Quantum electrodynamics in strong and supercritical Coulomb fields

Vladimir Shabaev

St. Petersburg State University

NRC “Kurchatov Institute” - PNPI

Outline of the talk

- Introduction
- QED in strong Coulomb field
- QED in supercritical Coulomb field
- How to observe the vacuum decay
- Conclusion

Introduction: QED

Early Quantum Electrodynamics (QED): Dirac, Heisenberg, Jordan, Pauli, Fermi, Born, Fock, Wigner, ... (1926-1934)

Discovery of the Lamb shift in Hydrogen: Lamb and Retherford (1947)

$$(E_{2s} - E_{2p_{1/2}})_{\rm exp} = 1062(5){\rm MHz}$$

First evaluation of the Lamb shift: Bethe (1947)

$$(E_{2s} - E_{2p_{1/2}})_{\rm theor} \approx 1040{\rm MHz}$$

Modern QED formalism: Dyson, Feynman, Schwinger, Tomonaga (1946-1950)

Mass and charge renormalization: m_0, $e_0 \to m$, e.

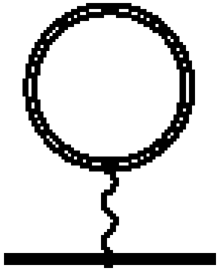
Perturbation theory in $\alpha \approx 1/137$.

Introduction: QED

Feynman diagrams

Lamb shift

Self energy (SE)



Vacuum polarization (VP)

Introduction: tests of QED with atomic systems

Light atoms ($\alpha Z \ll 1$, weak fields):

Tests of QED to lowest orders in α and αZ.

Heavy few-electron ions ($\alpha Z \sim 1$, strong fields):

Tests of QED in nonperturbative in αZ regime.

Low-energy heavy-ion collisions at $Z_1 + Z_2 > 173$ (supercritical fields):

Tests of QED in supercritical regime.

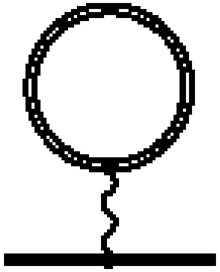
Binding energies in heavy few-electron ions

QED corrections

Calculations in the external field approximation $(M \to \infty)$

First-order QED corrections

P.J. Mohr, Ann. Phys., 1974



G. Soff and P.J. Mohr, PRA, 1988
N.L. Manakov, A.A Nekipelov,
A.G. Fainshtein, JETP, 1989

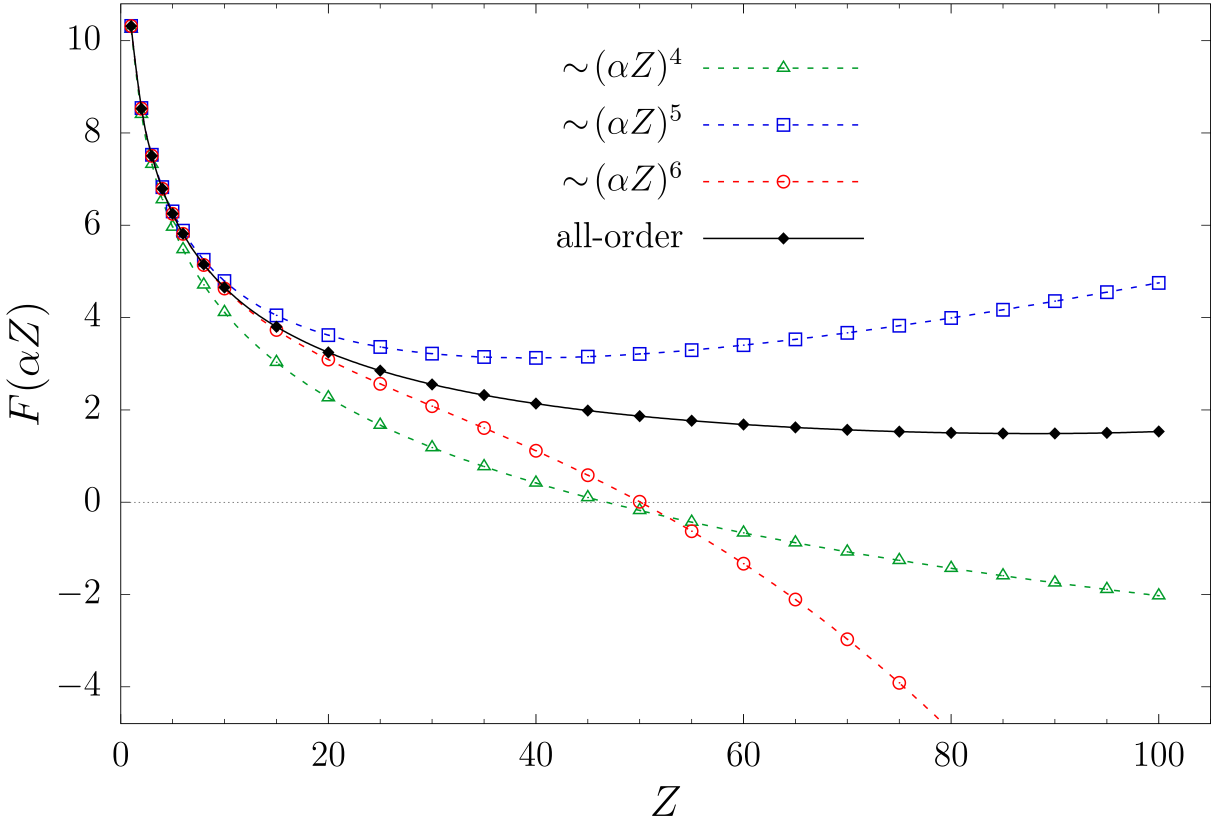
Binding energies in heavy few-electron ions

Self-energy correction for $1s$:

$$\Delta E_{\rm SE} = \frac{\alpha}{\pi}\frac{(\alpha Z)^4}{n^3}F(\alpha Z)mc^2$$

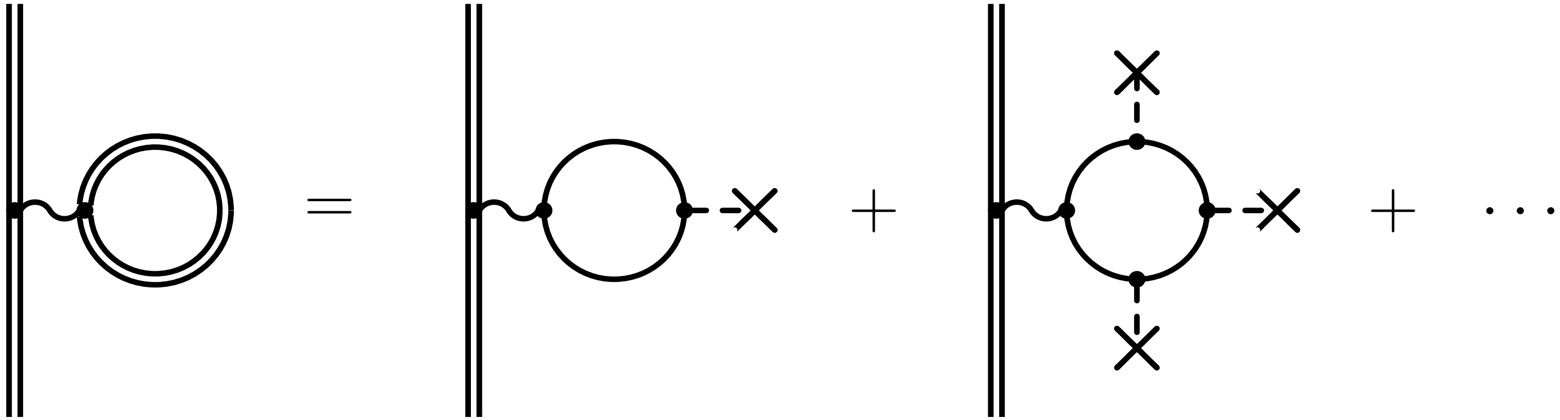
The αZ-expansion has the form ($L = \ln[(\alpha Z)^{-2}]$):

$$F(\alpha Z) = LA_{41} + A_{40} + (\alpha Z)A_{50} + (\alpha Z)^2\left[L^2A_{62} + LA_{61} + A_{60}\right] + \ldots$$



Binding energies in heavy few-electron ions

Evaluation of the one-loop vacuum-polarization diagram



The first term after the renormalization gives the Uehling potential:

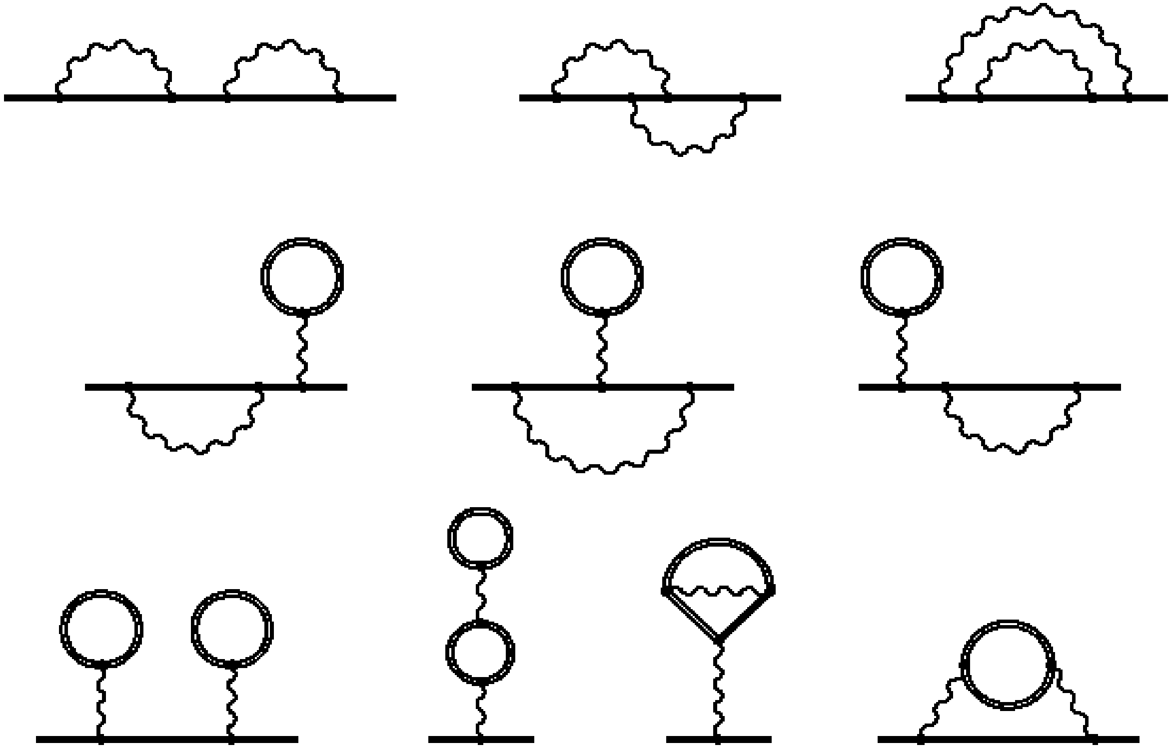
$$U_{\rm Uehl}(r) \quad = \quad -\alpha Z \frac{2\alpha}{3\pi} \int\limits_0^\infty dr' \; 4\pi r' \rho(r') \int\limits_1^\infty dt \; (1 + \frac{1}{2t^2}) \frac{\sqrt{t^2-1}}{t^2}$$

$$\times \frac{[\exp(-2m|r-r'|t) - \exp(-2m(r+r')t)]}{4mrt} \,.$$

where $|e|Z\rho(r)$ is the density of the nuclear charge distribution.

Binding energies in heavy few-electron ions

One-electron second-order QED corrections

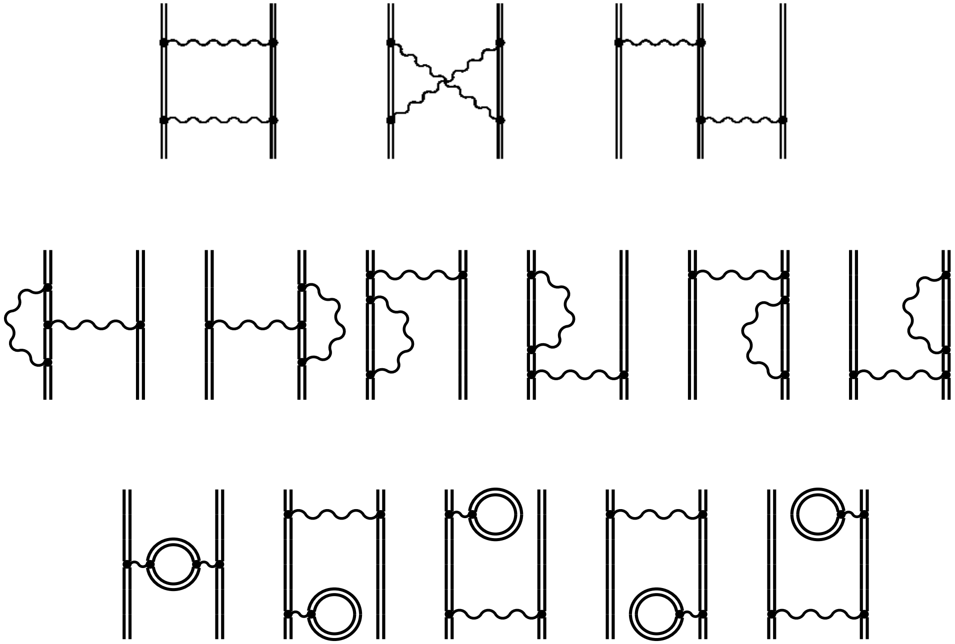


Evaluation of the two-loop self-energy diagrams:

V.A. Yerokhin, P. Indelicato, and V.M. Shabaev, PRL, 2006.

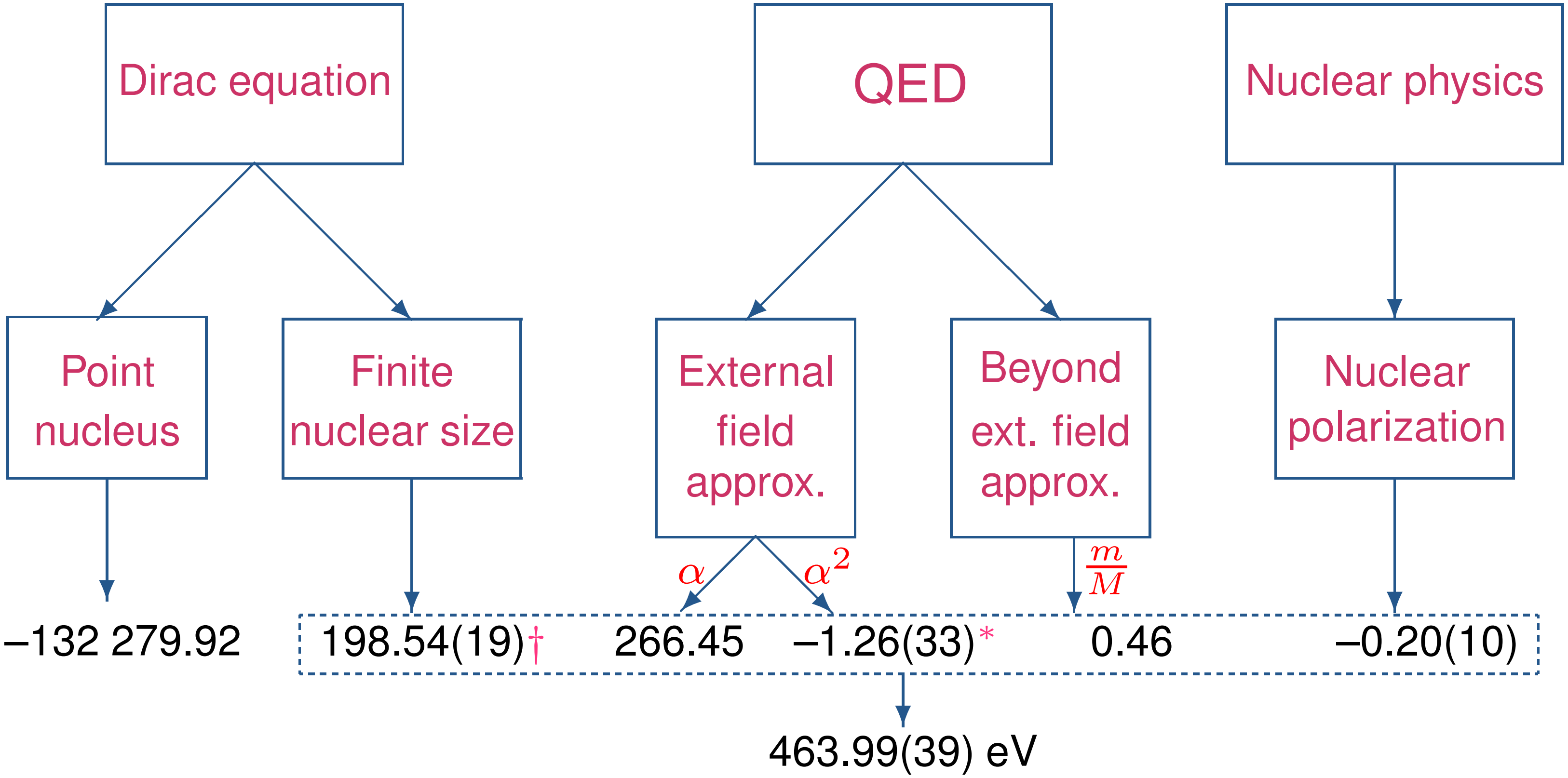
Binding energies in heavy few-electron ions

Two- and three-electron second-order QED corrections



Latest progress: Evaluations of all these diagrams for quasidegenerate states in Be-like ions *(A.V. Malyshev et al., PRL, 2021)*.

1s Lamb shift in H-like uranium, in eV



Dirac equation

QED

Nuclear physics

Point nucleus

Finite nuclear size

External field approx.

Beyond ext. field approx.

Nuclear polarization

α

α^2

$\frac{m}{M}$

–132 279.92

198.54(19)† 266.45 –1.26(33)* 0.46 –0.20(10)

463.99(39) eV

Experiment: 460.2(4.6) eV

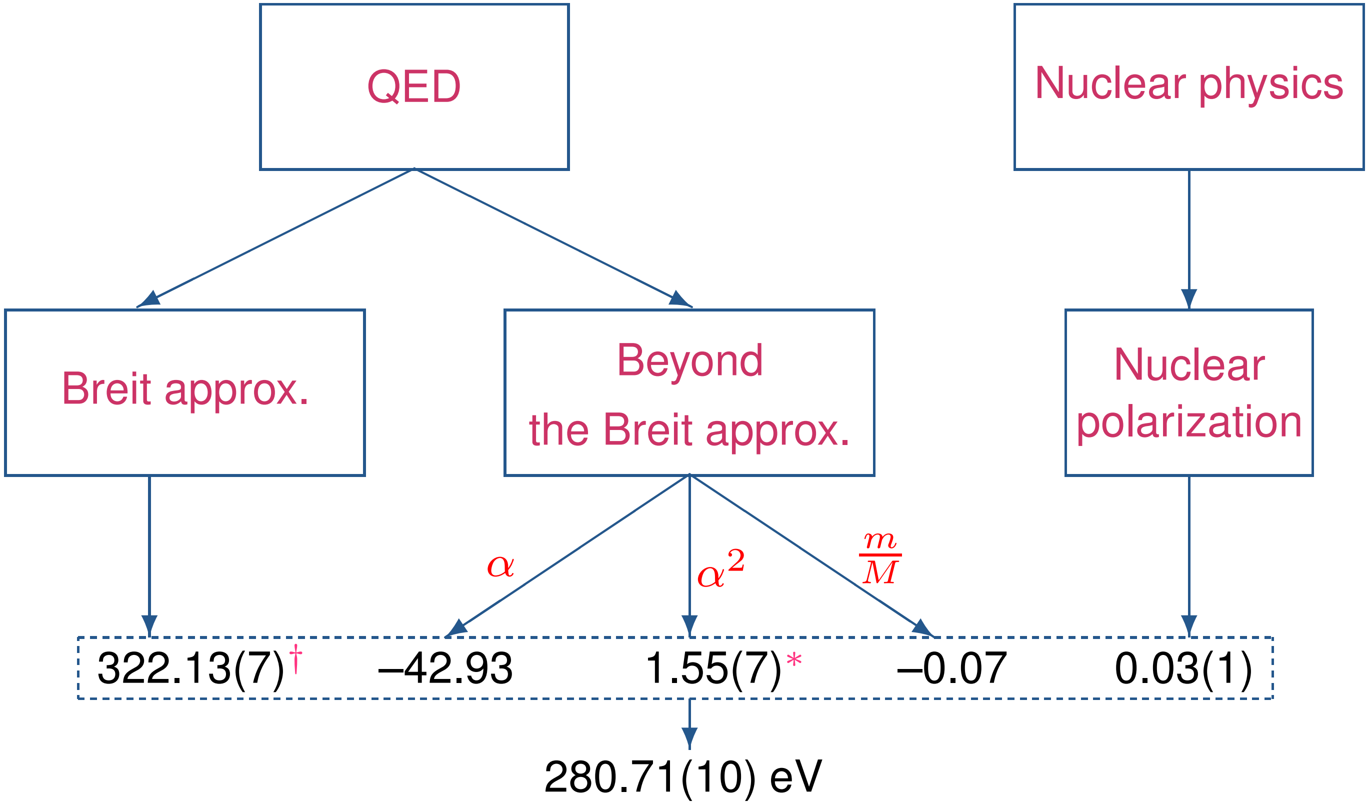
(A. Gumberidze, T. Stöhlker, D. Banas et al., PRL, 2005)

Test of QED: $\sim$ 2%

* V.A. Yerokhin, P. Indelicato, and V.M. Shabaev, PRL, 2006

† Y.S. Kozhedub, O.V. Andreev, V.M. Shabaev et al., PRA, 2008

$2p_{1/2}$-$2s$ *transition energy in Li-like uranium, in eV*



QED

Nuclear physics

Breit approx.

Beyond the Breit approx.

Nuclear polarization

322.13(7)† –42.93 1.55(7)* –0.07 0.03(1)

280.71(10) eV

Experiment: 280.59(10) eV (J. Schweppe et al., PRL, 1991)
280.52(10) eV (C. Brandau et al., PRL, 2003)
280.645(15) eV (P. Beiersdorfer et al., PRL, 2005)

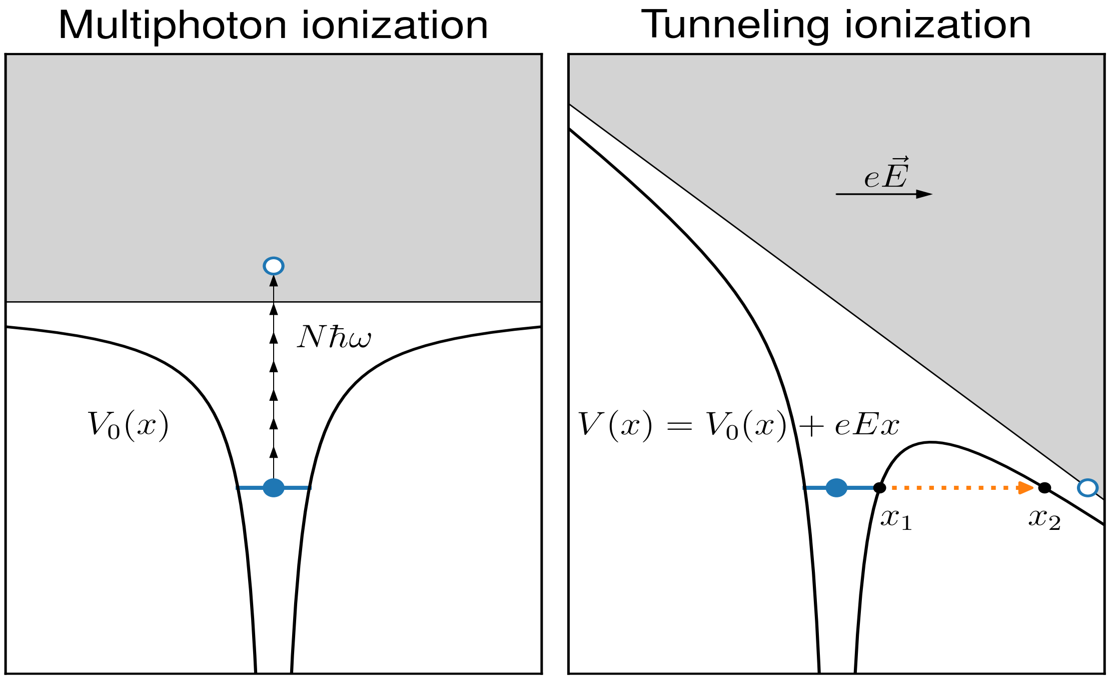
Test of QED: $\sim$ 0.2%

* V.A. Yerokhin, P. Indelicato, and V.M. Shabaev, PRL, 2006

† Y.S. Kozhedub, O.V. Andreev, V.M. Shabaev et al., PRA, 2008

QED at supercritical fields

Ionization in quantum mechanics



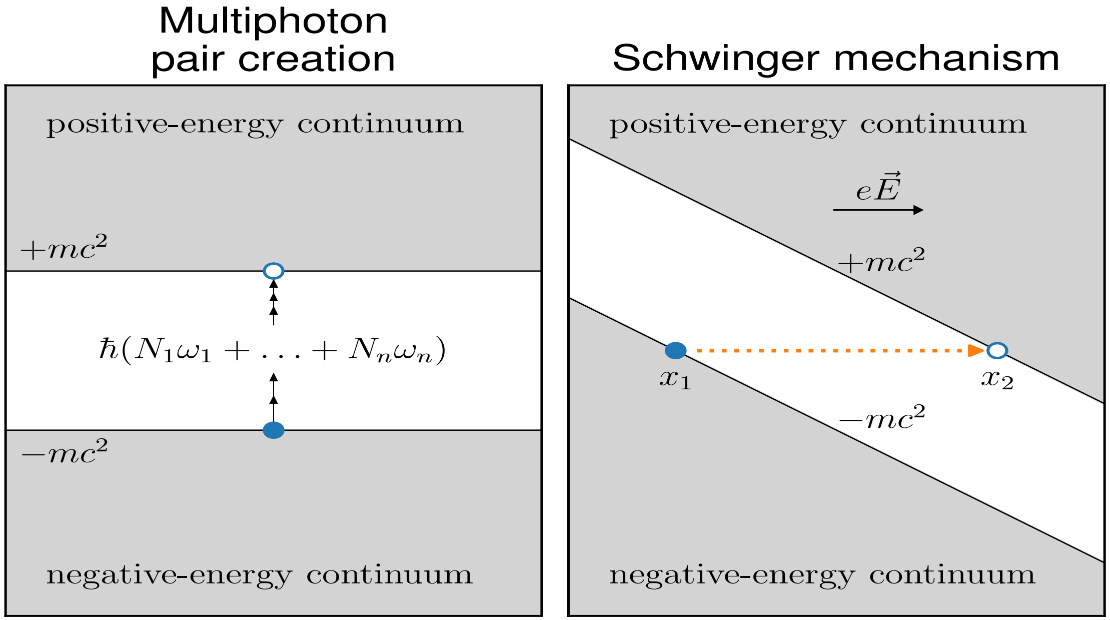
The tunneling probability for a static uniform electric field E:

$$W \sim \exp\Big\{-\frac{2}{\hbar}\int_{x_1}^{x_2} dx\sqrt{2m(V(x)-\mathcal{E})}\Big\}$$

where $V(x) = V_0(x) + eEx$ and $\mathcal{E}$ is the electron energy.

QED at supercritical fields

Electron-positron pair creation



The rate of pair production for a static uniform electric field E:

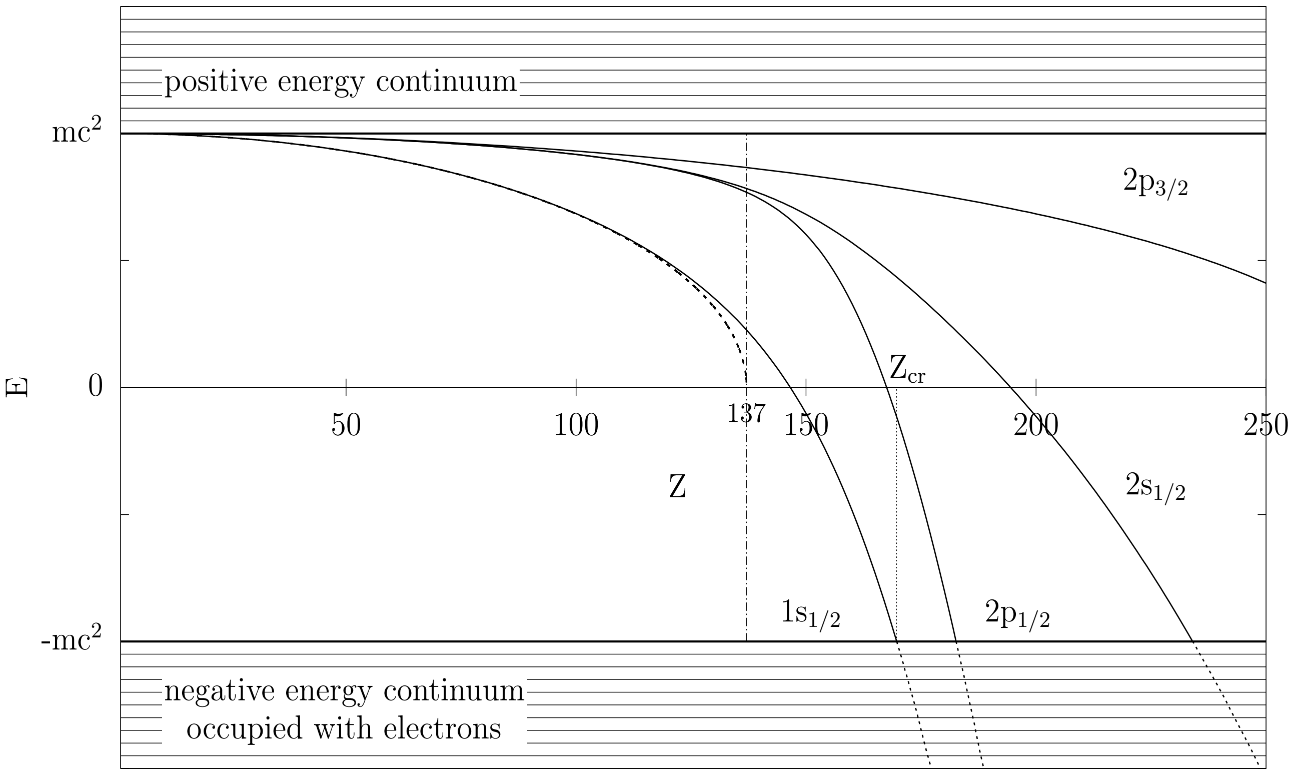
$$\frac{d^4 n_{e^+e^-}}{d^3x dt} \sim \frac{c}{4\pi^3 {\lambda_\mathrm{C}}^4} \exp\left(-\pi \frac{E_c}{E}\right)$$

where $\lambda_\mathrm{C} = \hbar/(mc)$ and $E_c = m^2c^3/(e\hbar) \approx 1.3 \times 10^{16}\mathrm{V/cm}$.

QED at supercritical Coulomb field

Supercritical Coulomb field

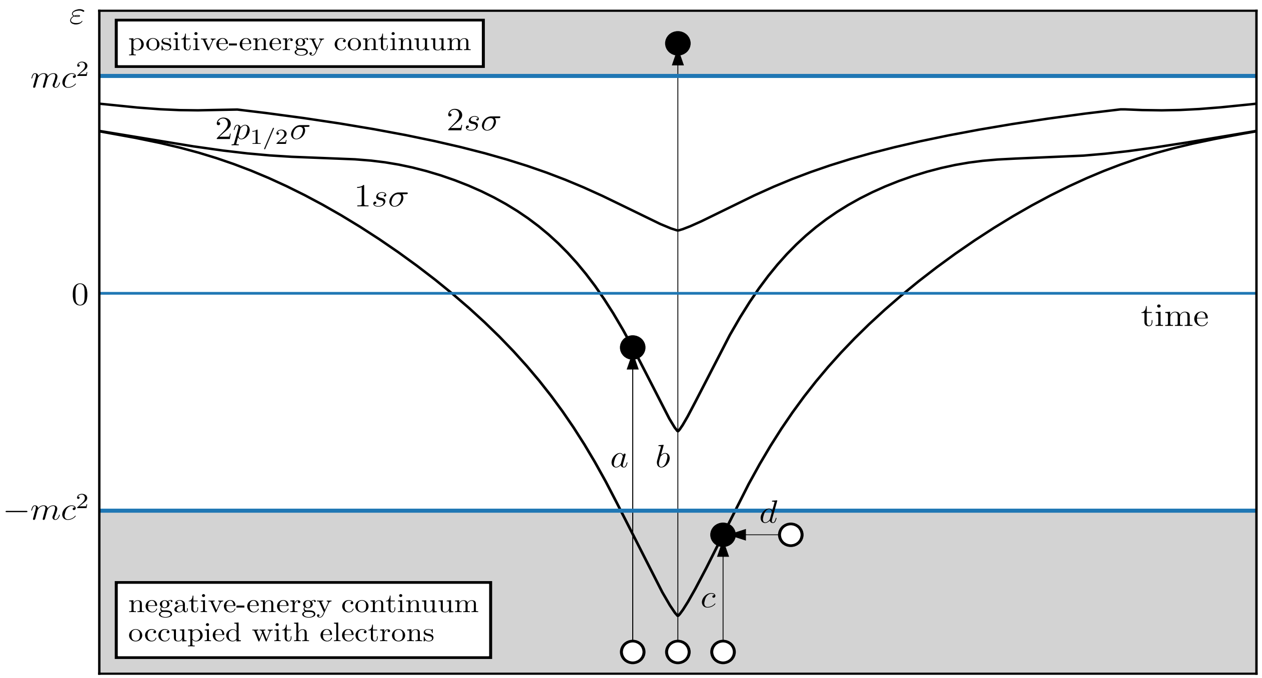
S.S. Gershtein, Ya.B. Zel'dovich, 1969; W. Pieper, W. Greiner, 1969



The $1s$ level dives into the negative-energy continuum at $Z_{\text{crit}} \approx 173$.

Low-energy heavy-ion collisions

Creation of electron-positron pairs in low-energy heavy-ion collisions, with $Z_1 + Z_2 > 173$

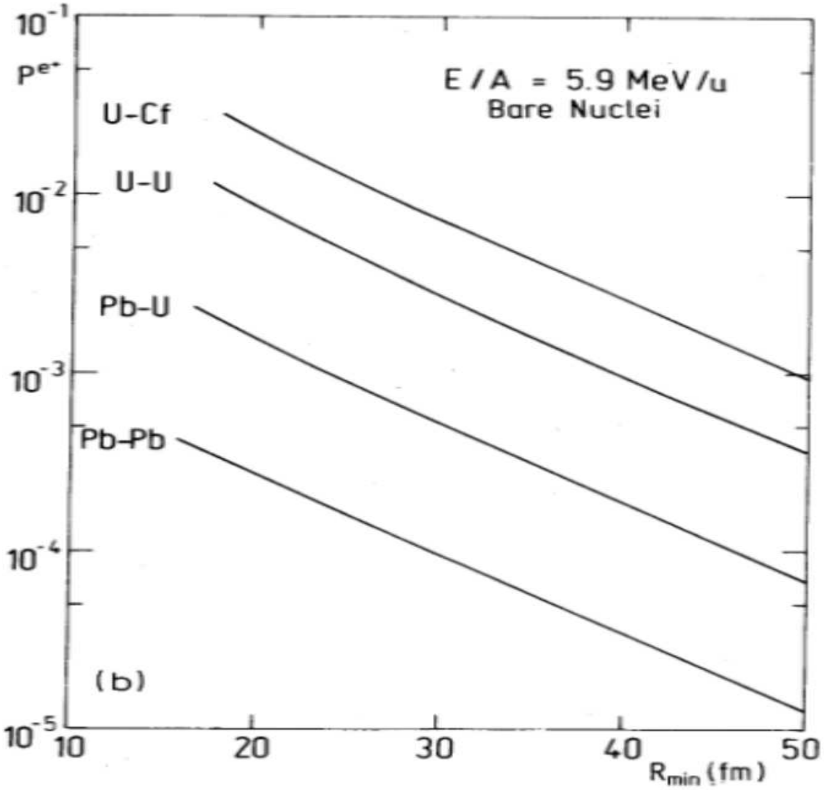


Dynamical mechanism: a),b),c). Spontaneous mechanism (vacuum decay): d). The $1s$ state dives into the negative-energy continuum for about 10^{-21} sec.

Low-energy heavy-ion collisions

Positron production probability in 5.9 MeV/u collisions of bare nuclei as a function of distance of closest approach $R_{\min}$

(J. Reinhardt, B. Müller, and W. Greiner, Phys. Rev. A, 1981).



Conclusion by Frankfurt's group (2005):The vacuum decay could only be observed in collisions with nuclear sticking, in which the nuclei are bound to each other for some period of time by nuclear forces.

Low-energy heavy-ion collisions

New methods for calculations of quantum dynamics of electron-positron field in low-energy heavy-ion collisions at subcritical and supercritical regimes have been developed:

- *I.I. Tupitsyn, Y.S. Kozhedub, V.M. Shabaev et al., Phys. Rev. A 82, 042701 (2010).*
- *I. I. Tupitsyn, Y. S. Kozhedub, V. M. Shabaev et al., Phys. Rev. A 85, 032712 (2012).*
- *G. B. Deyneka, I. A. Maltsev, I. I. Tupitsyn et al., Russ. J. of Phys. Chem. B 6, 224 (2012).*
- *G. B. Deyneka, I. A. Maltsev, I. I. Tupitsyn et al., Eur. Phys. J. D 67, 258 (2013).*
- *Y.S. Kozhedub, V.M. Shabaev, I.I. Tupitsyn et al., Phys. Rev. A 90, 042709 (2014).*
- *I.A. Maltsev, V.M. Shabaev, I.I. Tupitsyn et al., NIMB, 408, 97 (2017).*
- *R.V. Popov, A.I. Bondarev, Y.S. Kozhedub et al., Eur. Phys. J. D 72, 115 (2018).*
- *I.A. Maltsev, V.M. Shabaev, R.V. Popov et al., Phys. Rev. A 98, 062709 (2018).*

Low-energy heavy-ion collisions

Time-dependent Dirac equation

$$i\frac{\partial}{\partial t}\psi(\mathbf{r},t) = (\boldsymbol{\alpha}\cdot\mathbf{p} + \beta m_e + V(\mathbf{r},t))\psi(\mathbf{r},t)$$

with

$$V(\mathbf{r},t) = V_{\mathrm{A}}(|\mathbf{r}-\mathbf{R}_{\mathrm{A}}(t)|) + V_{\mathrm{B}}(|\mathbf{r}-\mathbf{R}_{\mathrm{B}}(t)|)\,.$$

We introduce two sets of the solutions *(see book: E.S. Fradkin, D.M. Gitman, S.M. Shvartsman, Quantum Electrodynamics with Unstable Vacuum, 1991):*

$$\psi_i^{(+)}(\mathbf{r},t_{\mathrm{in}}) = \phi_i^{\mathrm{in}}(\mathbf{r})\,, \qquad \psi_i^{(-)}(\mathbf{r},t_{\mathrm{out}}) = \phi_i^{\mathrm{out}}(\mathbf{r})\,,$$

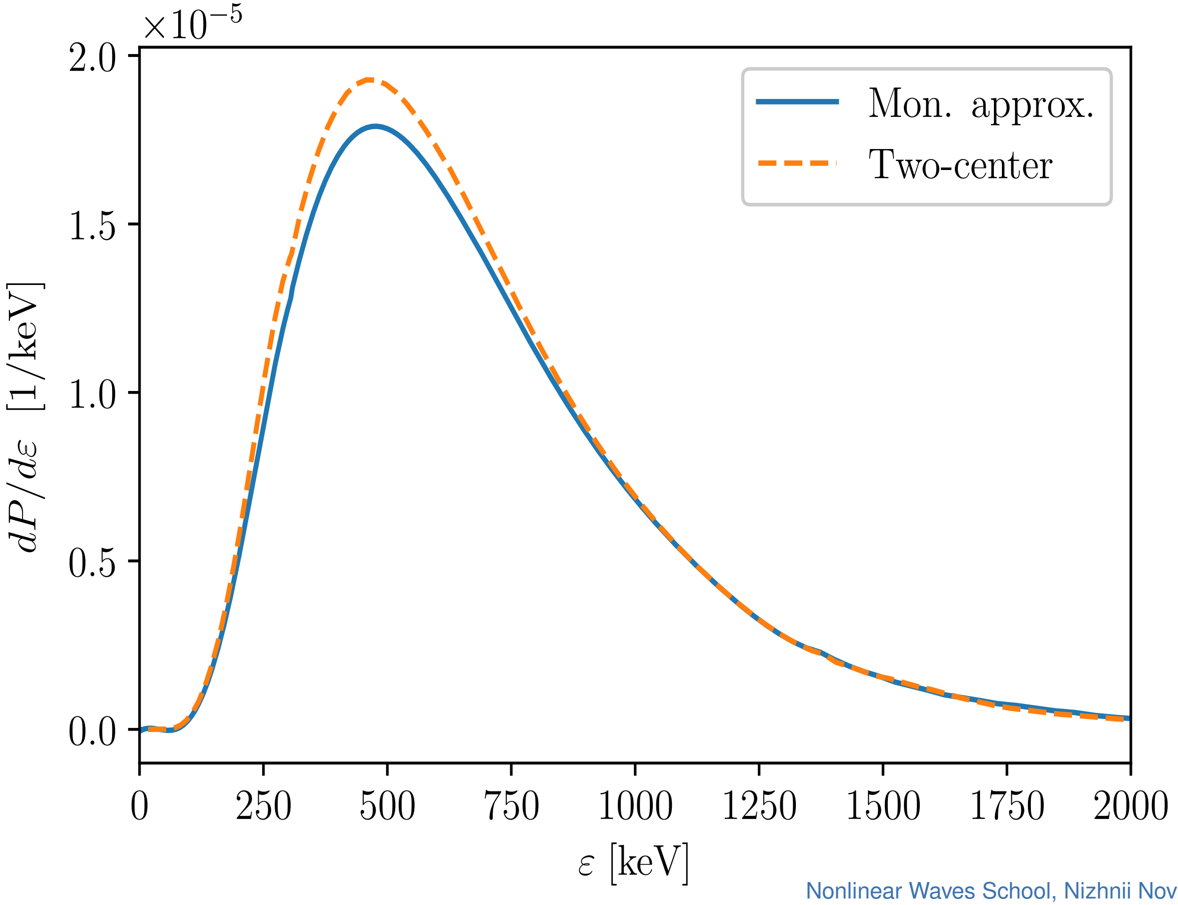
where $\phi_i^{\mathrm{in}}(\mathbf{r})$ and $\phi_i^{\mathrm{out}}(\mathbf{r})$ are the eigenfunctions of the Dirac Hamiltonian at the corresponding time moments. The number of created positrons in a state “p” is given by

$$\overline{n}_p = \sum_{i>F}\left|\int d\mathbf{r}\psi_p^{(-)\dagger}(\mathbf{r},t)\psi_i^{(+)}(\mathbf{r},t)\right|^2\,.$$

Low-energy heavy-ion collisions

Pair creation beyond the monopole approximation

Positron energy spectrum for the U$-$U head-on collision at energy E_{cm} = 740 MeV *(I.A. Maltsev, V.M. Shabaev, R.V. Popov et al., PRA, 2018; R.V. Popov, V.M. Shabaev, I.A. Maltsev et al., PRD, 2023)*



Low-energy heavy-ion collisions

Pair creation beyond the monopole approximation

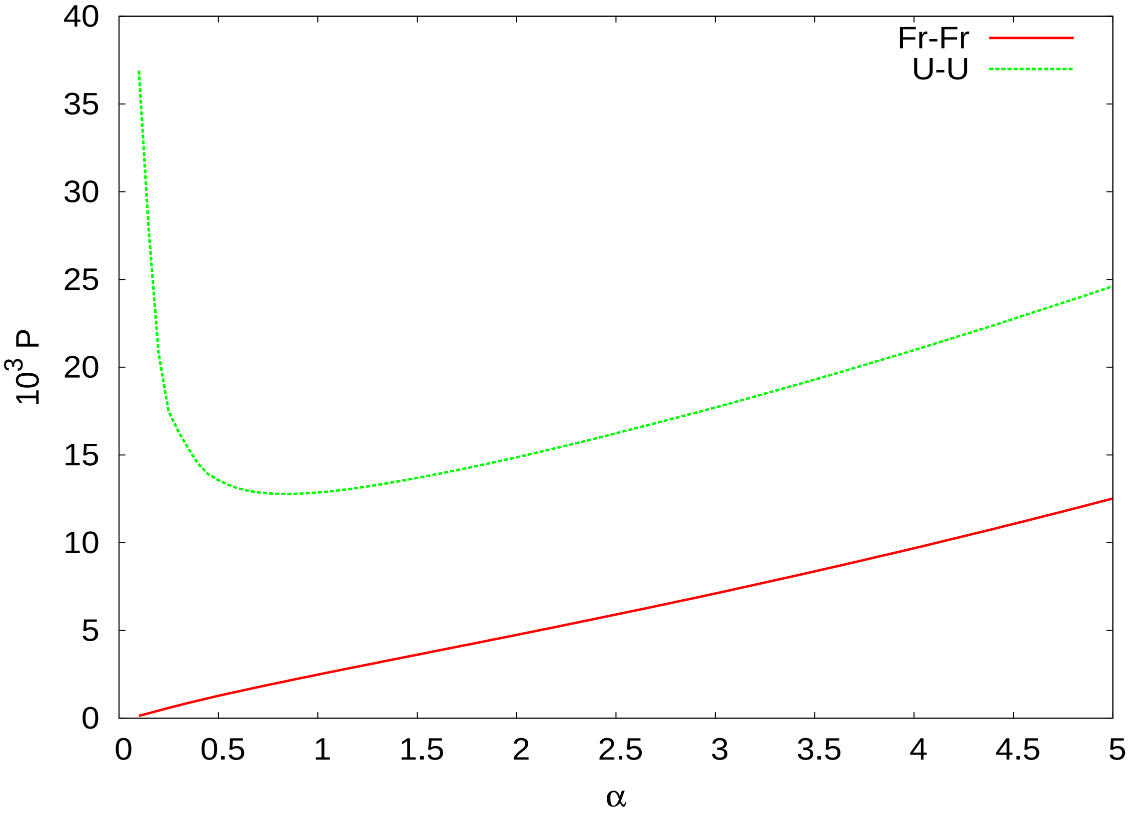
U-U, $E_{\rm cm}$ = 740 MeV

Expected number of created pairs as a function of the impact parameter b

(I.A. Maltsev, V.M. Shabaev, R.V. Popov et al., PRA, 2018;
R.V. Popov, V.M. Shabaev, I.A. Maltsev et al., PRD, 2023)

b (fm)	Monopole approximation	Two-center approach
0	1.29×10^{-2}	1.35×10^{-2}
10	7.26×10^{-3}	7.78×10^{-3}
20	2.75×10^{-3}	3.09×10^{-3}
30	1.04×10^{-3}	1.22×10^{-3}

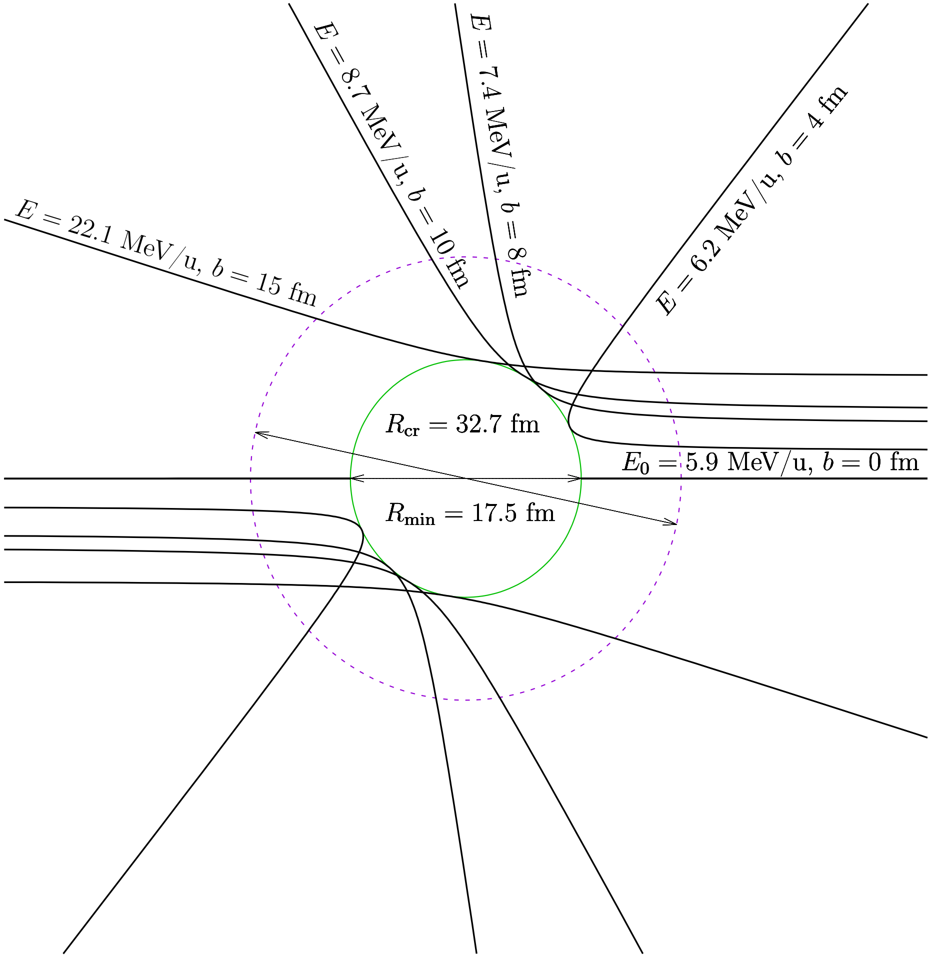
Low-energy heavy-ion collisions



Pair creation with artificial trajectories for the supercritical U$-$U and subcritical Fr$-$Fr head-on collisions at $E_{\rm cm}=674.5$ and $E_{\rm cm}=740$ MeV, respectively. The trajectory $R_\alpha(t)$ is defined by $\dot{R}_\alpha(t)=\alpha\dot{R}(t)$, where $R(t)$ is the classical Rutherford trajectory *(I.A. Maltsev, V.M. Shabaev, I.I. Tupitsyn et al., PRA, 2015)*.

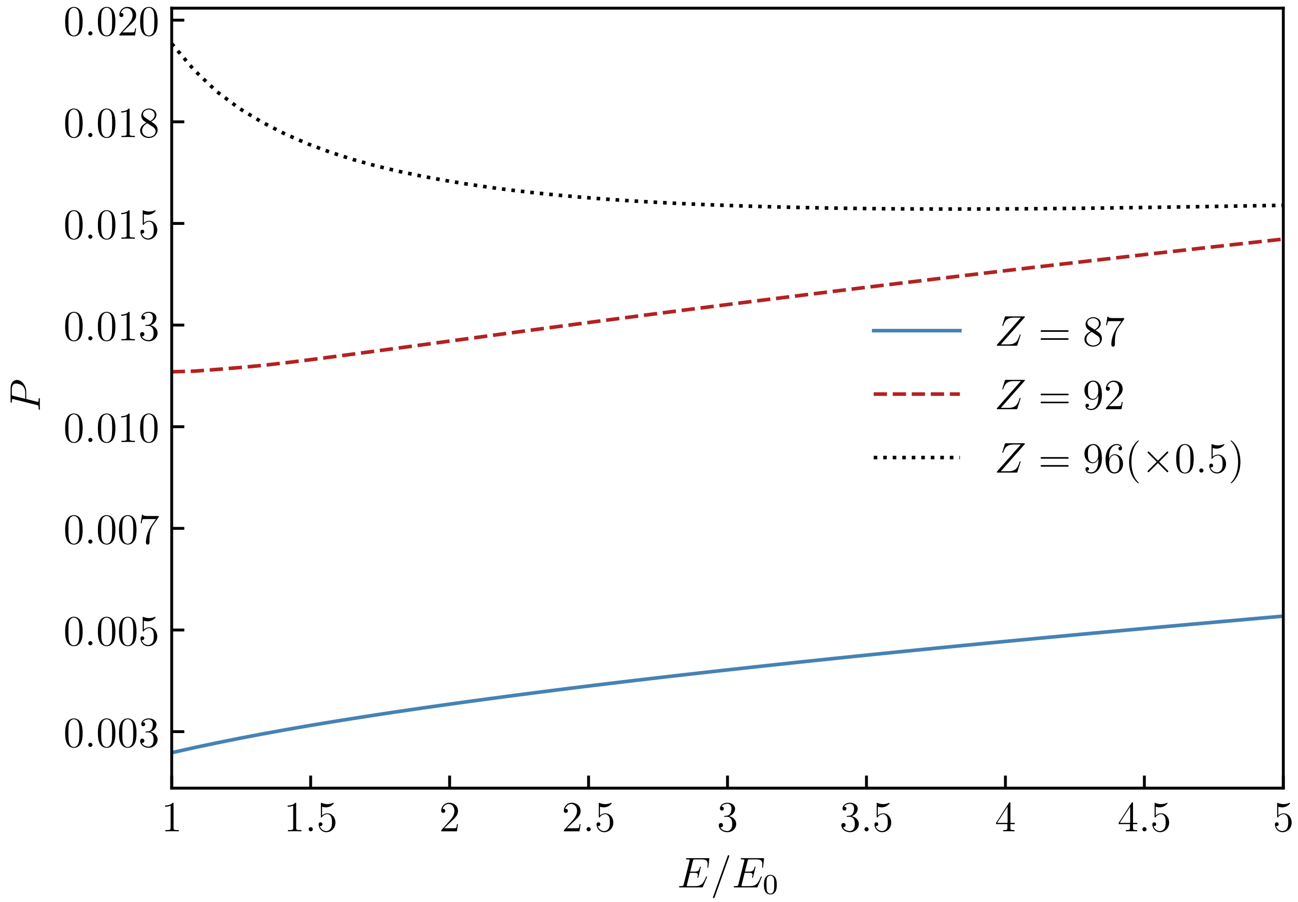
How to observe the vacuum decay

(I.A. Maltsev et al., PRL, 2019; R.V. Popov et al., PRD, 2020)



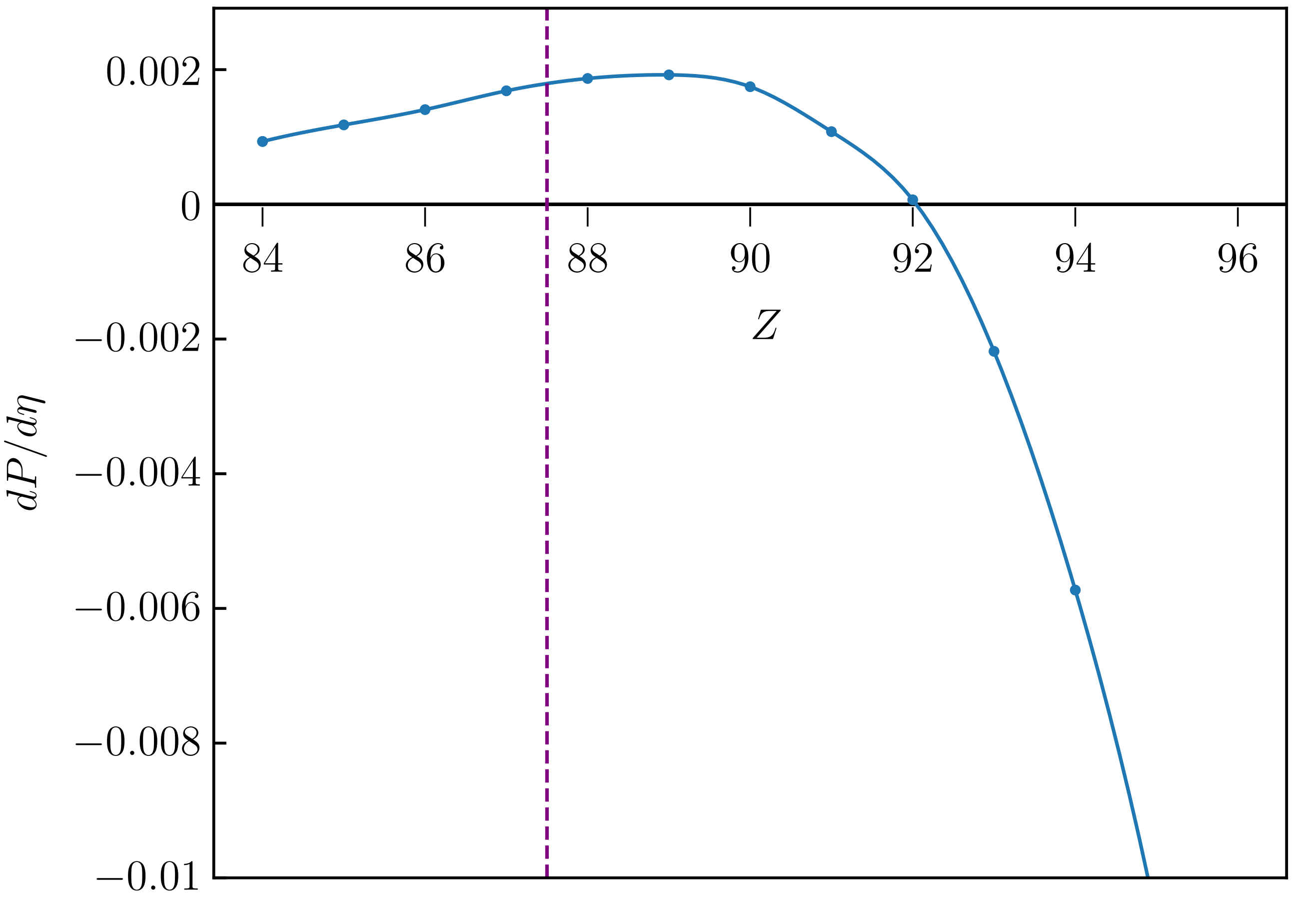
We consider only the trajectories for which the minimal internuclear distance is the same: $R_{\rm min} = 17.5$ fm. We introduce $\eta = E/E_0 \geq 1$.

How to observe the vacuum decay



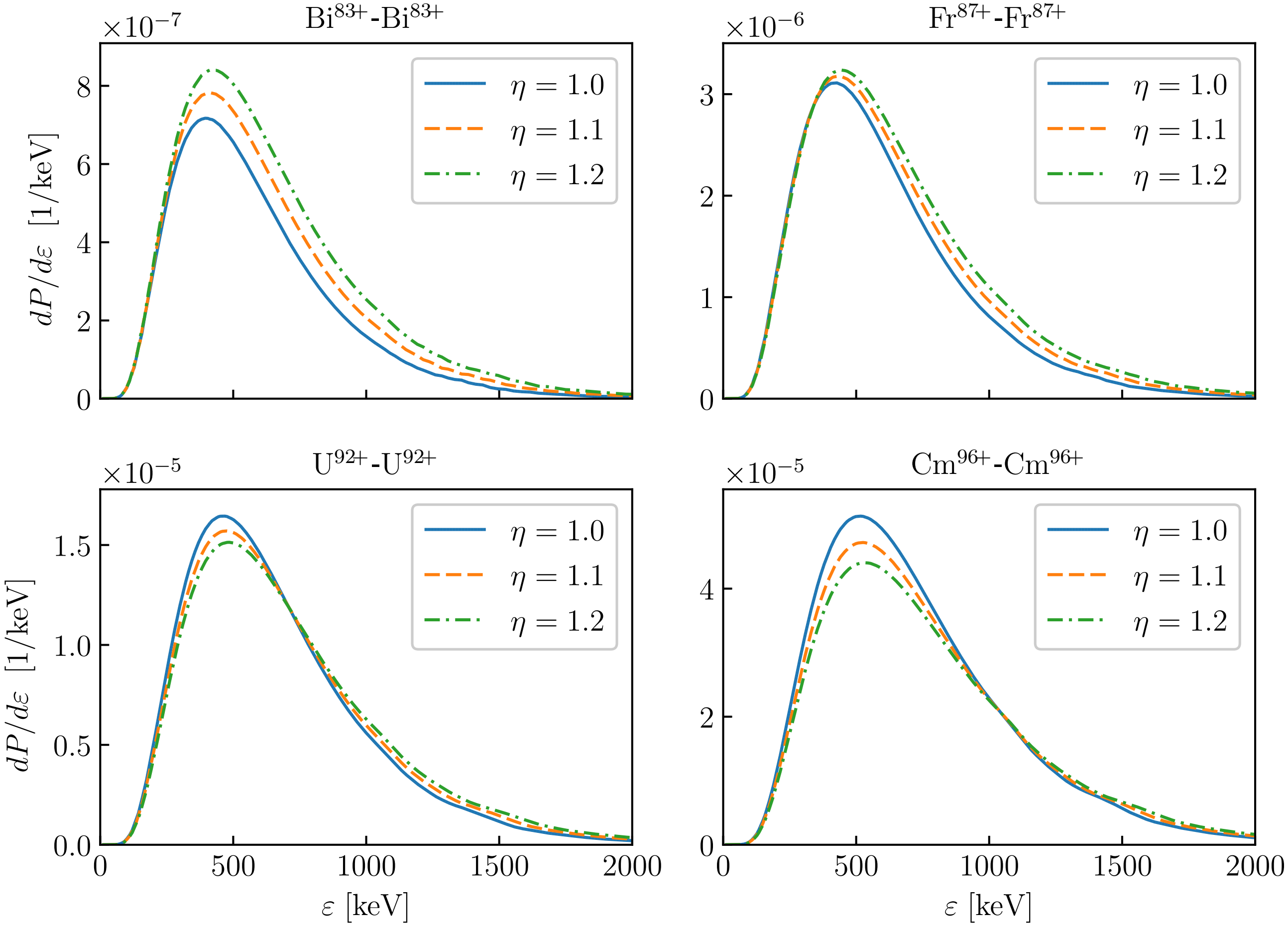
Total pair-production probability for symmetric ($Z = Z_1 = Z_2$) collisions as a function of the collision energy at $R_{\mathrm{min}} = 17.5$ fm.

How to observe the vacuum decay



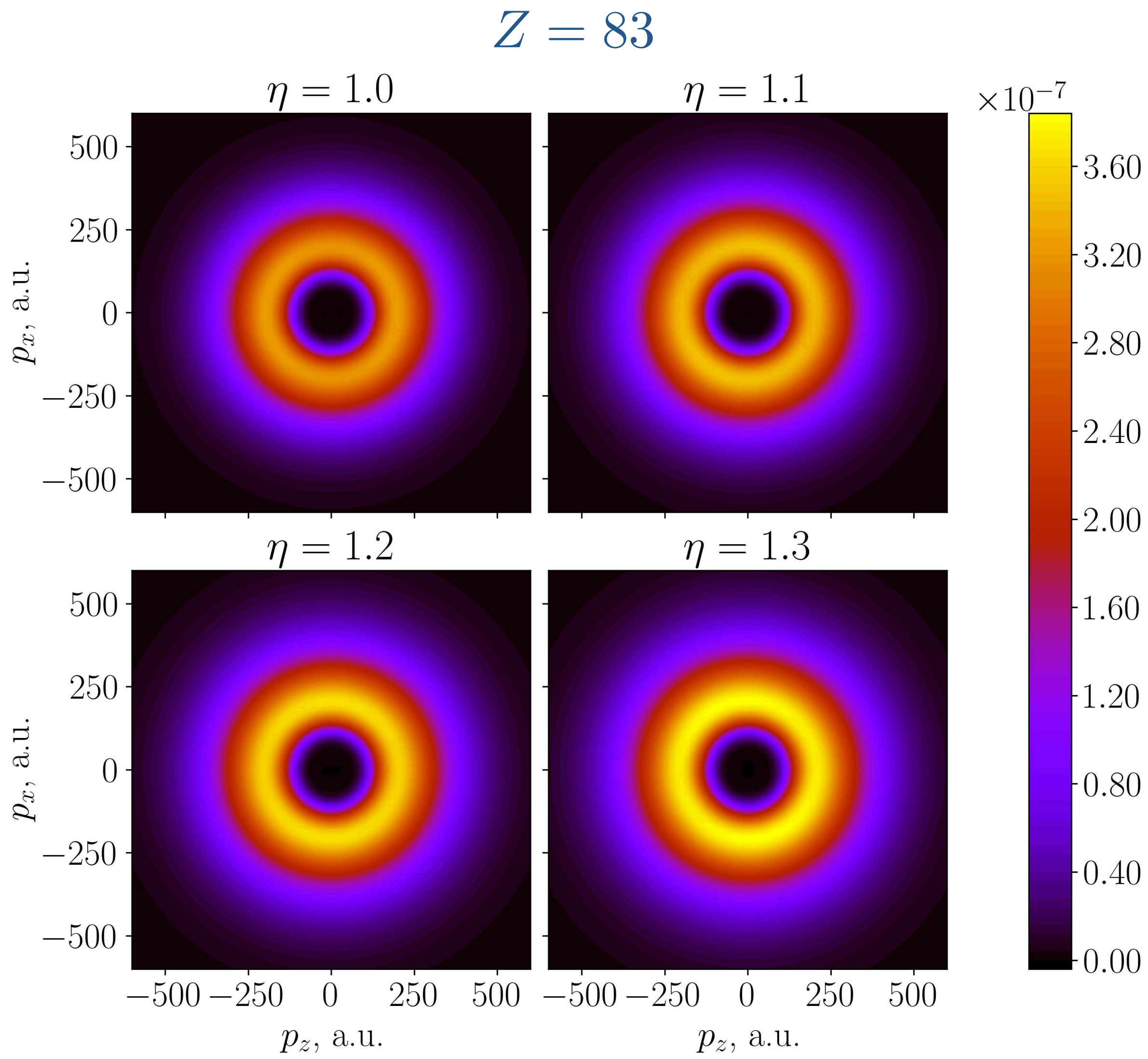
The derivative of the pair-production probability with respect to the energy $dP/d\eta$, where $\eta = E/E_0$, at the point $\eta = 1$ as a function of the nuclear charge number $Z = Z_1 = Z_2$ at $R_{\min} = 17.5$ fm.

How to observe the vacuum decay



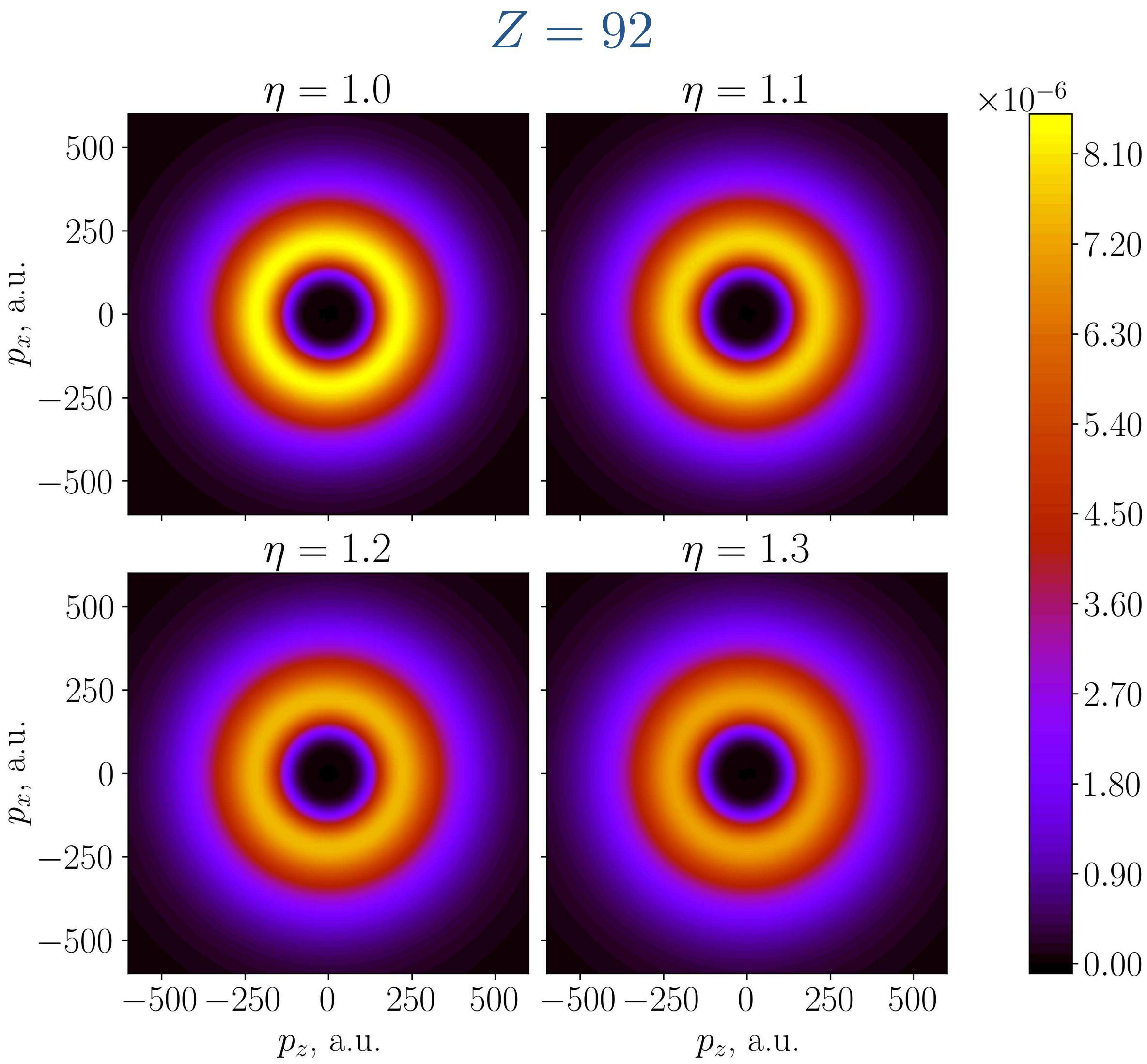
Positron spectra in symmetric ($Z = Z_1 = Z_2$) collisions for different collision energy $\eta = E/E_0$ at $R_{\min} = 17.5$ fm.

How to observe the vacuum decay



N.K. Dulaev, D.A. Telnov, V.M. Shabaev et al., PRD, 2024.

How to observe the vacuum decay



N.K. Dulaev, D.A. Telnov, V.M. Shabaev et al., PRD, 2024.

Conclusion

The experimental study of the proposed scenarios would either prove the vacuum decay in the supercritical Coulomb field or lead to discovery of a new physical phenomenon, which can not be described within the presently used QED formalism.

The same scenarios can be applied to observe the vacuum decay in collisions of bare nuclei with neutral atoms.

For details:

I.A. Maltsev, V.M. Shabaev, R.V. Popov et al., Phys. Rev. Lett. 123, 113401 (2019).

R.V. Popov, V.M. Shabaev, D.A. Telnov et al., Phys. Rev. D 102, 076005 (2020).

R.V. Popov, V.M. Shabaev, I.A. Maltsev et al., Phys. Rev. D 107, 116014 (2023).

N.K. Dulaev, D.A. Telnov, V.M. Shabaev et al., Phys. Rev. D 109, 036008 (2024).