c^2 + \sin^2\chi_c d\Omega^2\right)$$

$$ds^2 = dt^2 - a_1^2 e^{2H_0 t}\left(dr^2 + r^2 d\Omega^2\right), \; a_1 = const$$

$$ds^2 = dt_o^2 - H_0^{-2}\sinh^2(H_0 t_o)\left(d\chi_o^2 + \sinh^2\chi_o d\Omega^2\right)$$

$$ds^2 = (1 - H_0^2 R^2)d\tau^2 - (1 - H_0^2 R^2)^{-1}dR^2 - R^2 d\Omega^2$$

$$d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$$

Perturbative anomalous growth of light scalar fields in the de Sitter space-time

Background - fixed - de Sitter or, more interestingly, quasi-de Sitter space-time (slow roll inflation).
Occurs for $0 \leq m^2 \ll H^2$ where $H \equiv \frac{\dot{a}}{a}$, $a(t)$ is a FRW scale factor. The simplest and textbook example:
$m = 0, \; H = H_0 = const$ for $t \geq t_0$ and the initial quantum state of the scalar field at $t = t_0$ is the adiabatic vacuum for modes with $k/a(t_0) \gg H_0$ and some infrared finite state otherwise.

The wave equation:

$$\phi_{;\mu}^{;\mu} = 0$$

Quantization with the adiabatic vacuum initial condition:

$$\phi = (2\pi)^{-3/2} \int \left[\hat{a}_{\mathbf{k}}\, \phi_k(\eta)\, e^{-i\mathbf{kr}} + \hat{a}^{\dagger}_{\mathbf{k}}\, \phi^*_k\, e^{i\mathbf{kr}}\right] d^3k$$

$$\phi_k(\eta) = \frac{H_0 e^{-ik\eta}}{\sqrt{2k}}\left(\eta - \frac{i}{k}\right),\ a(\eta) = \frac{1}{H_0\eta},\ \eta_0 < \eta < 0,\ k = |\mathbf{k}|$$

Then

$$< \phi^2 >= \frac{H_0^2 N}{4\pi^2} + const$$

Here $N = \ln \frac{a}{a(t_0)} \gg 1$ is the number of e-folds from the beginning of inflation and the constant depends on the initial quantum state (Linde, 1982; Starobinsky, 1982; Vilenkin and Ford, 1982).
Straightforward generalization to the slow-roll case $|\dot{H}| \ll H^2$.

For $0 < m^2 \ll H^2$, the Bunch-Davies equilibrium value

$$< \phi^2 >= \frac{3H_0^4}{8\pi^2 m^2} \gg H_0^2$$

is reached after a large number of e-folds $N \gg \frac{H_0^2}{m^2}$.
Purely infrared effect - creation of real field fluctuations; renormalization is not important and does not affect it.

For the de Sitter inflation (gravitons only) (AS, 1979):

$$P_g(k) = \frac{16GH_0^2}{\pi}; \quad < h_{ik}h^{ik} >= \frac{16GH_0^2 N}{\pi} \ .$$

The assumption of small perturbations breaks down for $N \gtrsim 1/GH_0^2$. Still ongoing discussion on the final outcome of this effect. My opinion - no screening of the background cosmological constant, instead - stochastic drift through an infinite number of locally de Sitter, but globally non-equivalent vacua.

Reason: the de Sitter space-time is not the generic late-time asymptote of classical solutions of GR with a cosmological constant Λ both without and with hydrodynamic matter. The generic late-time (expanding) asymptote is (Starobinsky, 1983):

$$ds^2 = dt^2 - \gamma_{ik} dx^i dx^k$$

$$\gamma_{ik} = e^{2H_0 t} a_{ik} + b_{ik} + e^{-H_0 t} c_{ik} + ...$$

where $H_0^2 = \Lambda/3$ and the matrices a_{ik}, b_{ik}, c_{ik} are functions of spatial coordinates. a_{ik} contains two independent physical functions (after 3 spatial rotations and 1 shift in time + spatial dilatation) and can be made unimodular, in particular.

Generation of metric perturbations

One spatial Fourier mode $\propto e^{i\mathbf{kr}}$ is considered.

For scales of astronomical and cosmological interest, the effect of creation of metric perturbations occurs at the primordial de Sitter (inflationary) stage when $k \sim a(t)H(t)$ where $k \equiv |\mathbf{k}|$ (the first Hubble radius crossing).

After that, for a very long period when $k \ll aH$ until the second Hubble radius crossing (which occurs rather recently at the radiation or matter dominated stages), there exist one mode of scalar (adiabatic, density) perturbations and two modes of tensor perturbations (primordial gravitational waves) for which metric perturbations are constant (in some gauge) and independent of (unknown) local microphysics due to the causality principle.

Classical-to-quantum transition for the leading modes of perturbations

In the superhorizon regime in the coordinate representation:

$$ds^2 = dt^2 - a^2(t)(\delta_{lm} + h_{lm})dx^l dx^m, \quad l, m = 1, 2, 3$$

$$h_{lm} = 2\zeta(\mathbf{r})\delta_{lm} + \sum_{a=1}^{2} g^{(a)}(\mathbf{r})\, e_{lm}^{(a)}$$

$$e_l^{l(a)} = 0, \;\; g^{(a)}_{\;,l}\, e_m^{l(a)} = 0, \;\; e_{lm}^{(a)}\, e^{lm(a)} = 1$$

ζ describes primordial scalar perturbations, g – primordial tensor perturbations (primordial gravitational waves (GW)).

In fact, metric perturbations h_{lm} are quantum (operators in the Heisenberg representation) and remain quantum up to the present time. But, after omitting of a very small part, decaying with time, they become commuting and, thus, equivalent to classical (c-number) stochastic quantities with the Gaussian statistics (up to small terms quadratic in $\zeta,\ g$).

Remaining quantum coherence: deterministic correlation between $\mathbf{k}$ and $-\mathbf{k}$ modes - shows itself in the appearance of acoustic oscillations (primordial oscillations in case of GW).

Visualizing small differences in the number of e-folds

Local duration of inflation in terms of $N_{tot} = \ln\left(\frac{a(t_{fin})}{a(t_{in})}\right)$ is different is different point of space: $N_{tot} = N_{tot}(\mathbf{r})$. Then

$$\zeta(\mathbf{r}) = \delta N_{tot}(\mathbf{r}) = \left(\frac{\partial N_{tot}}{\partial \phi}\right)_b \delta\phi(\mathbf{r})$$

– δN formalism.

Correct generalization to the non-linear case: the space-time metric after the end of inflation at super-Hubble scales

$$ds^2 = dt^2 - a^2(t)e^{2N_{tot}(\mathbf{r})}(dx^2 + dy^2 + dz^2)$$

First derived in A. A. Starobinsky, Phys. Lett. B **117**, 175 (1982) in the case of one-field inflation.

CMB temperature anisotropy

$$T_\gamma = (2.72548 \pm 0.00057)\mathrm{K}$$

$$\Delta T(\theta, \phi) = \sum_{\ell m} a_{\ell m} Y_{\ell m}(\theta, \phi)$$

$$< a_{\ell m} a_{\ell' m'} >= C_\ell \delta_{\ell \ell'} \delta_{m m'}$$

Theory: averaging over realizations.
Observations: averaging over the sky for a fixed ℓ.

For scalar perturbations, generated mainly at the last scattering surface (the surface or recombination) at $z_{LSS} \approx 1090$ (the Sachs-Wolfe, Silk and Doppler effects), but also after it (the integrated Sachs-Wolfe effect).
For GW – only the ISW works.

For $\ell \lesssim 50$, neglecting the Silk and Doppler effects, as well as the ISW effect due the presence of dark energy,

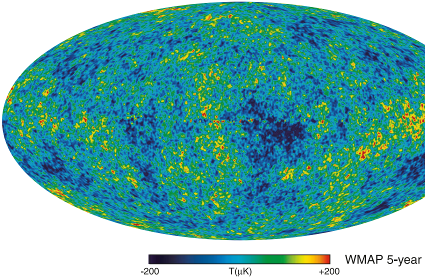
$$\frac{\Delta T(\theta,\phi)}{T_\gamma} = -\frac{1}{5}\zeta(r_{LSS},\theta,\phi) = -\frac{1}{5}\delta N_{tot}(r_{LSS},\theta,\phi)$$

For $n_s = 1$,

$$\ell(\ell+1)C_{\ell,s} = \frac{2\pi}{25}P_\zeta$$

For $\frac{\Delta T}{T} \sim 10^{-5}$, $\delta N \sim 5\times 10^{-5}$, and for $H \sim 10^{14}\,\mathrm{GeV}$, $\delta t \sim 5 t_{Pl}$!

Planck time intervals are seen by the naked eye!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WMAP 5-year

Inflationary models in GR

Based on a minimally coupled scalar field with a potential. In the absence of spatial curvature and other matter:

$$H^2 = \frac{\kappa^2}{3}\left(\frac{\dot{\phi}^2}{2} + V(\phi)\right)$$

$$\dot{H} = -\frac{\kappa^2}{2}\dot{\phi}^2$$

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

where $\kappa^2 = 8\pi G$ ($\hbar = c = 1$).

Reduction to the first order equation

It can be reduced to the first order Hamilton-Jacobi-like equation for $H(\phi)$. From the equation for $\dot{H}$, $\frac{dH}{d\phi} = -\frac{\kappa^2}{2}\dot{\phi}$. Insering this into the equation for H^2, we get

$$\frac{2}{3\kappa^2}\left(\frac{dH}{d\phi}\right)^2 = H^2 - \frac{\kappa^2}{3}V(\phi)$$

Time dependence is determined using the relation

$$t = -\frac{\kappa^2}{2}\int\left(\frac{dH}{d\phi}\right)^{-1} d\phi$$

However, during oscillations of ϕ, $H(\phi)$ acquires non-analytic behaviour of the type $const + \mathcal{O}(|\phi - \phi_1|^{3/2})$ at the points where $\dot{\phi} = 0$, and then the correct matching with another solution is needed.

Inflationary slow-roll dynamics

The crucial element: slow-roll.

Occurs if: $|\ddot{\phi}| \ll H|\dot{\phi}|, \quad \dot{\phi}^2 \ll V$, and then $|\dot{H}| \ll H^2$.

Necessary conditions: $|V'| \ll \kappa V, \quad |V''| \ll \kappa^2 V$. Then

$$H^2 \approx \frac{\kappa^2 V}{3}, \quad \dot{\phi} \approx -\frac{V'}{3H}, \quad N \equiv \ln \frac{a_f}{a} \approx \kappa^2 \int_{\phi_f}^{\phi} \frac{V}{V'}\, d\phi$$

First obtained in A. A. Starobinsky, Sov. Astron. Lett. 4, 82 (1978) in the $V = \frac{m^2\phi^2}{2}$ case and for a bouncing model which has "slow climb" first and "slow roll" after the bounce.

Spectral predictions of the one-field inflationary scenario in GR

Scalar (adiabatic) perturbations:

$$P_\zeta(k) = \frac{H_k^4}{4\pi^2\dot\phi^2} = \frac{GH_k^4}{\pi|\dot H|_k} = \frac{128\pi G^3 V_k^3}{3V_k'^2}$$

where the index k means that the quantity is taken at the moment $t = t_k$ of the Hubble radius crossing during inflation for each spatial Fourier mode $k = a(t_k)H(t_k)$. Through this relation, the number of e-folds from the end of inflation back in time $N(t)$ transforms to $N(k) = \ln \frac{k_f}{k}$ where $k_f = a(t_f)H(t_f)$, t_f denotes the end of inflation.
The spectral slope

$$n_s(k) - 1 \equiv \frac{d\ln P_\zeta(k)}{d\ln k} = \frac{1}{\kappa^2}\left(2\frac{V_k''}{V_k} - 3\left(\frac{V_k'}{V_k}\right)^2\right)$$

Tensor perturbations (A. A. Starobinsky, JETP Lett. 50, 844 (1979)):

$$P_g(k) = \frac{16 G H_k^2}{\pi}; \quad n_g(k) \equiv \frac{d \ln P_g(k)}{d \ln k} = -\frac{1}{\kappa^2}\left(\frac{V_k'}{V_k}\right)^2$$

The consistency relation:

$$r(k) \equiv \frac{P_g}{P_\zeta} = \frac{16|\dot{H}_k|}{H_k^2} = 8|n_g(k)|$$

Tensor perturbations are always suppressed by at least the factor $\sim 8/N(k)$ compared to scalar ones. For the present Hubble scale, $N(k_H) = (50 - 60)$.

Inflation in $f(R)$ gravity

The simplest model of modified gravity (= geometrical dark energy) considered as a phenomenological macroscopic theory in the fully non-linear regime and non-perturbative regime.

$$S = \frac{1}{16\pi G} \int f(R)\sqrt{-g}\, d^4x + S_m$$

$$f(R) = R + F(R), \quad R \equiv R_\mu^\mu$$

Here $f''(R)$ is not identically zero. Usual matter described by the action S_m is minimally coupled to gravity.

Vacuum one-loop corrections depending on R only (not on its derivatives) are assumed to be included into $f(R)$. The normalization point: at laboratory values of R where the scalaron mass (see below) $m_s \approx \mathrm{const}$.

Metric variation is assumed everywhere. Palatini variation leads to a different theory with a different number of degrees of freedom.

Field equations

$$\frac{1}{8\pi G}\left(R_\mu^\nu - \frac{1}{2}\,\delta_\mu^\nu R\right) = -\left(T^\nu_{\mu\,(vis)} + T^\nu_{\mu\,(DM)} + T^\nu_{\mu\,(DE)}\right)\ ,$$

where $G = G_0 = const$ is the Newton gravitational constant measured in laboratory and the effective energy-momentum tensor of DE is

$$8\pi G T^\nu_{\mu\,(DE)} = F'(R)\,R_\mu^\nu - \frac{1}{2}\,F(R)\delta_\mu^\nu + \left(\nabla_\mu\nabla^\nu - \delta_\mu^\nu\nabla_\gamma\nabla^\gamma\right)F'(R)\,.$$

Because of the need to describe DE, de Sitter solutions in the absence of matter are of special interest. They are given by the roots $R = R_{dS}$ of the algebraic equation

$$Rf'(R) = 2f(R)\ .$$

The special role of $f(R) \propto R^2$ gravity: admits de Sitter solutions with any curvature.

Transformation to the Einstein frame and back

In the Einstein frame, free particles of usual matter do not follow geodesics and atomic clocks do not measure proper time.

From the Jordan (physical) frame to the Einstein one:

$$g^E_{\mu\nu} = f' g^J_{\mu\nu}, \quad \kappa\phi = \sqrt{\frac{3}{2}} \ln f', \quad V(\phi) = \frac{f'R - f}{2\kappa^2 f'^2}$$

where $\kappa^2 = 8\pi G$.

Inverse transformation:

$$R = \left(\sqrt{6}\kappa \frac{dV(\phi)}{d\phi} + 4\kappa^2 V(\phi)\right) \exp\left(\sqrt{\frac{2}{3}}\kappa\phi\right)$$

$$f(R) = \left(\sqrt{6}\kappa \frac{dV(\phi)}{d\phi} + 2\kappa^2 V(\phi)\right) \exp\left(2\sqrt{\frac{2}{3}}\kappa\phi\right)$$

$V(\phi)$ should be at least C^1.

Background FRW equations in $f(R)$ gravity

$$ds^2 = dt^2 - a^2(t)\left(dx^2 + dy^2 + dz^2\right)$$

$$H \equiv \frac{\dot{a}}{a}\,, \quad R = 6(\dot{H} + 2H^2)$$

The trace equation (4th order)

$$\frac{3}{a^3}\frac{d}{dt}\left(a^3\frac{df'(R)}{dt}\right) - Rf'(R) + 2f(R) = 8\pi G(\rho_m - 3p_m)$$

The 0-0 equation (3d order)

$$3H\frac{df'(R)}{dt} - 3(\dot{H} + H^2)f'(R) + \frac{f(R)}{2} = 8\pi G\rho_m$$

Reduction to the first order equation

In the absence of spatial curvature and $\rho_m = 0$, it is always possible to reduce these equations to a first order one using the transformation to the Einstein frame and the Hamilton-Jacobi-like equation for a minimally coupled scalar field in a spatially flat FLRW metric:

$$\frac{2}{3\kappa^2}\left(\frac{dH_E(\phi)}{d\phi}\right)^2 = H_E^2 - \frac{\kappa^2}{3}V(\phi)$$

where

$$H_E \equiv \frac{d}{dt_E}\ln a_E = \frac{1}{\sqrt{f'}}\frac{d}{dt}\left(\ln a + \frac{1}{2}\ln f'\right)$$

$$= \frac{1}{2\sqrt{f'}}\left(3H + \frac{\dot{H}}{H} - \frac{f}{6Hf'}\right)$$

From a solution $H_E(\phi(R))$ of this equation, the scale factor $a(t)$ follows in the parametric form:

$$\ln a = -\frac{1}{2}\ln f'(R) - \frac{3}{4}\int \left(\frac{f''}{f'}\right)^2 H_E(R) \left(\frac{dH_E(R)}{dR}\right)^{-1} dR$$

$$t = -\frac{3}{4}\int \left(\frac{f''}{f'}\right)^2 \left(\frac{dH_E(R)}{dR}\right)^{-1} dR$$

Analogues of large-field (chaotic) inflation: $F(R) \approx R^2 A(R)$ for $R \to \infty$ with $A(R)$ being a slowly varying function of R, namely

$$|A'(R)| \ll \frac{A(R)}{R}\ ,\ |A''(R)| \ll \frac{A(R)}{R^2}\ .$$

In particular,

$$f(R) \approx \frac{R^2}{6m^2 \ln^2(R/m^2)}$$

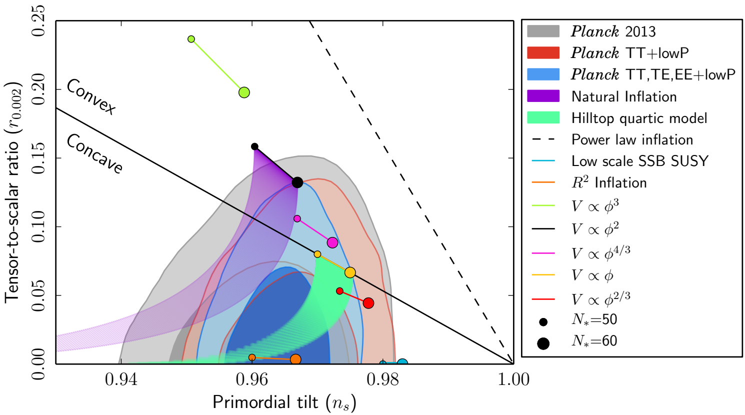
for $R \gg m^2$ to have the same n_s, r as for $V = m^2\phi^2/2$.

Analogues of small-field (new) inflation, $R \approx R_1$:

$$F'(R_1) = \frac{2F(R_1)}{R_1}\ ,\ F''(R_1) \approx \frac{2F(R_1)}{R_1^2}\ .$$

Thus, all inflationary models in $f(R)$ gravity are close to the simplest one over some range of R.

Comparison with some simple models



The simplest models producing the observed scalar slope

I. In the Einstein gravity:

$$V(\phi) = \frac{m^2\phi^2}{2}$$

$$m \approx 1.3 \times 10^{-6} \left(\frac{55}{N}\right) M_{Pl} \approx 1.6 \times 10^{13}\,\mathrm{GeV}$$

$$n_s - 1 = -\frac{2}{N} \approx -0.036, \quad r = \frac{8}{N} \approx 0.15$$

$$H_{dS}(N = 55) = 1.0 \times 10^{14}\,\mathrm{GeV}$$

Practically excluded by the latest BICEP2/Keck Array/Planck data: $r < 0.07$ at 95% c.f.
(P. A. R. Ade et al., arXiv:1510.09217).

II. In the modified $f(R)$ gravity:

$$f(R) = R + \frac{R^2}{6M^2}$$

$$M = 2.6 \times 10^{-6} \left(\frac{55}{N}\right) M_{Pl} \approx 3.2 \times 10^{13}\,\mathrm{GeV}$$

$$n_s - 1 = -\frac{2}{N} \approx -0.036, \quad r = \frac{12}{N^2} \approx 0.004$$

$$H_{dS}(N = 55) = 1.4 \times 10^{14}\,\mathrm{GeV}$$

The same prediction from a scalar field model with $V(\phi) = \frac{\lambda\phi^4}{4}$ at large ϕ and strong non-minimal coupling to gravity $\xi R\phi^2$ with $\xi < 0,\ |\xi| \gg 1$, including the Brout-Englert-Higgs inflationary model.

Note similar predictions for the masses m and M and for $H_{dS}(N = 55)$.

The Lagrangian density for this model

$$\mathcal{L} = \frac{R}{16\pi G} + \frac{N^2}{288\pi^2 P_\zeta(k)} R^2 = \frac{R}{16\pi G} + 5\times 10^8\, R^2$$

1. The specific case of the fourth order gravity in 4D

$$\mathcal{L} = \frac{R}{16\pi G} + AR^2 + BC_{\alpha\beta\gamma\delta}C^{\alpha\beta\gamma\delta}$$

for which $A \gg 1,\ A \gg |B|$.

2. Another, completely different way: a non-minimally coupled scalar field with a large negative coupling ξ ($\xi_{conf} = \frac{1}{6}$):

$$L = \frac{R}{16\pi G} - \frac{\xi R\phi^2}{2} + \frac{1}{2}\phi_{,\mu}\phi^{,\mu} - V(\phi), \quad \xi < 0, \quad |\xi| \gg 1 \ .$$

In this limit, the Higgs-like scalar tree level potential $V(\phi) = \frac{\lambda(\phi^2-\phi_0^2)^2}{4}$ just produces $f(R) = \frac{1}{16\pi G}\left(R + \frac{R^2}{6M^2}\right)$ with $M^2 = \lambda/24\pi\xi^2 G$ and $\phi^2 = |\xi|R/\lambda$ (plus small corrections $\propto |\xi|^{-1}$).

Smooth potential reconstruction from scalar power spectrum in GR

In the slow-roll approximation:

$$\frac{V^3}{V'^2} = CP_\zeta(k(t(\phi))), \quad C = \frac{12\pi^2}{\kappa^6}$$

Changing variables for ϕ to $N(\phi)$ and integrating, we get:

$$\frac{1}{V(N)} = -\frac{\kappa^4}{12\pi^2}\int \frac{dN}{P_\zeta(N)}$$

$$\kappa\phi = \int dN\sqrt{\frac{d\ln V}{dN}}$$

First derived in H. M. Hodges and G. R. Blumenthal, Phys. Rev. D 42, 3329 (1990).

An ambiguity in the form of $V(\phi)$ because of an integration constant in the first equation. Information about $P_g(k)$ (even a negative one) helps to remove this ambiguity.

"Scale-free" reconstruction

Numerical coincidence between $2/N(k_H)$ and $1 - n_s$.

Let us assume that it is not a coincidence but happens for all $1 \ll N \lesssim 60$:

$$P_\zeta = P_0 N^2$$

Then:

$$V = V_0 \frac{N}{N + N_0} = V_0 \tanh^2 \frac{\kappa\phi}{2\sqrt{N_0}}$$

$$r = \frac{8N_0}{N(N + N_0)}$$

$r \sim 0.003$ for $N_0 \sim 1$. From the upper limit on r: $N_0 < 100$ for $N = 57$.

On the other hand, $m_{infl}(0) = \sqrt{6\pi^2 P_0}\,\kappa^{-1} \approx 1.6 \times 10^{13}\,\mathrm{GeV}$ and does not depend on N_0 and r.

Inflation reconstruction in $f(R)$ gravity

$$f(R) = R^2 A(R)$$

$$A = const - \frac{\kappa^2}{96\pi^2} \int \frac{dN}{P_\zeta(N)}$$

$$\ln R = const + \int dN \sqrt{-\frac{2\, d \ln A}{3\, dN}}$$

Here, the additional assumptions that $P_\zeta \propto N^\beta$ and that the resulting $f(R)$ can be analytically continued to the region of small R without introducing a new scale, and it has the linear (Einstein) behaviour there, leads to $\beta = 2$ and the $R + R^2$ inflationary model with $r = \frac{12}{N^2} = 3(n_s - 1)^2$ unambiguously.

GW from inflation

The typical inflationary prediction that $r < 8(1 - n_s) \approx 0.3$ is confirmed.

Moreover: no sign of GW in the CMB temperature anisotropy power spectrum. For $1 \ll \ell \lesssim 50$, the Sachs-Wolfe plateau occurs for the contribution from GW, too:

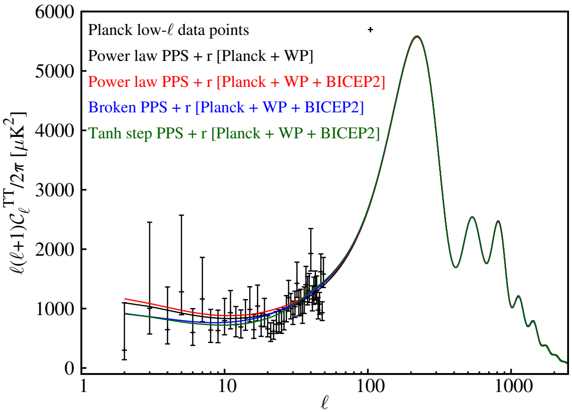
$$\ell(\ell+1)C_{\ell,g} = \frac{\pi}{36}\left(1 + \frac{48\pi^2}{385}\right) P_g$$

assuming $n_t = 1$ (A. A. Starobinsky, Sov. Astron. Lett. 11, 133 (1985)). So,

$$C_\ell = C_{\ell,s} + C_{\ell,g} = (1 + 0.775r)C_{\ell,s}$$

.

For larger $\ell > 50$, $\ell(\ell+1)C_{\ell,s}$ grows and the first acoustic peak forms at $\ell \approx 200$, while $\ell(\ell+1)C_{\ell,g}$ decreases quickly. Thus, the presence of GW should lead to a step-like enhancement of $\ell(\ell+1)C_\ell$ for $\ell \lesssim 50$.



Planck low-ℓ data points
Power law PPS + r [Planck + WP]
Power law PPS + r [Planck + WP + BICEP2]
Broken PPS + r [Planck + WP + BICEP2]
Tanh step PPS + r [Planck + WP + BICEP2]
$\ell(\ell+1)\mathcal{C}_\ell^{TT}/2\pi\ [\mu K^2]$
ℓ
0
1000
2000
3000
4000
5000
6000
1
10
100
1000

Small features in the power spectrum

Contrary, a $\sim 10\%$ depression has been discovered for $20 \lesssim \ell \lesssim 40$.

The effect of at least the same order: an upward wiggle at $\ell \approx 40$ (the Archeops feature) and a downward one at $\ell \approx 22$.

Lesson: irrespective of the search for primordial GW from inflation, features in the anisotropy spectrum for $20 \lesssim \ell \lesssim 40$ confirmed by WMAP and Planck should be taken into account and studied seriously. Some new physics beyond one slow-rolling inflaton may show itself through them.

For a more elaborated class of models suggested by previous studies of sharp features in the inflaton potential caused, e.g. by a fast phase transition occurred in another field coupled to the inflaton during inflation, see
D. K. Hazra, A. Shafieloo, G. F. Smoot and A. A. Starobinsky, JCAP 1408, 048 (2014).

Conclusions

- Inflation is being transformed into a normal physical theory, based on some natural assumptions confirmed by observations and used to obtain new theoretical knowledge from them.
- First quantitative observational evidence for small quantities of the first order in the slow-roll parameters: the measurement of $n_s(k) - 1$ and the upper limit on $r(k)$. The generic inflationary predictions $|n_s - 1| \ll 1$ and $r \ll 1$ are confirmed. Typical consequences from this: $H_{55} \sim 10^{14}\,\mathrm{GeV}$, $m_{infl} \sim 10^{13}\,\mathrm{GeV}$.
- The quantitative theoretical prediction of these quantities is based on gravity (space-time metric) quantization and requires very large space-time curvature in the past of our Universe with a characteristic length only five orders of magnitude larger than the Planck one.

- Using the measured value of $n_s - 1$ and assuming a scale-free scalar power spectrum leads to the prediction that the region $r > 10^{-3}$ is well expected. Under the same assumptions, r can be even larger and close to its present observational upper limit in two-parametric inflationary models having large, but not too large $N_0 \sim N$. However, this requires a moderate amount of parameter tuning.
- Regarding CMB temperature anisotropy, small features in the multipole range $20 \lesssim \ell \lesssim 40$ at the accuracy level $\sim 1\ \mu\mathrm{K}$ which mask the GW contribution to CMB temperature anisotropy have to be investigated and understood. They may reflect some fine structure of inflation (i.e. fast phase transitions in other quantum fields coupled to an inflaton during inflation).
- Though the Einstein gravity plus a minimally coupled inflaton remains sufficient for description of inflation with existing observational data, modified (in particular, scalar-tensor or $f(R)$) gravity can do it as well.