Уравнения умнее тех, кто их вывел.

Генрих Герц (1857-1894)

ГИДРОДИНАМИЧЕСКИЕ УРАВНЕНИЯ КРУПНОМАСШТАБНОЙ АТМОСФЕРЫ

Р.И. Нигматулин

МОСКОВСКИЙ ГОСУДАРСТВЕННЫЙ УНИВЕРСИТЕТ им. М.В. Ломоносова

Механико-математический факультет

РОССИЙСКАЯ АКАДЕМИЯ НАУК

Институт океанологии им П.П. Ширшова РАН

XXI Научная школа
НЕЛИНЕЙНЫЕ ВОЛНЫ
Нижний Новгород, 09.11.2024

Гидротермодинамические уравнения для атмосферы (уравнения Навье-Стокса)

$$\frac{d\rho}{dt} = -\frac{1}{\rho}\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right)$$ – уравнение сохранения массы

$$\frac{d}{dt} \equiv \frac{\partial}{\partial t} + v_x\frac{\partial}{\partial x} + v_y\frac{\partial}{\partial y} + v_z\frac{\partial}{\partial z}$$

$$\frac{dv_x}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + f_x^{(cor)} + \frac{1}{\rho}\left(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z}\right)$$

$$\frac{dv_y}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + f_y^{(cor)} + \frac{1}{\rho}\left(\frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z}\right)$$

$$\frac{dv_z}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + f_z^{(cor)} - \boldsymbol{g} + \frac{1}{\rho}\left(\frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zy}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z}\right)$$

– уравнения сохранения импульса

– уравнения сохранения импульса

$$\frac{dT}{dt} = \frac{Q^*}{\rho c_v} + \frac{p}{\rho^2 c_v}\frac{d\rho}{dt}$$ – уравнение притока тепла – $$\frac{1}{\rho}\frac{d\rho}{dt} = \frac{1}{\gamma p}\frac{dp}{dt} - \frac{\gamma-1}{\gamma}\frac{Q^*}{p}$$

$$Q^* \equiv -\frac{\partial q}{\partial z} + Q^{(R)} - J^{(e)} l^{(e)} - J^{(m)} l^{(m)}$$

Уравнения состояния совершенного газа

$$p = R\rho T \qquad u = c_v T + \text{const} \qquad c_v = R/(\gamma - 1)$$

MOISTURE CONTENT

$$\rho = \rho_A + \rho_W \qquad \rho_W = \rho_V + \rho_L$$

Dry air (ρ_A), Water (ρ_W); Vapor (ρ_V), Liquid Droplets (Snow, Ice) (ρ_L)

Температурный ресурс влаги за счет испарения (конденсации)

$$\rho_W \ll \rho \quad \left(\frac{\rho_W}{\rho} \sim 10^{-2}\right) \Rightarrow \quad \rho_W l^{(e)} / \rho c_p = 2{,}3\ \mathrm{K}, \qquad \rho_W l^{(m)} / \rho c_p = 0{,}34\ \mathrm{K}$$

Mass Conservation Equation for Water Vapor

$$\frac{\partial \rho_V}{\partial t} = -\frac{\partial \rho_V v_x}{\partial x} - \frac{\partial \rho_V v_y}{\partial y} - \frac{\partial \rho_V (v_z + w_V)}{\partial z} + J^{(e)}$$

$$v_{Vx} = v_x, \quad v_{Vy} = v_y, \quad v_{Vz} = v_z + w_V$$

Turbulent Diffusion

$$w_V = -\nu^{(d)} \frac{\partial}{\partial z}\left(\frac{\rho_V}{\rho}\right)$$

Mass Conservation Equation for water Droplets

$$\frac{\partial \rho_L}{\partial t} = -\frac{\partial \rho_L v_x}{\partial x} - \frac{\partial \rho_L v_y}{\partial y} - \frac{\partial \rho_L (v_z + w_L)}{\partial z} - J^{(e)}$$

$$v_{Lx} = v_x, \quad v_{Ly} = v_y, \quad v_{Lz} = v_z + w_L$$

Sedimentation of Droplets

$$\lambda_A \frac{\rho w_L^2}{2} \frac{\pi d^2}{4} = \rho_L \frac{\pi d^3}{6} g$$

$$\lambda_A = \lambda_A(\mathrm{Re}), \qquad \mathrm{Re} = \frac{\rho w_L d}{\mu_m}$$

Aerodynamic Drag

ДЛЯ ЗАМЫКАНИЯ СИСТЕМЫ УРАВНЕНИЯ НЕОБХОДИМЫ УРАВНЕНИЯ:

$$Q^* \equiv -\frac{\partial q}{\partial z} + Q^{(R)} - J^{(e)} l^{(e)} - J^{(m)} l^{(m)}$$

- для вертикального теплового потока q (турбулентная теплопроводность).

- для выделения тепла из-за поглощения радиации $Q^{(R)}$, определяемого уравнениями для потоков коротковолновой радиации G, длинноволновой радиации U.

- для интенсивности испарения (конденсации) $J^{(e)}$, плавления льда или снега) $J^{(m)}$, и тепла $J^{(e)} l^{(e)} + J^{(m)} l^{(m)}$ фазовых переходов.

КЛИМАТИЧЕСКИЕ и МЕТЕОРОЛОГИЧЕСКИЕ МАСШТАБЫ

$\tau > 10^2$ s

$V_{hor} < 30$ m/s,

$L_{hor} \sim V_{hor}\tau > 10^3$ m

$V_{ver} < 3$ m/s,

$L_{ver} \sim V_{ver}\tau > 10^2$ m

КЛИМАТ, ПОГОДА

$\tau \sim 10^0$ s

$V_{hor} \sim 10^2$ m/s,

$L_{hor} \sim 10^2$ m,

$V_{ver} \sim 30$ m/s,

$L_{ver} \sim 10^2$ m,

ТАЙФУН, ШТОРМ, БОРА

$$\frac{\partial v_x}{\partial t} \sim \frac{\partial v_y}{\partial t} \sim \frac{V_{hor}}{\tau}$$

$$v_j \frac{\partial v_i}{\partial x_j} \sim \frac{V_{hor}^2}{L_{hor}} \sim \frac{V_{hor}}{\tau} \quad (i, j = x, y)$$

$$\frac{\partial v_z}{\partial t} \sim \frac{V_{ver}}{\tau}$$

$$v_z \frac{\partial v_z}{\partial x_z} \sim \frac{V_{ver}^2}{L_{ver}} \sim \frac{V_{ver}}{\tau}$$

$$\bar{t} \equiv \frac{t}{\boldsymbol{\tau}}, \quad \bar{x} \equiv \frac{x}{\boldsymbol{L}_{\text{hor}}}, \quad \bar{y} \equiv \frac{y}{\boldsymbol{L}_{\text{hor}}}, \quad \bar{z} \equiv \frac{z}{\boldsymbol{L}_{\text{ver}}},$$

$$\bar{v}_x \equiv \frac{v_x}{\boldsymbol{V}_{\text{hor}}}, \quad \bar{v}_y \equiv \frac{v_x}{\boldsymbol{V}_{\text{hor}}}, \quad \bar{v}_z \equiv \frac{v_z}{\boldsymbol{V}_{\text{ver}}} = \mathcal{O}(1)$$

Масштабы ускорений

$$\frac{dv_x}{dt} = \frac{\boldsymbol{V}_{\text{hor}}}{\boldsymbol{\tau}}\left(\frac{\partial \bar{v}_x}{\partial \bar{t}} + \bar{v}_x\frac{\partial \bar{v}_x}{\partial \bar{x}} + \bar{v}_y\frac{\partial \bar{v}_x}{\partial \bar{y}} + \bar{v}_z\frac{\partial \bar{v}_x}{\partial \bar{z}}\right) = \boldsymbol{A}_{\text{hor}}\frac{d\bar{v}_x}{d\bar{t}} = \boldsymbol{A}_{\text{hor}}\mathcal{O}(1)$$

$$\frac{dv_y}{dt} = \frac{\boldsymbol{V}_{\text{hor}}}{\boldsymbol{\tau}}\left(\frac{\partial \bar{v}_y}{\partial \bar{t}} + \bar{v}_x\frac{\partial \bar{v}_y}{\partial \bar{x}} + \bar{v}_y\frac{\partial \bar{v}_y}{\partial \bar{y}} + \bar{v}_z\frac{\partial \bar{v}_y}{\partial \bar{z}}\right) = \boldsymbol{A}_{\text{hor}}\frac{d\bar{v}_y}{d\bar{t}} = \boldsymbol{A}_{\text{hor}}\mathcal{O}(1)$$

$$\frac{dv_z}{dt} = \frac{\boldsymbol{V}_{\text{ver}}}{\boldsymbol{\tau}}\left(\frac{\partial \bar{v}_z}{\partial \bar{t}} + \bar{v}_x\frac{\partial \bar{v}_z}{\partial \bar{x}} + \bar{v}_y\frac{\partial \bar{v}_z}{\partial \bar{y}} + \bar{v}_z\frac{\partial \bar{v}_z}{\partial \bar{z}}\right) = \boldsymbol{A}_{\text{ver}}\frac{d\bar{v}_z}{d\bar{t}} = \boldsymbol{A}_{\text{ver}}\mathcal{O}(1)$$

$$|-\boldsymbol{a}_{\text{cor}}| = |\mathbf{f}_{\text{cor}}| = \left|2[\vec{\boldsymbol{\Omega}} \times \vec{\mathbf{v}}]\right| = 2\boldsymbol{V}_{\text{hor}}\Omega\left|[\vec{\mathbf{e}}_{\text{o}} \times \vec{\bar{\mathbf{v}}}]\right|$$

$$\Omega = \frac{2\pi}{24\ 3600\ \text{c}} \approx 0{,}73 \times 10^{-4}\text{c}^{-1}, \quad \boldsymbol{A}_{\text{cor}} = 2\boldsymbol{V}_{\text{hor}}\,\Omega$$

Безразмерные функции

$$\left(\bar{\upsilon}_i\,, \quad \frac{d\bar{\upsilon}_i}{d\bar{t}}\right) = \mathcal{O}(1)$$

Масштабы ускорений

$$\boldsymbol{A}_{\mathbf{hor}} = \frac{\boldsymbol{V}_{\mathbf{hor}}}{\boldsymbol{\tau}} < 10^{-1}\ \text{м/с}^2$$

$$\boldsymbol{A}_{\mathbf{ver}} = \frac{\boldsymbol{V}_{\mathbf{ver}}}{\boldsymbol{\tau}} < 10^{-2}\ \text{м/с}^2$$

$$\boldsymbol{A}_{\mathbf{cor}} = 2\Omega \boldsymbol{V}_{\mathbf{hor}} < 1{,}4 \times 10^{-3}\ \text{м/с}^2$$

$$\varepsilon = \left(\frac{\boldsymbol{A}_{\mathbf{hor}}}{g}, \quad \frac{\boldsymbol{A}_{\mathbf{ver}}}{g}, \quad \frac{\boldsymbol{A}_{\mathbf{cor}}}{g}\right) < (10^{-2}, \quad 10^{-3}, \quad 10^{-4})$$

$$\varepsilon = \left(\frac{\boldsymbol{A}_{\mathbf{hor}}}{g}, \quad \frac{\boldsymbol{A}_{\mathbf{ver}}}{g}, \quad \frac{\boldsymbol{A}_{\mathbf{cor}}}{g}\right) \to \ 0$$

УРАВНЕНИЯ МПУЛЬСА

$$\frac{\partial p}{\partial x} = \rho\mathcal{O}(\boldsymbol{A}_{\mathrm{hor}} + \boldsymbol{A}_{\mathrm{cor}})$$

$$\frac{\partial p}{\partial y} = \rho\mathcal{O}(\boldsymbol{A}_{\mathrm{hor}} + \boldsymbol{A}_{\mathrm{cor}})$$

$$\frac{\partial p}{\partial z} = \rho\mathcal{O}(\boldsymbol{A}_{\mathrm{ver}} + \boldsymbol{A}_{\mathrm{cor}}) - \rho \boldsymbol{g} = -\rho \boldsymbol{g}\left(1 + \mathcal{O}\left(\frac{\boldsymbol{A}_{\mathrm{ver}} + \boldsymbol{A}_{\mathrm{cor}}}{g}\right)\right)$$

$$\frac{\partial p}{\partial z} = -\rho \boldsymbol{g}\,(1 + \mathcal{O}(\varepsilon))$$

$$p\,(t,\ x,\ y,\ z) \underset{\varepsilon \to 0}{\longrightarrow} \boldsymbol{g}\int_{z}^{\infty} \boldsymbol{\rho}\,(t,x,y,z')\,\mathrm{d}z = \boldsymbol{g}M$$

$$M(t,x,y,z) = \int_{z}^{\infty} \rho\,(t,x,y,z')\,\mathrm{d}z'$$

$$\rho\frac{\mathrm{d}v_x}{\mathrm{d}t}$$

$$\frac{\partial p}{\partial x} \underset{\varepsilon \to 0}{\longrightarrow} g\frac{\partial M}{\partial x} = g\int_{z}^{\infty} \frac{\partial \rho}{\partial x} dz'$$

$$\rho\frac{\mathrm{d}v_x}{\mathrm{d}t}$$

$$\frac{\partial p}{\partial y} \underset{\varepsilon \to 0}{\longrightarrow} g\frac{\partial M}{\partial y} = g\int_{z}^{\infty} \frac{\partial \rho}{\partial y} dz'$$

Как рассчитать вертикальную

ВЕРТИКАЛЬНЫЕ ПОТОКИ. Дождь (фото с борта самолета)

ВЕРТИКАЛЬНЫЕ ПОТОКИ. Образование грозы (gM го с борта самолета на высоте 11 000 м).

УРАВНЕНИЕ СОХРАНЕНИЯ МАССЫ

$$\frac{1}{\rho}\frac{d\rho}{dt} = \operatorname{div}\mathbf{v} \equiv -\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right)$$

$$\frac{\partial \rho}{\partial t} = -\left(\frac{\partial \rho v_z}{\partial z} + \frac{\partial \rho v_y}{\partial y} + \frac{\partial \rho v_x}{\partial x}\right)$$

$$\frac{\partial v_z}{\partial z} = -\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y}\right) + \frac{1}{\rho}\frac{d\rho}{dt}$$

Уравнение для распределения вертикальной скорости по вертикали

ОКЕАН: $\Delta\rho/\rho \sim 10^{-4} - 10^{-3}$

$$\frac{1}{\rho}\frac{d\rho}{dt} = \frac{1}{\rho}\left(\frac{\partial\rho}{\partial p}\right)_{S,T}\frac{dp}{dt} + \frac{1}{\rho}\left(\frac{\partial\rho}{\partial S}\right)_{p,T}\frac{dS}{dt} + \frac{1}{\rho}\left(\frac{\partial\rho}{\partial T}\right)_{S,T}\frac{dT}{dt} \ll \left|\frac{\partial v_x}{\partial x}, \frac{\partial v_y}{\partial y}, \frac{\partial v_z}{\partial z}\right|$$

$$\frac{\partial v_z}{\partial z} = -\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y}\right) + \frac{1}{\rho}\frac{d\rho}{dt}$$

$$\frac{\partial v_z}{\partial z} = -\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y}$$

- Квазинесжимаемость

"Обыкновенное" дифференциальное уравнение для распределения вертикальной скорости по вертикали

Но плотность меняется: $\rho = \rho(p, T, S)$,

$p(t, x, y, z) \rightarrow g\int_z^\infty \rho(t,x,y,z')\,dz'$ ...еляет... си... ные

$$\frac{\partial p}{\partial x} = g\int_z^\infty \frac{\partial \rho}{\partial x} dz'$$

$$\frac{\partial p}{\partial y} = g\int_z^\infty \frac{\partial \rho}{\partial y} dz'$$

Атмосфера: Δρ/ ρ ~ 10⁰

$$\frac{1}{\rho}\frac{d\rho}{dt} = \frac{1}{\gamma p}\frac{dp}{dt} - \frac{\gamma - 1}{\gamma p} Q^*$$

- уравнение притока тепла для совершенного газа

$$\frac{\partial v_z}{\partial z} = -\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y}\right) - \frac{1}{\rho}\frac{d\rho}{dt}$$

$$\frac{\partial v_z}{\partial z} = -\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y}\right) - \frac{1}{\gamma p}\frac{dp}{dt} + \frac{\gamma - 1}{\gamma p} Q^*$$

$$p(t,x,y,z) \underset{\varepsilon \to 0}{\to} gM(t,x,y,z)$$

$$M(t,x,y,z) = \int_z^{\infty} \rho(t,x,y,z')\, dz'$$

$$p(t,x,y,z) \underset{\varepsilon\to 0}{\to} gM(t,x,y,z) \qquad M(t,x,y,z)=\int_z^{\infty}\rho(t,x,y,z')\,dz'$$

$$\frac{dp}{dt}=\frac{\partial p}{\partial t}+v_z\frac{\partial p}{\partial z}+v_x\frac{\partial p}{\partial x}+v_y\frac{\partial p}{\partial y}$$

(with gM written above each p)

$$\frac{\partial\rho}{\partial t}=-\left(\frac{\partial\rho v_z}{\partial z}+\frac{\partial\rho v_y}{\partial y}+\frac{\partial\rho v_x}{\partial x}\right)$$

$$\dot M\equiv\frac{\partial M}{\partial t}+v_z\frac{\partial M}{\partial z}=\frac{\partial}{\partial t}\int_z^H\rho\,dz'+v_z\frac{\partial}{\partial z}\int_z^H\rho\,dz'=\int_z^H\frac{\partial\rho}{\partial t}dz'-\rho v_z=-\rho v_z-\int_z^H\left(\frac{\partial\rho v_z}{\partial z}+\frac{\partial\rho v_y}{\partial y}+\frac{\partial\rho v_x}{\partial x}\right)dz=$$

$$=-\rho v_z+\rho v_z-\int_z^H\left(\frac{\partial\rho v_y}{\partial y}+\frac{\partial\rho v_x}{\partial x}\right)dz'$$

Горизонтальная дивергенция

Горизонтальный перенос давления

$$\frac{dp}{dt}=\frac{\partial p}{\partial t}+v_z\frac{\partial p}{\partial z}+v_x\frac{\partial p}{\partial x}+v_y\frac{\partial p}{\partial y}=-g\int_z^H\left(\frac{\partial(\rho v_x)}{\partial x}+\frac{\partial(\rho v_y)}{\partial y}\right)dz'+\left(v_x\frac{\partial p}{\partial x}+v_y\frac{\partial p}{\partial y}\right)$$

$O(\varepsilon)$

Горизонтальная дивергенция

$$\left(M=\int_z^H\rho(t,x,y,z')\,dz',\quad \dot M\equiv-\int_z^H\left(\frac{\partial(\rho v_x)}{\partial x}+\frac{\partial(\rho v_y)}{\partial y}\right)dz'=\frac{\partial M}{\partial t}-\rho v_z\right)$$

$$\frac{\partial M}{\partial z}=-\rho(z),\qquad \frac{\partial\dot M}{\partial z}=\left(\frac{\partial(\rho v_x)}{\partial x}+\frac{\partial(\rho v_y)}{\partial y}\right)$$

Горизонтальная дивергенция

Горизонтальный перенос давления

$$\frac{dp}{dt}=\frac{\partial p}{\partial t}+v_z\frac{\partial p}{\partial z}+v_x\frac{\partial p}{\partial x}+v_y\frac{\partial p}{\partial y}=-g\int_z^H\left(\frac{\partial(\rho v_x)}{\partial x}+\frac{\partial(\rho v_y)}{\partial y}\right)dz'+\left(v_x\frac{\partial p}{\partial x}+v_y\frac{\partial p}{\partial y}\right)$$

O(ε)

$$\frac{\partial v_z}{\partial z}=-\left(\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}\right)-\frac{1}{\gamma p}\frac{dp}{dt}+\frac{\gamma-1}{\gamma p}Q^*$$

$$p=gM\equiv g\int_z^H\rho(t,x,y,z')\,dz'$$

$$\boldsymbol{\frac{\partial v_z}{\partial z}=-\left(\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}\right)-\frac{1}{\gamma}\frac{\dot{M}}{M}+\frac{\gamma-1}{\gamma}\frac{Q^*}{gM}}+\left\{\frac{1}{\gamma p}\left(v_x\frac{\partial p}{\partial x}+v_y\frac{\partial p}{\partial y}\right)\right\}$$

$$\left(M=\int_z^H\rho(t,x,y,z')\,dz',\quad \dot{M}\equiv-\int_z^H\left(\frac{\partial(\rho v_x)}{\partial x}+\frac{\partial(\rho v_y)}{\partial y}\right)dz'\right)$$

$$\frac{\partial M}{\partial z}=-\rho(z),\qquad \frac{\partial \dot{M}}{\partial z}=\left(\frac{\partial(\rho v_x)}{\partial x}+\frac{\partial(\rho v_y)}{\partial y}\right)$$

$$\frac{\partial v_z}{\partial z} = -\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y}\right) - \frac{1}{\gamma}\frac{\dot{M}}{M} + \frac{\gamma - 1}{\gamma}\frac{Q^*}{g M} + \left\{\frac{1}{\gamma p}\left(v_x \frac{\partial p}{\partial x} + v_y \frac{\partial p}{\partial y}\right)\right\}$$

$$\frac{\left\{v_x\left(\frac{\partial p}{\partial x}\right) + v_y\left(\frac{\partial p}{\partial y}\right)\right\} / \gamma p}{\dot{M} / \gamma} =$$

$$\left(v_x \frac{\partial p}{\partial x},\ v_y \frac{\partial p}{\partial y}\right) = V_{\mathrm{hor}}\, \rho\, O\left(A_{\mathrm{hor}} + A_{\mathrm{cor}}\right) = V_{\mathrm{hor}}\, \rho\, O\left(\frac{V_{\mathrm{hor}}^2}{L_{\mathrm{hor}}} + \frac{V_{\mathrm{hor}}^2}{L_{\mathrm{cor}}}\right)$$

$$\frac{\dot{M}}{\gamma M} = \frac{\int_z^H \left(\frac{\partial(\rho v_x)}{\partial x} + \frac{\partial(\rho v_y)}{\partial y}\right) dz'}{\gamma \int_z^H \rho(t, x, y, z')\, dz'} \sim \frac{\frac{\hat{\rho} V_{\mathrm{hor}}(H - z)}{L_{\mathrm{hor}}}}{\tilde{\gamma}\rho(H - z)} = O\left(\frac{V_{\mathrm{hor}}}{L_{\mathrm{hor}}}\right)$$

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} \sim O\left(\frac{V_{\mathrm{hor}}}{L_{\mathrm{hor}}}\right)$$

$$V_{hor}\rho\,\mathcal{O}\left(\frac{V_{hor}^2}{L_{hor}}+\frac{V_{hor}^2}{L_{cor}}\right)$$

$$\frac{\gamma p}{\rho}=C^2$$

$$\frac{\left\{v_x\left(\frac{\partial p}{\partial x}\right)+v_y\left(\frac{\partial p}{\partial y}\right)\right\}\Big/\gamma p}{\dot{M}/\gamma}=\frac{\frac{V_{hor}\rho}{\gamma p}\,\mathcal{O}\left(\frac{V_{hor}^2}{L_{hor}}+\frac{V_{hor}^2}{L_{cor}}\right)}{\mathcal{O}\left(\frac{V_{hor}}{L_{hor}}\right)}=$$

$$\dot{M}/\gamma \rightarrow \mathcal{O}\left(\frac{V_{hor}}{L_{hor}}\right)$$

$$=\frac{L_{hor}}{C^2}\,\mathcal{O}\left(\frac{V_{hor}^2}{L_{hor}}+\frac{V_{hor}^2}{L_{cor}}\right)=\frac{V_{hor}^2}{C^2}\,\mathcal{O}\left(1+\frac{L_{hor}}{L_{cor}}\right)=\left(1+\frac{L_{hor}}{L_{cor}}\right)\mathcal{O}(\mathbf{M}^2)$$

$$\left(\mathbf{M}\equiv\frac{V_{hor}}{C},\qquad C=\left(\frac{\gamma p}{\rho}\right)^{\frac{1}{2}}=300-350\ \text{м/с}\right)$$

$$\frac{\partial v_z}{\partial z} = -\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y}\right) - \frac{1}{\gamma}\frac{\dot{M}}{M} + \frac{\gamma - 1}{\gamma}\frac{Q^*}{gM}$$

O(ε)→ 0

$$\frac{\partial v_z}{\partial z} = -\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y}\right) - \frac{1}{\gamma}\frac{\dot{M}}{M} + \frac{\gamma - 1}{\gamma}\frac{Q^*}{gM} + \left\{\frac{1}{\gamma p}\left(v_x\frac{\partial p}{\partial x} + v_y\frac{\partial p}{\partial y}\right)\right\}$$

Edwards Lorentz (1967)

Р.И. Нигматулин (2015)

Горизонтальная дивергенция

$$\frac{dp}{dt} = -g\int_z^H \left(\frac{\partial(\rho v_x)}{\partial x} + \frac{\partial(\rho v_y)}{\partial y} \right) dz'$$

$$\frac{dp}{dt} = -g\rho v_z$$

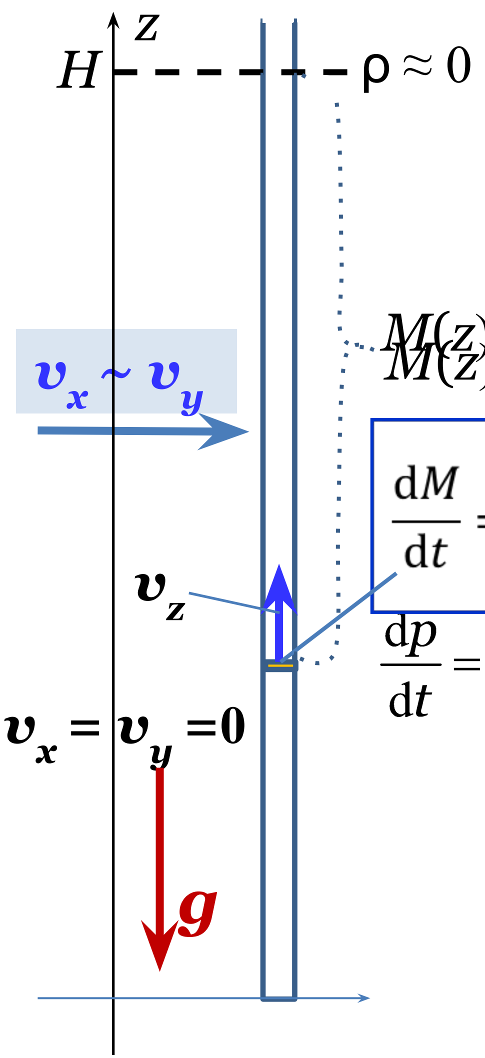
Text book J. Holton "An Introduction to Dynamic Meteorology" (2004)

$$\frac{d\boldsymbol{p}}{d\boldsymbol{t}} = \frac{\partial \boldsymbol{p}}{\partial \boldsymbol{t}} + \boldsymbol{v}_z \frac{\partial \boldsymbol{p}}{\partial \boldsymbol{z}}$$

$\frac{\partial p}{\partial t} \to$ **0**

$v_z \frac{\partial p}{\partial z} \to -g\rho v_z$

Both are principal errors



$M(z)$ = variable

$M(z)$ = const

$$\frac{dM}{dt} = -\int_z^H \left(\frac{\partial(\rho v_x)}{\partial x} + \frac{\partial(\rho v_y)}{\partial y} \right) dz'$$

$$\frac{dp}{dt} = 0, \text{ but not } \frac{dp}{dt} = -\rho g v_z$$

Marchuk (1976) used

$$\frac{\partial \rho}{\partial t} = 0$$

to "filter acoustic" ???

Уравнения сохранения для атмосферы с вертикальной квазистатикой (относительно ρ, v_x, v_y)

$$\frac{\partial}{\partial t}:$$

$$\frac{\partial \rho}{\partial t} = -v_x \frac{\partial \rho}{\partial x} - v_y \frac{\partial \rho}{\partial y} - v_z \frac{\partial \rho}{\partial z} + \underbrace{\frac{1}{\gamma}\frac{\rho \dot{M}}{M} - \frac{\gamma - 1}{\gamma}\frac{\rho Q^*}{g}}_{-\rho \,\mathrm{div}\, \mathbf{v}}$$

$$\frac{\partial v_x}{\partial t} = -v_x \frac{\partial v_x}{\partial x} - v_y \frac{\partial v_x}{\partial y} - v_z \frac{\partial v_x}{\partial z} - \frac{g}{\rho}\frac{\partial M}{\partial x} + f_x^{(\mathrm{cor})}$$

$$\frac{\partial v_y}{\partial t} = -v_x \frac{\partial v_y}{\partial x} - v_y \frac{\partial v_y}{\partial y} - v_z \frac{\partial v_y}{\partial z} - \frac{g}{\rho}\frac{\partial M}{\partial y} + f_y^{(\mathrm{cor})}$$

$$\frac{\partial}{\partial z}:$$

$$\frac{\partial M}{\partial z} = -\rho$$

$$\frac{\partial \dot{M}}{\partial z} = -\frac{\partial(\rho v_x)}{\partial x} - \frac{\partial(\rho v_y)}{\partial y}$$

$$\frac{\partial v_z}{\partial z} = -\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y}\right) - \frac{1}{\gamma}\frac{\dot{M}}{M} + \frac{\gamma - 1}{\gamma}\frac{Q^*}{g\,M}$$

$$p = gM, \qquad T = \frac{g}{R}\frac{M}{\rho} \qquad \left(M = \int_z^H \rho(t, x, y, z')\,\mathrm{d}z', \qquad \dot{M} \equiv -\int_z^H \left(\frac{\partial(\rho v_x)}{\partial x} + \frac{\partial(\rho v_y)}{\partial y}\right)\mathrm{d}z' \right)$$

$$Q^* \equiv -\frac{\partial q}{\partial z} + Q^{(\mathrm{R})} J l$$

~~$\frac{\partial p}{\partial x} = C^2 \frac{\partial \rho}{\partial x}$~~

~~$\frac{\partial p}{\partial y} = C^2 \frac{\partial \rho}{\partial y}$~~

$C \to \infty$

$(\otimes)$

$$\boldsymbol{\varepsilon} = \left(\varepsilon_{\text{ver}} = \frac{A_{\text{ver}}}{g}, \quad \varepsilon_{\text{hor}} = \frac{A_{\text{hor}}}{g}, \quad \varepsilon_{\text{cor}} = \frac{A_{\text{cor}}}{g}, \quad \varepsilon_{\text{C}} = \mathbf{M}^2 \right)$$

$$\left(A_{\text{ver}} = \frac{V_{\text{ver}}}{\tau} + \frac{V_{\text{ver}}^2}{L_{\text{ver}}}, \quad A_{\text{hor}} = \frac{V_{\text{hor}}}{\tau} + \frac{V_{\text{hor}}^2}{L_{\text{hor}}}, \quad A_{\text{cor}} = \frac{V_{\text{hor}}}{\tau_{\text{cor}}}, \quad \mathbf{M}^2 = \frac{V_{\text{ver}}^2}{C^2} \right)$$

Теорема. Уравнения ($\otimes$) асимптотически точные уравнения для $\varepsilon_{\text{ver}} \to 0, \quad \varepsilon_{\text{hor}} \to 0, \quad \varepsilon_{\text{cor}} \to 0, \quad \varepsilon_C \to 0$

$$\frac{\partial v_z}{\partial z} = -\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} \right) - \frac{1}{\gamma} \frac{\dot{M}}{M} + \frac{\gamma - 1}{\gamma} \frac{Q^*}{g M}$$

SHARPENNING !!!

$$\frac{dp}{dt} = -g\int_z^H \left(\frac{\partial(\rho v_x)}{\partial x} + \frac{\partial(\rho v_y)}{\partial y} \right) dz'$$

$$\frac{\partial v_z}{\partial z} = -\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} \right) - \frac{1}{\gamma}\frac{\dot{M}}{M} + \frac{\gamma - 1}{\gamma}\frac{Q^*}{gM}$$

Часто использую т

$$v_z = 0$$

$$v_z = 0$$

Часто используют

$$\frac{d\rho}{dt} = 0$$

$$\operatorname{div} \mathbf{V} = 0$$

$$\frac{\partial v_z}{\partial z} = -\frac{\partial v_x}{\partial x} - \frac{\partial v_y}{\partial y}$$

Г.И. Марчук

$$\frac{\partial \rho}{\partial t} = 0$$

чтобы «отфильтровать акустику» ???

$$\operatorname{div}(\rho \mathbf{V}) = 0$$

$$\frac{\partial(\rho v_z)}{\partial z} = -\frac{\partial(\rho v_x)}{\partial x} - \frac{\partial(\rho v_y)}{\partial y}$$

Дж. Холтон (учебник)

$$\frac{\partial p}{\partial t} = 0$$

$$\frac{dp}{dt} = -g\rho v_z$$

$$\rho v_z = \int_z^H \left(\frac{\partial(\rho v_x)}{\partial x} + \frac{\partial(\rho v_y)}{\partial y} \right) dz'$$

Схема распределения параметров в тропосфере над межфазной поверхностью

МЕЖФАЗНАЯ ГРАНИЦА

ξ - поток испаряющейся массы на нижней границе атмосферы

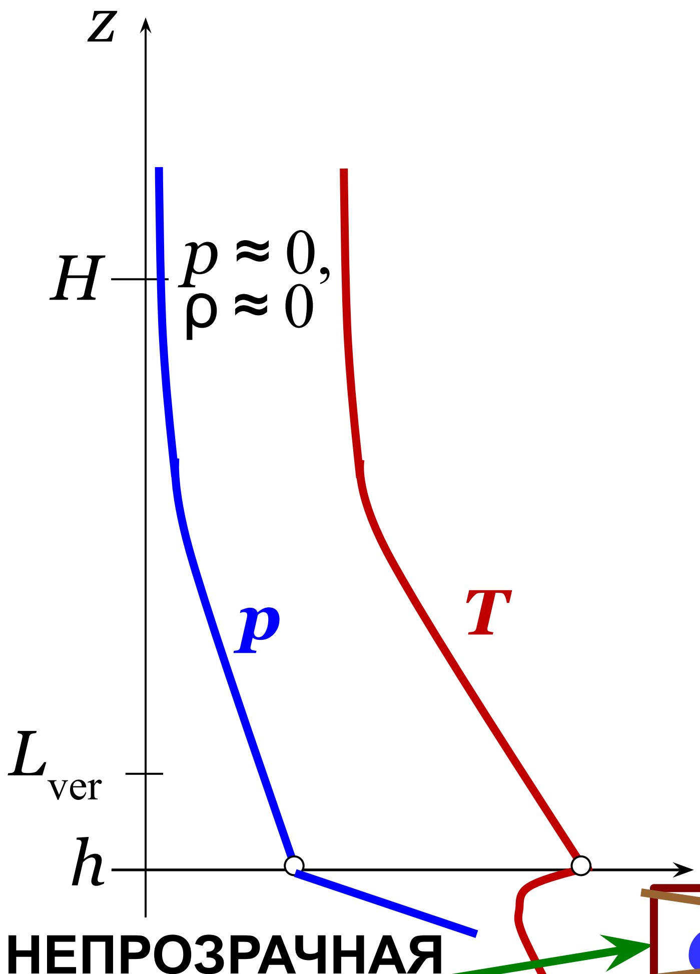
$\xi l^{(e)}$ - поток энергии испарения

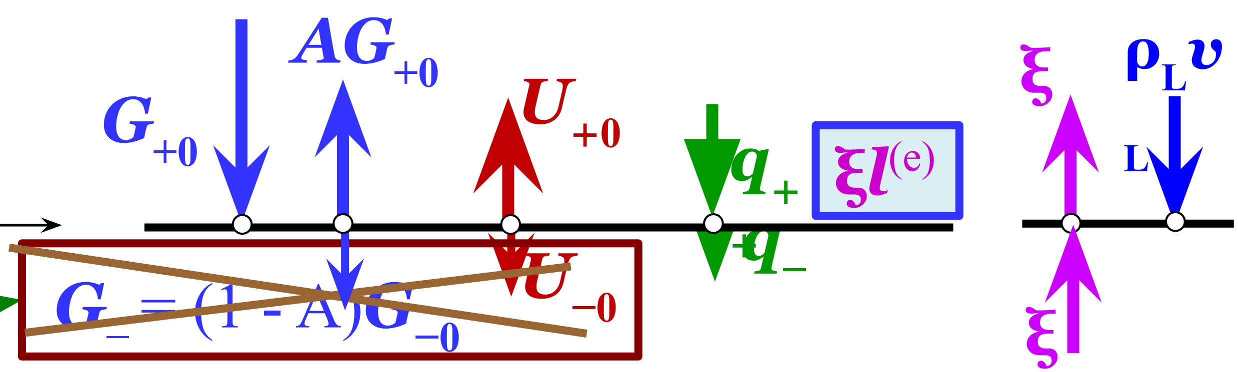
$\rho_L \upsilon_L$ – дождевой (снежный) поток

q - поток тепла

G - коротковолновая солнечная радиация

U - длинноволновое тепловое излучение





НЕПРОЗРАЧНАЯ (твердая) фаза:

$G_- = U_- = 0$

$$q_- - q_+ + (1-A)G_{+0} - G_{-0} - U_{+0} - U_{-0} = \xi^{(e)} l^{(e)} + \xi$$

$^{(m)} l^{(m)}$

$$\mathbf{B}_t \frac{\partial \bar{\mathbf{U}}}{\partial \bar{t}} + \mathbf{B}_x \frac{\partial \bar{\mathbf{U}}}{\partial \bar{x}} + \mathbf{B}_y \frac{\partial \bar{\mathbf{U}}}{\partial \bar{y}} + \mathbf{B}_z \frac{\partial \bar{\mathbf{U}}}{\partial \bar{z}} + \mathbf{B} = 0,$$

$$\mathbf{U} = \begin{pmatrix} \bar{\rho} \\ \bar{v}_x \\ \bar{v}_y \\ \bar{v}_z \\ \frac{\dot{\bar{M}}}{\bar{M}} \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} \bar{\rho}\left(\gamma\bar{Q}^{(R)} - \dot{\bar{M}}\right)/(\gamma\bar{M}) \\ -\bar{v}_y\tau f \\ \bar{v}_x\tau f \\ -\left(\gamma\bar{Q}^{(R)} - \dot{\bar{M}}\right)/(\gamma\bar{M}) \\ 0 \\ \bar{\rho} \end{pmatrix} \quad \mathbf{B}_t = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\mathbf{B}_x = \begin{pmatrix} \bar{v}_x & 0 & 0 & 0 & 0 & 0 \\ 0 & \bar{v}_x & 0 & 0 & 0 & \bar{g}/\bar{\rho} \\ 0 & 0 & \bar{v}_x & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ -\bar{v}_x & -\bar{\rho} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \mathbf{B}_y = \begin{pmatrix} \bar{v}_y & 0 & 0 & 0 & 0 & 0 \\ 0 & \bar{v}_y & 0 & 0 & 0 & 0 \\ 0 & 0 & \bar{v}_y & 0 & 0 & \bar{g}/\bar{\rho} \\ 0 & 0 & 1 & 0 & 0 & 0 \\ -\bar{v}_y & 0 & -\bar{\rho} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \mathbf{B}_z = \begin{pmatrix} \bar{v}_z & 0 & 0 & 0 & 0 & 0 \\ 0 & \bar{v}_z & 0 & 0 & 0 & 0 \\ 0 & 0 & \bar{v}_z & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

1. В **дифференциальном операторе** системы уравнений **квазистатического по вертикали движения нет скорости звука** C, даже в уравнениях горизонтального движения,

т.е. уже «акустика уже отфильтрована»

(вопреки утверждению Г.И. Марчука).

2. Система уравнений **только с вертикальной квазистатичностью негиперболична** ($C \to \infty$) даже при отсутствии теплопроводности, т.е. при отсутствии параболического члена

$$\frac{\partial}{\partial x_k}\left(\lambda^{(t)} \frac{\partial T}{\partial x_k}\right)$$

Некоторое решение

$$\bar{\mathbf{U}} = \bar{\mathbf{U}}\left(\bar{t}, \bar{x}, \bar{y}, \bar{z}\right)$$

Другое решение, отличающееся $\bar{\mathbf{U}}$ малым возмущением

$$\bar{\mathbf{U}}^{(\mathrm{d})} = \bar{\mathbf{U}} + \bar{\mathbf{U}}', \qquad \left(|\bar{\mathbf{U}}| \sim 1, \quad |\bar{\mathbf{U}}'| \ll 1\right).$$

Квазилинейные уравнения с переменными коэффициентами, определяемые значениями $\bar{\mathrm{U}}$ для возмущений

$$\mathbf{B}_t \frac{\partial \bar{\mathbf{U}}'}{\partial \bar{t}} + \mathbf{B}_x \frac{\partial \bar{\mathbf{U}}'}{\partial \bar{x}} + \mathbf{B}_y \frac{\partial \bar{\mathbf{U}}'}{\partial \bar{y}} + \mathbf{B}_z \frac{\partial \bar{\mathbf{U}}'}{\partial \bar{z}} + \mathbf{B}'\bar{\mathbf{U}}' = \mathbf{F}'$$

$$\bar{\mathbf{U}}' = \begin{pmatrix} \bar{\rho}' \\ \bar{v}_x' \\ \bar{v}_y' \\ \bar{v}_z' \\ \dot{\bar{M}}' \\ \bar{M}' \end{pmatrix}, \qquad \mathbf{F}' = \begin{pmatrix} -\bar{\rho}\bar{Q}^{(R)'}/\bar{M} \\ 0 \\ 0 \\ \bar{\rho}\bar{Q}^{(R)'}/\bar{M} \\ 0 \\ \bar{\rho} \end{pmatrix}$$

$$
\mathbf{B'} = \begin{pmatrix}
B_{11} & \dfrac{\partial\bar{\rho}}{\partial\bar{x}} & \dfrac{\partial\bar{\rho}}{\partial\bar{y}} & \dfrac{\partial\bar{\rho}}{\partial\bar{z}} & -\dfrac{\bar{\rho}}{\gamma\bar{M}} & -\dfrac{\bar{\rho}B_{11}}{\bar{M}} \\
B_{21} & \dfrac{\partial\bar{v}_x}{\partial\bar{x}} & \dfrac{\partial\bar{v}_x}{\partial\bar{y}} - \tau f & \dfrac{\partial\bar{v}_x}{\partial\bar{z}} & 0 & 0 \\
B_{31} & \dfrac{\partial\bar{v}_y}{\partial\bar{x}}\tau f & \dfrac{\partial\bar{v}_y}{\partial\bar{y}} & \dfrac{\partial\bar{v}_y}{\partial\bar{z}} & 0 & 0 \\
0 & 0 & 0 & 0 & \dfrac{1}{\gamma\bar{M}} & \dfrac{B_{11}}{\bar{M}} \\
-\dfrac{\partial\bar{v}_x}{\partial\bar{x}} - \dfrac{\partial\bar{v}_y}{\partial\bar{y}} & -\dfrac{\partial\bar{\rho}}{\partial\bar{x}} & -\dfrac{\partial\bar{\rho}}{\partial\bar{y}} & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
$$

$$
B_{11} = \frac{\overline{Q}^{(R)}}{\overline{M}} - \frac{\dot{\overline{M}}}{\gamma\overline{M}}, \qquad B_{21} = \frac{1}{\bar{\rho}}\left(\frac{\mathrm{d}\bar{v}_x}{\mathrm{d}\bar{t}} - \bar{v}_y\tau f\right),
$$

$$
B_{31} = \frac{1}{\bar{\rho}}\left(\frac{\mathrm{d}\bar{v}_y}{\mathrm{d}\bar{t}} + \bar{v}_x\tau f\right),
$$

Исходное гармоническое возмущение

$$t = 0: \quad \bar{\mathbf{U}}' = \mathbf{A} \sin\left(\bar{k}_x \bar{x} + \bar{k}_y \bar{y} + \bar{k}_z \bar{z}\right)$$

$$\mathbf{A} = \begin{pmatrix} A^{(\rho)} \\ A^{(vx)} \\ A^{(vy)} \\ A^{(vz)} \\ A^{(\dot{M})} \\ A^{(M)} \end{pmatrix},$$

Коротковолновые возмущения с $Q^{(\mathrm{R})\prime} = 0$:

$$\bar{k}_x = k_x L_{\mathrm{hor}}, \; \bar{k}_y = k_y L_{\mathrm{hor}}, \; \bar{k}_z = k_z L_{\mathrm{hor}} \gg 1,$$

$$l'_x = \frac{2\pi}{k_x} \ll L_{\mathrm{hor}}, \quad l'_y = \frac{2\pi}{k_y} \ll L_{\mathrm{hor}}, \quad l'_z = \frac{2\pi}{k_z} \ll L_{\mathrm{ver}}$$

Коротковолновые возмущения

$$\bar{k}_x = k_x L_{\text{hor}}, \quad \bar{k}_y = k_y L_{\text{hor}}, \quad \bar{k}_z = k_z L_{\text{ver}} \gg 1,$$

$$l'_x = \frac{2\pi}{k_x} \ll L_{\text{hor}}, \quad l'_y = \frac{2\pi}{k_y} \ll L_{\text{hor}}, \quad l'_z = \frac{2\pi}{k_z} \ll L_{\text{ver}}$$

$$\overbrace{\mathbf{B}_t, \ \mathbf{B}_x, \ \mathbf{B}_y, \ \mathbf{B}_z, \ \mathbf{B}'}\ \approx \text{const}, \quad \mathbf{F}' = 0$$

if $\bar{Q}^{(R)\prime} = 0$

$$\mathbf{B}_t \frac{\partial \bar{\mathbf{U}}'}{\partial \bar{t}} + \mathbf{B}_x \frac{\partial \bar{\mathbf{U}}'}{\partial \bar{x}} + \mathbf{B}_y \frac{\partial \bar{\mathbf{U}}'}{\partial \bar{y}} + \mathbf{B}_z \frac{\partial \bar{\mathbf{U}}'}{\partial \bar{z}} + \mathbf{B}'\bar{\mathbf{U}}' = 0$$

Однородная система линейных уравнений с постоянными коэффициентами

$$\boldsymbol{E}_* = \exp\left\{\mathbf{i}\left(\bar{k}_{x*}\bar{x} + \bar{k}_{y*}\bar{y} + \bar{k}_{z*}\bar{z} - \bar{\omega}_*\bar{t}\right)\right\}$$

$$\bar{\omega}_* = \bar{\omega} + \mathbf{i}\,\bar{\omega}_{**}, \quad \bar{k}_* = \bar{k} + \mathbf{i}\,\bar{k}_{**} \quad (\mathbf{i}^2 = -1)$$

$$\bar{v}_x = A_*^{(vx)} \boldsymbol{E}_*, \quad \bar{v}_y = A_*^{(vy)} \boldsymbol{E}_*, \quad \bar{v}_z = A_*^{(vz)} \boldsymbol{E}_*$$

$$\bar{\rho} = A_*^{(\rho)} \boldsymbol{E}_*, \quad \dot{\bar{M}} = A_*^{(\dot{M})} \boldsymbol{E}_*, \quad \bar{M} = A_*^{(M)} \boldsymbol{E}_*$$

$$\bar{k}_{x*} = \bar{k}_x \ (\bar{k}_{x**} = 0), \quad \bar{k}_{y*} = \bar{k}_y, \quad \bar{k}_{z*} = \bar{k}_z \ , \quad \bar{\omega}_* = \bar{\omega} + \mathbf{i}\, \bar{\omega}_{**}$$

$$\boldsymbol{E}_* = \exp\{\mathbf{i}(\bar{k}_x \bar{x} + \bar{k}_y \bar{y} + \bar{k}_z \bar{z} - \bar{\omega}_* \bar{t})\} =$$
$$= \exp(\bar{\omega}_{**} \bar{t}) \exp\{\mathbf{i}(\bar{k}_x \bar{x} + \bar{k}_y \bar{y} + \bar{k}_z \bar{z} - \bar{\omega}\, \bar{t})\}$$

$\bar{\omega}_{**} < 0$ – **устойчивость** (*коротковолновая*) решения $\bar{\mathbf{U}}(\bar{t},\ \bar{x},\ \bar{y},\ \bar{z})$

$\bar{\omega}_{**} > 0$ – **неустойчивость** (*коротковолновая*) решения $\bar{\mathbf{U}}(\bar{t},\ \bar{x},\ \bar{y},\ \bar{z})$

$\bar{\omega}_{**} \xrightarrow[\bar{k} \to \infty]{} +\infty$ – ***абсолютная*** (*коротковолновая*) ***неустойчивость*** *решения* $\bar{\mathbf{U}}(\bar{t},\ \bar{x},\ \bar{y},\ \bar{z})$

некорректно поставленная задача (по Адамару)

$$\mathbf{D}\mathbf{A}_* = 0, \qquad \mathbf{A}_* = \left(A_*^{(\rho)}, A_*^{(vx)}, A_*^{(vy)}, A_*^{(vz)}, A_*^{(\dot{M})}, A_*^{(M)}\right)^{\mathrm{T}}.$$

$$\mathbf{D} = i\left(-\bar{\omega}_*\mathbf{B}_t + \bar{k}_x\mathbf{B}_x + \bar{k}_y\mathbf{B}_y + \bar{k}_z\mathbf{B}_z\right) + \mathbf{B}'$$

$$\det \mathbf{D} = 0$$

$$\bar{\omega}_*^3 + b\bar{\omega}_*^2 + c\bar{\omega}_* + d = 0$$

b, c, d – определяется значениями компонент $\mathbf{U}$,

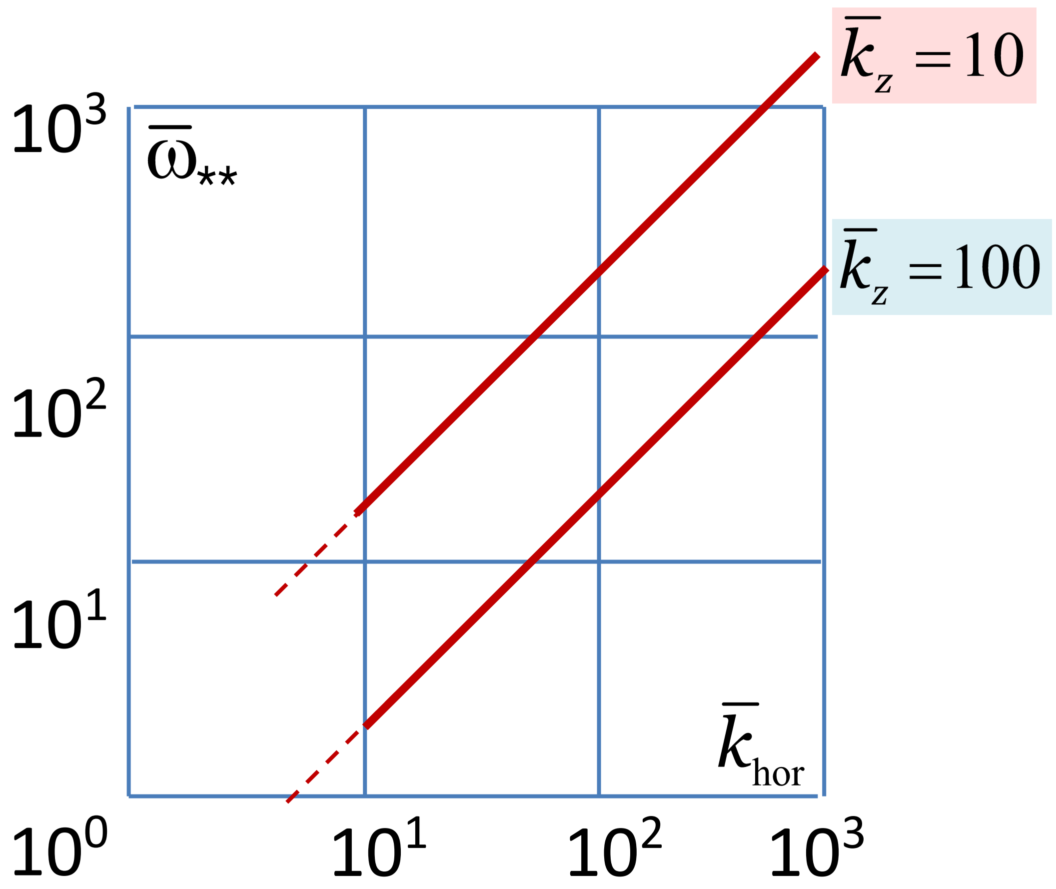
а именно $\bar{\rho}, \bar{v}_x, \bar{v}_y, \bar{v}_z, \dfrac{\partial \bar{v}_x}{\partial x}, \dfrac{\partial \bar{v}_y}{\partial x}, \ldots$

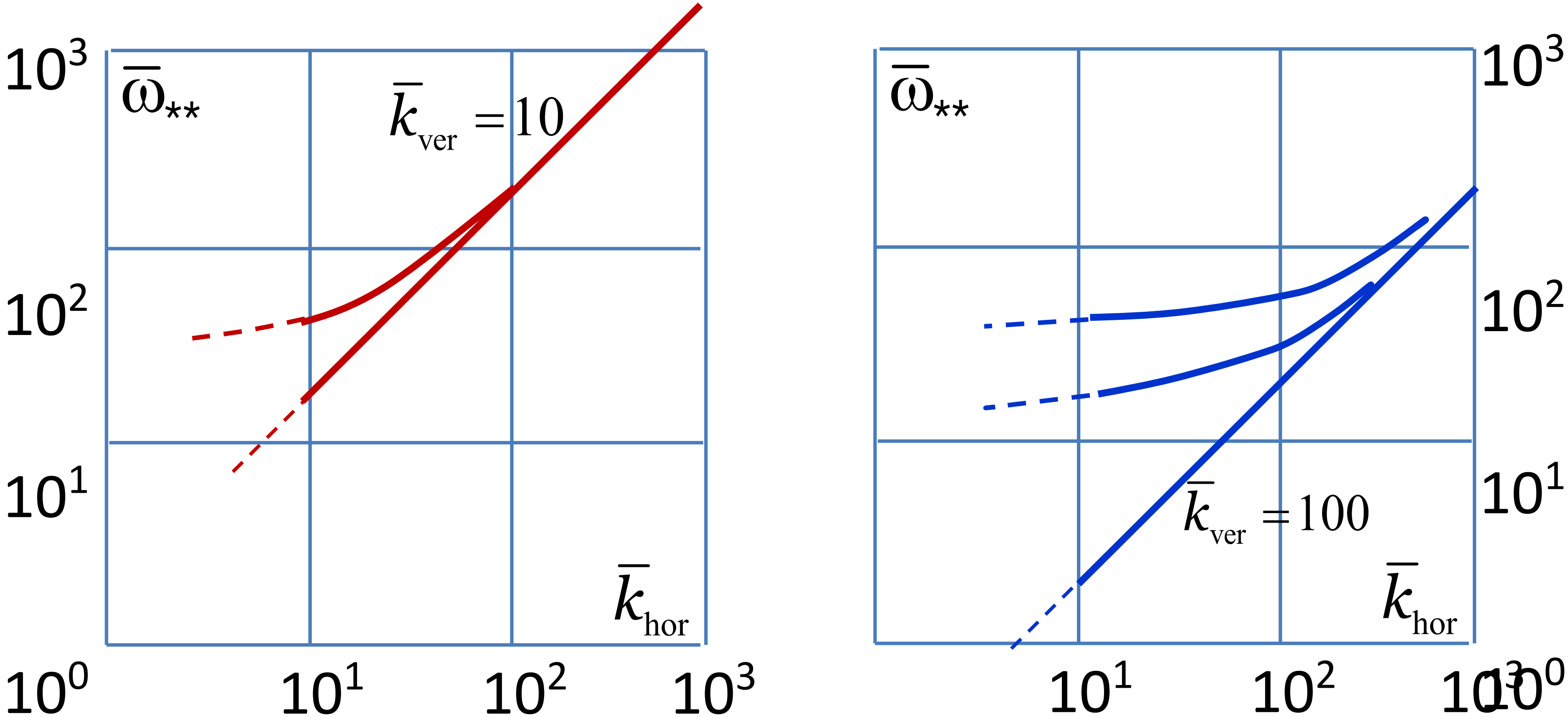
Неустойчивость покоя

$$\bar{v}_x = \bar{v}_y = \bar{v}_z = 0, \quad \frac{\partial \bar{\rho}}{\partial \bar{x}} = \frac{\partial \bar{\rho}}{\partial \bar{y}} = 0, \quad \dot{\bar{M}} = 0, \quad \bar{Q}^{(R)} = 0$$

$$\frac{\partial \bar{v}_x}{\partial \bar{t}} = \frac{\partial \bar{v}_y}{\partial \bar{t}} = 0, \quad \frac{\partial \bar{v}_x}{\partial \bar{x}} = \frac{\partial \bar{v}_x}{\partial \bar{y}} = \frac{\partial \bar{v}_x}{\partial \bar{z}} = \frac{\partial \bar{v}_y}{\partial \bar{x}} = \frac{\partial \bar{v}_y}{\partial \bar{y}} = \frac{\partial \bar{v}_y}{\partial \bar{z}} = \frac{\partial \bar{v}_z}{\partial \bar{z}} = 0,$$

+ стандартная атмосфера $\rho(z)$





$$\overline{v}_x = \overline{v}_y = \overline{v}_z = 10, \quad \overline{Q} = 0,$$

$$\frac{\partial \overline{v}_y}{\partial \overline{x}} = \frac{\partial \overline{v}_x}{\partial \overline{y}} = 10, \quad \frac{\partial \overline{v}_x}{\partial \overline{x}} = \frac{\partial \overline{v}_y}{\partial \overline{y}} = -10,$$

$$\frac{\partial \overline{v}_x}{\partial \overline{z}} = \frac{\partial \overline{v}_y}{\partial \overline{z}} = \frac{\partial \overline{v}_z}{\partial \overline{z}} = 0, \quad \frac{\partial \overline{\rho}}{\partial \overline{x}} = \frac{\partial \overline{\rho}}{\partial \overline{y}} = 0,$$

$$\frac{\mathrm{d}p}{\mathrm{d}t} = -g\int_z^H \left(\frac{\partial(\rho \upsilon_x)}{\partial x} + \frac{\partial(\rho \upsilon_y)}{\partial y} \right) \mathrm{d}z'$$

$$\frac{\partial \boldsymbol{\upsilon}_z}{\partial \boldsymbol{z}} = -\left(\frac{\partial \boldsymbol{\upsilon}_x}{\partial \boldsymbol{x}} + \frac{\partial \boldsymbol{\upsilon}_y}{\partial \boldsymbol{y}} \right) - \frac{1}{\gamma}\frac{\dot{\boldsymbol{M}}}{\boldsymbol{M}} + \frac{\gamma - 1}{\gamma}\frac{\boldsymbol{Q}^*}{\boldsymbol{g}\boldsymbol{M}}$$

$$\left(\frac{\mathrm{d}p}{\mathrm{d}t} = -g\rho \upsilon_z \right)$$

Часто используют

$$\upsilon_z = 0$$

$$\upsilon_z = 0$$

$$\dot{\omega}_{**} \underset{k_{\text{hor}}\to\infty}{\to}$$

$$\to \bar{G}\frac{k_{\text{hor}}}{k_{\text{ver}}} \to \infty$$

Часто используют

$$\frac{\mathrm{d}\rho}{\mathrm{d}\boldsymbol{t}} = \mathbf{0}$$

$$\frac{\partial \upsilon_z}{\partial z} = -\frac{\partial \upsilon_x}{\partial x} - \frac{\partial \upsilon_y}{\partial y}$$

$$\bar{\boldsymbol{\omega}}_*^2 = -\frac{\bar{g}}{\bar{\rho}}\frac{\partial \bar{\rho}}{\partial \bar{z}}\frac{\bar{k}_{\text{hor}}^2}{\bar{k}_{\text{ver}}^2}$$

$$\bar{\boldsymbol{\omega}}_{**} = 0, \ \text{if} \ \frac{\partial \bar{\rho}}{\partial \bar{z}} < 0$$

$$\bar{\boldsymbol{\omega}}_{**} \underset{\bar{k}_{\text{hor}}\to\infty}{\longrightarrow} \infty, \ \text{if} \ \frac{\partial \bar{\rho}}{\partial \bar{z}} > 0$$

Г.И. Марчук

$$\frac{\partial \rho}{\partial \boldsymbol{t}} = \mathbf{0}$$

чтобы «отфильтровать акустику» ???

$$\frac{\partial(\rho \upsilon_z)}{\partial z} = -\frac{\partial(\rho \upsilon_x)}{\partial x} - \frac{\partial(\rho \upsilon_y)}{\partial y}$$

$$\bar{\boldsymbol{\omega}}_{**} = 0$$

Учебник Дж. Холтона

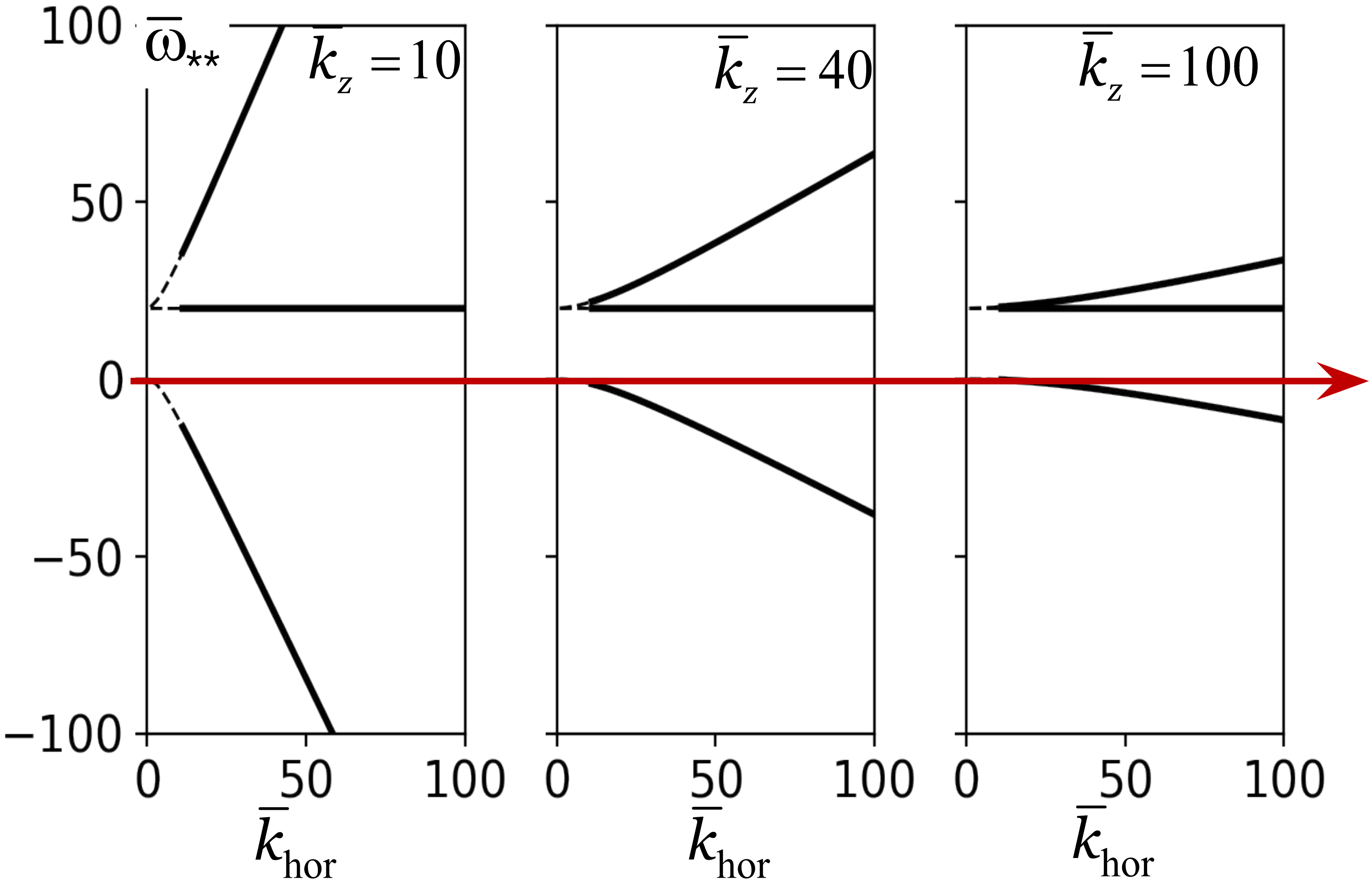
$$\frac{\partial \boldsymbol{p}}{\partial \boldsymbol{t}} = 0$$

$$\rho \upsilon_z = -\dot{M} \equiv \int_z^H \left(\frac{\partial(\rho \upsilon_x)}{\partial x} + \frac{\partial(\rho \upsilon_y)}{\partial y} \right) \mathrm{d}z'$$

$$\bar{\boldsymbol{\omega}}_*^2 = -\frac{\bar{g}}{\bar{\rho}}\frac{\partial \bar{\rho}}{\partial \bar{z}}\frac{\bar{k}_{\text{hor}}^2}{\bar{k}_{\text{ver}}^2},$$

$$\bar{\boldsymbol{\omega}}_{**} = 0, \ \text{if} \ \frac{\partial \bar{\rho}}{\partial \bar{z}} < 0,$$

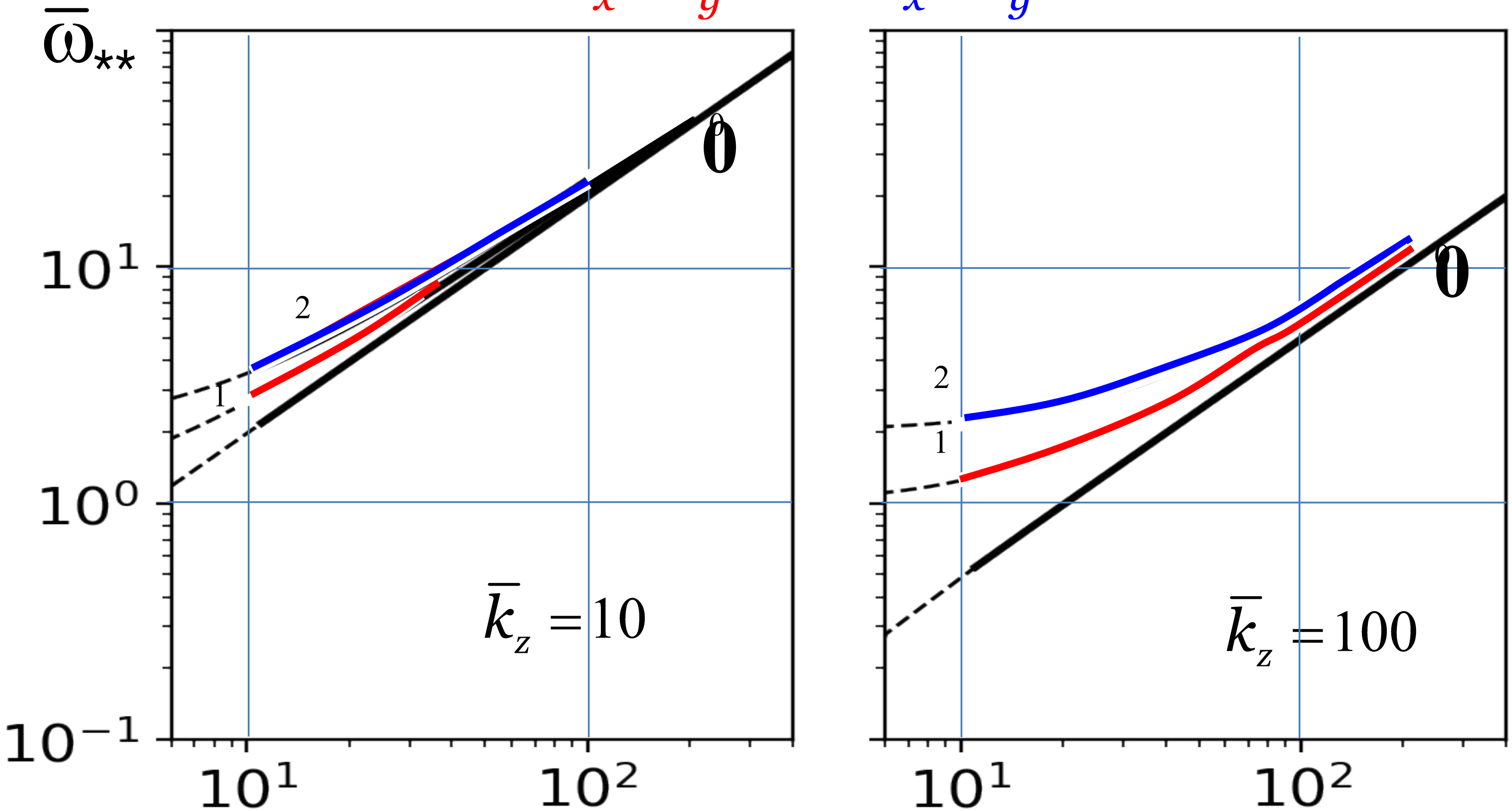
$$\bar{\boldsymbol{\omega}}_{**} \underset{\bar{k}_{\text{hor}}\to\infty}{\longrightarrow} \infty, \ \text{if} \ \frac{\partial \bar{\rho}}{\partial \bar{z}} > 0$$



$$\bar{v}_x = \bar{v}_y = \bar{v}_z = 1, \quad \bar{Q} = 0,$$

$$\frac{\partial \bar{v}_y}{\partial \bar{x}} = \frac{\partial \bar{v}_x}{\partial \bar{y}} = 0.5, \quad \frac{\partial \bar{v}_x}{\partial \bar{x}} = \frac{\partial \bar{v}_y}{\partial \bar{y}} = -0.5, \quad \frac{\partial \bar{v}_x}{\partial \bar{z}} = \frac{\partial \bar{v}_y}{\partial \bar{z}} = \frac{\partial \bar{v}_z}{\partial \bar{z}} = 0, \quad \frac{\partial \bar{\rho}}{\partial \bar{x}} = \frac{\partial \bar{\rho}}{\partial \bar{y}} = 0$$

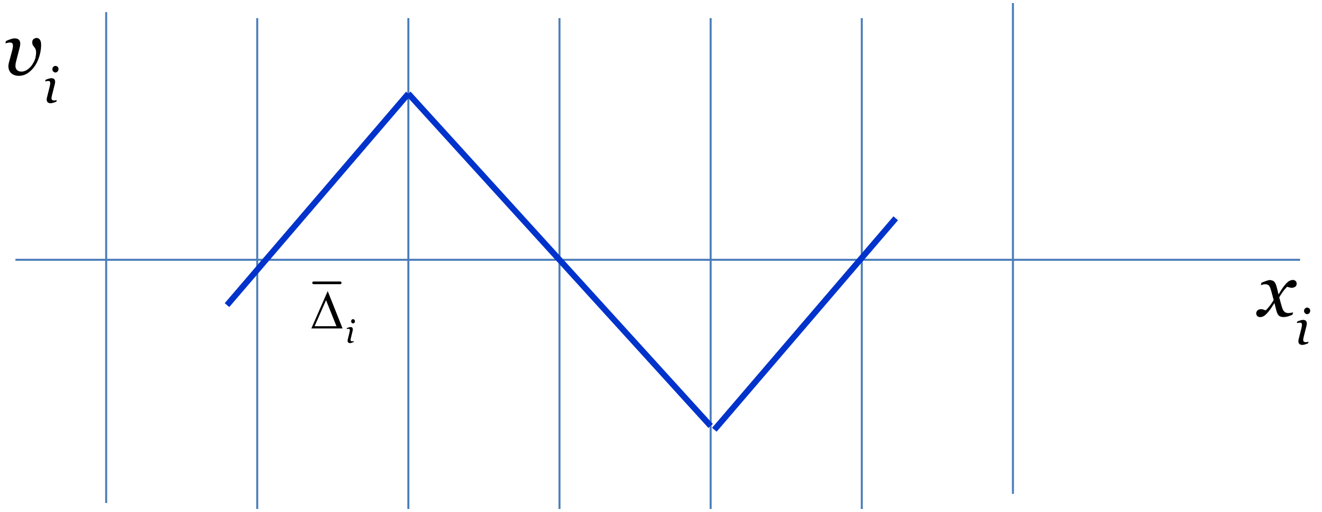
$\upsilon_x = \upsilon_y = 1, \ \upsilon_x = \upsilon_y = 2$



$$\overline{\upsilon}_x = \overline{\upsilon}_y = \overline{\upsilon}_z = 1 \text{ или } 2, \quad \overline{Q} = 0,$$

$$\frac{\partial \overline{\upsilon}_y}{\partial \overline{x}} = \frac{\partial \overline{\upsilon}_x}{\partial \overline{y}} = 1, \quad \frac{\partial \overline{\upsilon}_x}{\partial \overline{x}} = \frac{\partial \overline{\upsilon}_y}{\partial \overline{y}} = -1, \quad \frac{\partial \overline{\upsilon}_x}{\partial \overline{z}} = \frac{\partial \overline{\upsilon}_y}{\partial \overline{z}} = \frac{\partial \overline{\upsilon}_z}{\partial \overline{z}} = 0, \quad \frac{\partial \overline{\rho}}{\partial \overline{x}} = \frac{\partial \overline{\rho}}{\partial \overline{y}} = 0,$$

РАЗНОСТНАЯ СЕТКА



$$\bar{l}_i^{\,\min} = 4\bar{\Delta}_i, \qquad \bar{k}_i^{\max}\bar{l}_i^{\,\min} = 2\pi, \qquad \bar{k}_i^{\max} = k_i^{\max} L_i = \tfrac{1}{2}\pi N_i$$

$$\left(\bar{\Delta}_i = \Delta\bar{x}, \Delta\bar{y}, \Delta\bar{z}, \qquad N_i = \frac{L_i}{\Delta_i}, \qquad i = x, y, z \right),$$

$$k_i < \bar{k}_i^{\max} = \tfrac{1}{2}\pi N_i$$

Устранение коротковолновой неустойчивости с помощью искусственной вязкости

чтобы «убить» коротковолновые возмущения, которые не соответствуют квазистатике, положенной в исходные уравнения

$$\frac{dv_i}{dt} = \ \dots \ + \frac{\mu}{\rho}\left(\frac{\partial^2 v_i'}{\partial x^2} + \frac{\partial^2 v_i'}{\partial y^2} + \frac{\partial^2 v_i'}{\partial z^2}\right) + \frac{\lambda}{\rho}\frac{\partial}{\partial x_i}\left(\frac{\partial v_x'}{\partial x} + \frac{\partial v_y'}{\partial y} + \frac{\partial v_z'}{\partial z}\right)$$

$$\mu\left\{A^{(vi)}\bar{k}_x^2 + A^{(vi)}\bar{k}_y^2 + A^{(vi)}\bar{k}_z^2\right\} + \lambda\bar{k}_i \left\{A^{(vx)}\bar{k}_x + A^{(vy)}\bar{k}_y + A^{(vz)}\bar{k}_z\right\}$$

$$(i = x, y, z)$$

$$\mu_{\text{air}} = 1{,}8\times10^{-5}\ \text{кг}/(\text{м}\cdot\text{с})$$

$$\bar{\mu}_{\text{air}} = \mu_{\text{air}}\frac{\tau}{\rho_0 L_{\text{hor}}^2} = 1{,}8\times10^{-5}\frac{10^3}{1{,}19\times10^6} = 1{,}5\times10^{-8}$$

$$\bar{\mu} = c_\mu \bar{\mu}_{\text{air}} \bar{k}_x^n \bar{k}_y^n \bar{k}_z^n \qquad \bar{\lambda} = c_\lambda \bar{\lambda}_{\text{air}} \bar{k}_x^m \bar{k}_y^m \bar{k}_z^m$$

Негиперболичность приводит к **некорректной** (ill posed) постановке задачи Коши

при отсутствии диссипации, при которой **коротковолновые** возмущения

$$\delta W_k\big|_{t=0} = A(k)\left[\sin(kx)\right] \qquad \text{при } k \to \infty$$

растут неограниченно быстро:

$$\delta W = A(k)\times\exp\left(\omega_{**}(k)t\right)\left[\sin(kx)\right], \qquad \omega_{**}(k)\underset{k\to\infty}{\longrightarrow}\infty$$

$L_* = \dfrac{p_0}{\rho_0 g} \sim H \sim 10^4$ м **- линейный размер, следующий из дифференциального оператора и начальных условий**

$k_* \approx \dfrac{2\pi}{L_*} = \dfrac{2\pi\rho_0 g}{p_0}$ **- характерное волновое число метеорологического процесса**

Конечно-разностная схема **генерирует «паразитные»** коротковолновые возмущения с длинами волн $l \approx 4\Delta x$ и волновым

$$k_x \approx \frac{2\pi}{l_x} \text{ и } k_z \approx \frac{2\pi}{l_z}$$

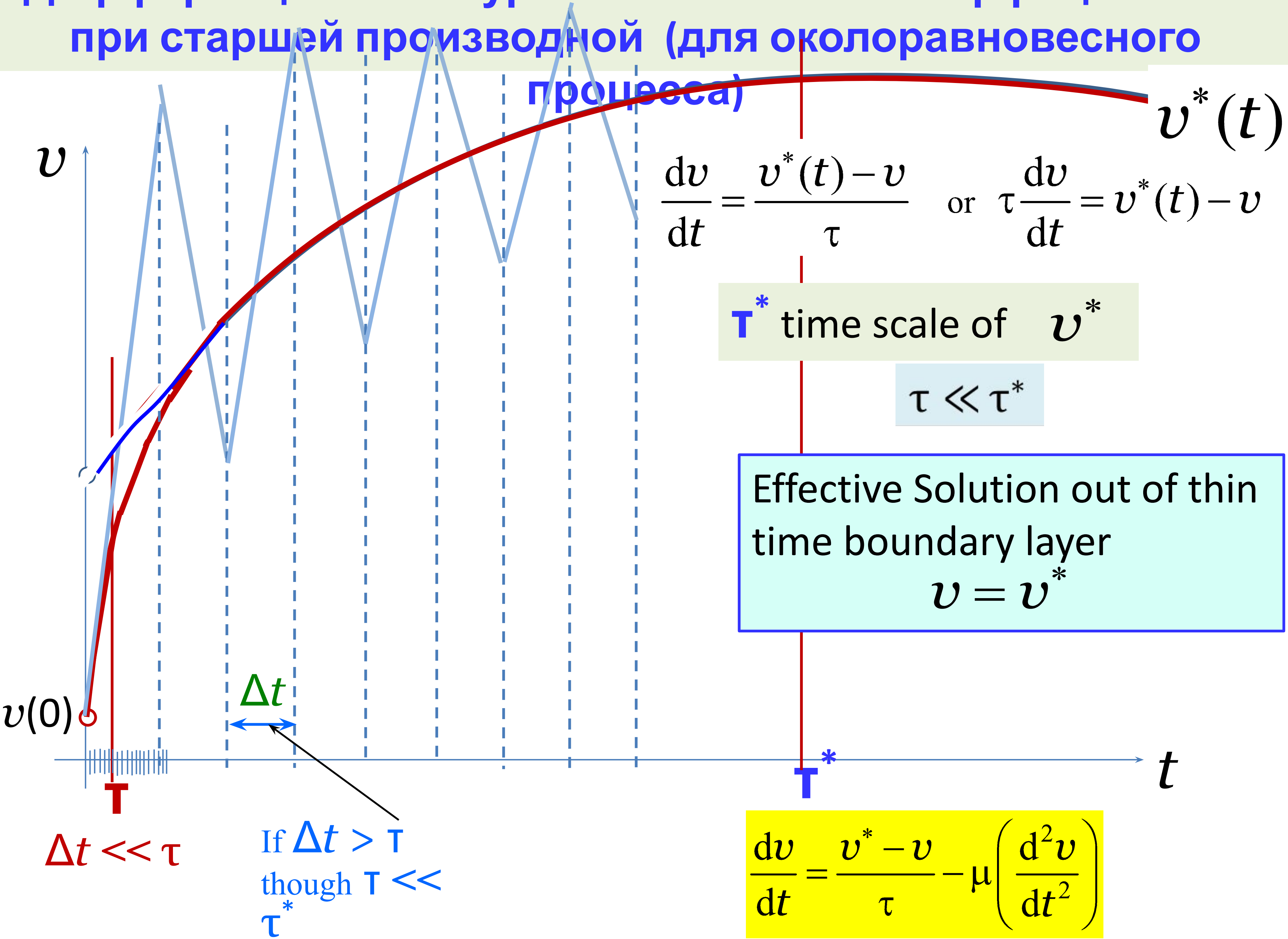
$$\text{если } k \approx \left(\frac{2\pi}{4\Delta z} \text{ или } \frac{2\pi}{4\Delta x}\right) >> k_* = \frac{2\pi\rho_0 g}{p_0}, \text{ то } \omega_{**}\tau >> 1, \quad \exp\left(\omega_{**}(k)\tau\right) >> 1$$

Необходим фильтр (численная диссипация) для возмущений типа

$$A(k)\times\sin(kx) \quad \text{при} \quad k\,\Delta x > 1$$

$$A(k) \to 0 \qquad \text{при } k \to \infty$$

Дифференциальное уравнение малым коэффициентом при старшей производной (для околоравновесного процесса)



$$\frac{dv}{dt} = \frac{v^*(t) - v}{\tau} \quad \text{or} \quad \tau \frac{dv}{dt} = v^*(t) - v$$

τ^* time scale of v^*

$$\tau \ll \tau^*$$

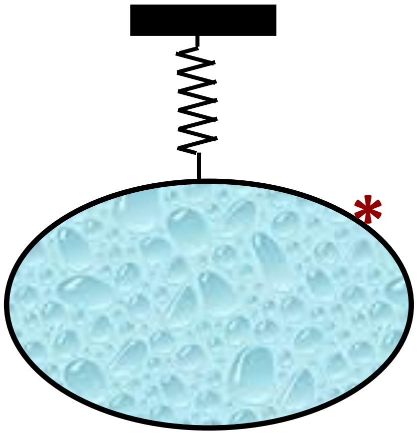
Effective Solution out of thin time boundary layer

$$v = v^*$$

$$\frac{dv}{dt} = \frac{v^* - v}{\tau} - \mu\left(\frac{d^2 v}{dt^2}\right)$$

“Stupid Method” for the Fly Weight Measurement *
(m(f) ~ 10^{-1} g) using a body m(B) ~ 10^4 g

1. To Weigh the Body B without the fly: m(B) ~ 10^4 g



2. To Weigh the Body B with the fly : m(B + f) ~ 10^4 g

3. Then $m(\mathrm{f}) = m(\mathrm{B+f}) - m(\mathrm{B})$

$$\rho\frac{\mathrm{d}v_z}{\mathrm{d}t} = -\frac{\partial p}{\partial z} - g$$

Луна и Земля

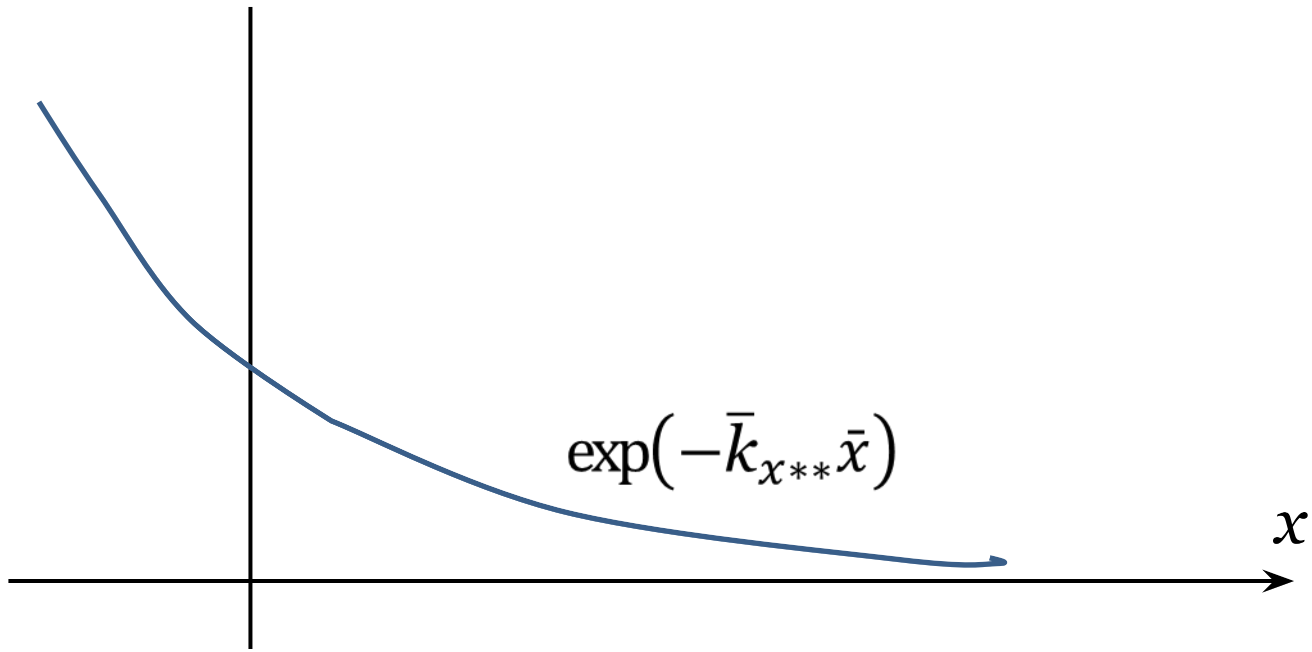
Снимок с борта Discovery 16.07.15 из точки Лагранжа (1,5 млн км от

СПАСИБО ЗА ВНИМАНИЕ !

Локальное возмущение для $x, y, z > 0$

$$\overline{\omega}_* = \overline{\omega} \quad (\overline{\omega}_{**} = 0), \qquad \overline{k}_{j*} = \overline{k}_j + i\overline{k}_{j**} \quad (j = x, y, z)$$

$$\boldsymbol{E}_* = \exp\left(i\left(\overline{k}_{x*}\overline{x} + \overline{k}_{y*}\overline{y} + \overline{k}_{z*}\overline{z} - \overline{\omega}_*\overline{t}\right)\right) =$$
$$= \exp\left(-\bar{k}_{x**}\bar{x} - \bar{k}_{y**}\bar{y} - \bar{k}_{z**}\bar{z}\right) \times \exp\left(i\left(\bar{k}_x\bar{x} + \bar{k}_y\bar{y} + \bar{k}_z\bar{z} - \overline{\omega}\bar{t}\right)\right)$$



Устойчивость покоя

$$\bar{v}_x = \bar{v}_y = \bar{v}_z = 0, \quad \frac{\partial\bar{\rho}}{\partial\bar{x}} = \frac{\partial\bar{\rho}}{\partial\bar{y}} = 0, \quad \dot{\bar{M}} = 0, \quad \bar{Q} = 0$$

$$\frac{\partial\bar{v}_x}{\partial\bar{t}} = \frac{\partial\bar{v}_y}{\partial\bar{t}} = 0, \quad \frac{\partial\bar{v}_x}{\partial\bar{x}} = \frac{\partial\bar{v}_x}{\partial\bar{y}} = \frac{\partial\bar{v}_x}{\partial\bar{z}} = \frac{\partial\bar{v}_y}{\partial\bar{x}} = \frac{\partial\bar{v}_y}{\partial\bar{y}} = \frac{\partial\bar{v}_y}{\partial\bar{z}} = \frac{\partial\bar{v}_z}{\partial\bar{z}} = 0,$$

+ стандартная атмосфера $\rho(z)$

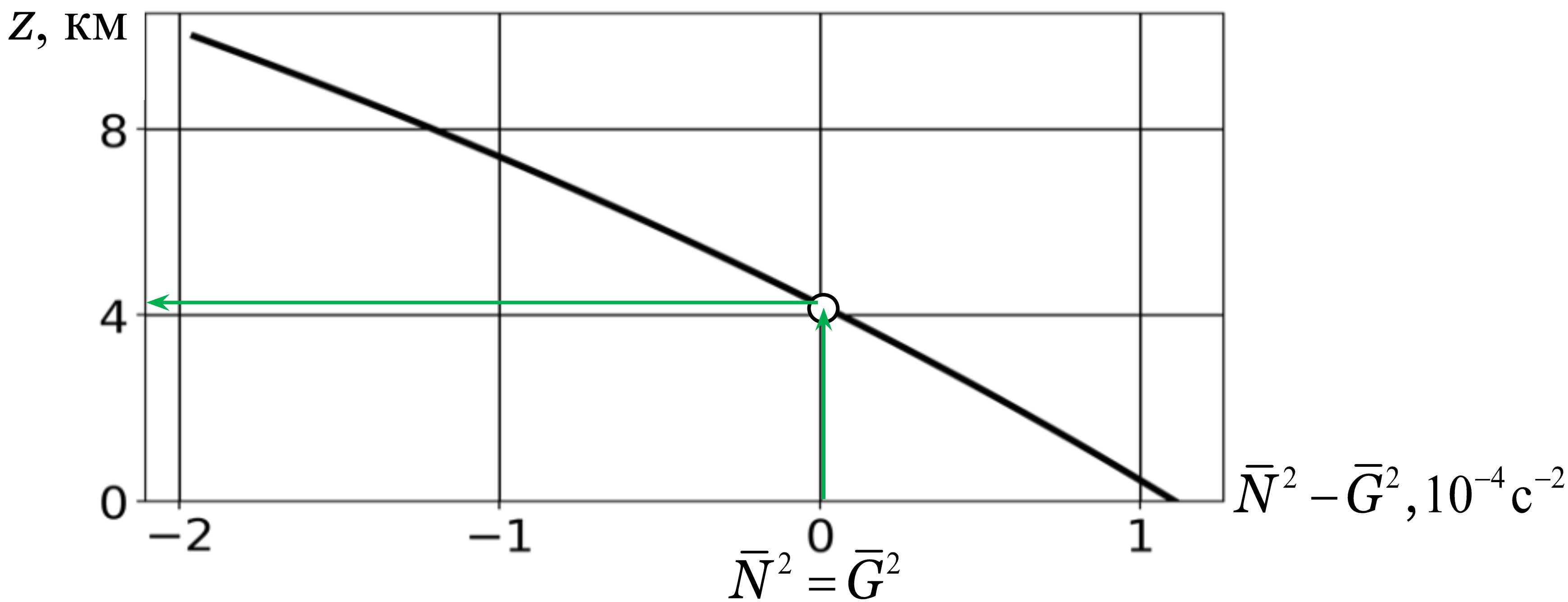
$$\bar{\omega}_*\left[\bar{\omega}_*^2 - \left(\left(\bar{N}^2 - \bar{G}^2\right)\frac{\bar{k}_{\text{hor}}^2}{\bar{k}_{\text{ver}}^2} + \bar{f}_{\text{cor}}^2 - \frac{\bar{N}^2\bar{G}^2}{\bar{g}}\frac{\bar{k}_{\text{hor}}^2}{\bar{k}_{\text{ver}}^3}\cdot i\right)\right] = 0,$$

$$\bar{N}^2 = -\frac{\bar{g}}{\bar{\rho}}\frac{\partial\bar{\rho}}{\partial\bar{z}} = \left(\frac{L_{\text{ver}}\tau}{L_{\text{hor}}}\right)^2 N^2 \qquad \left(N^2 = -\frac{g}{\rho}\frac{\partial\rho}{\partial z}\right), \qquad \bar{f}_{\text{cor}} = 2\Omega\,\tau\sin\theta$$

N частота Бр ента-Вяйсяля

$$\bar{G}^2 = \frac{\bar{\rho}\bar{g}}{\gamma\bar{M}} = \left(\frac{L_{\text{ver}}\tau}{L_{\text{hor}}}g\right)^2\frac{\rho}{\gamma p} \qquad \left(G^2 = \left(\frac{g}{C}\right)^2 = \frac{g^2\rho}{\gamma p}\right),$$

$$\bar{\omega}_*\left[\bar{\omega}_*^2-\left(\left(\bar{N}^2-\bar{G}^2\right)\frac{\bar{k}_{\text{hor}}^2}{\bar{k}_{\text{ver}}^2}+\bar{f}^2-\frac{\bar{N}^2\bar{G}^2}{\bar{g}}\frac{\bar{k}_{\text{hor}}^2}{\bar{k}_{\text{ver}}^3}\cdot i\right)\right]=0,$$



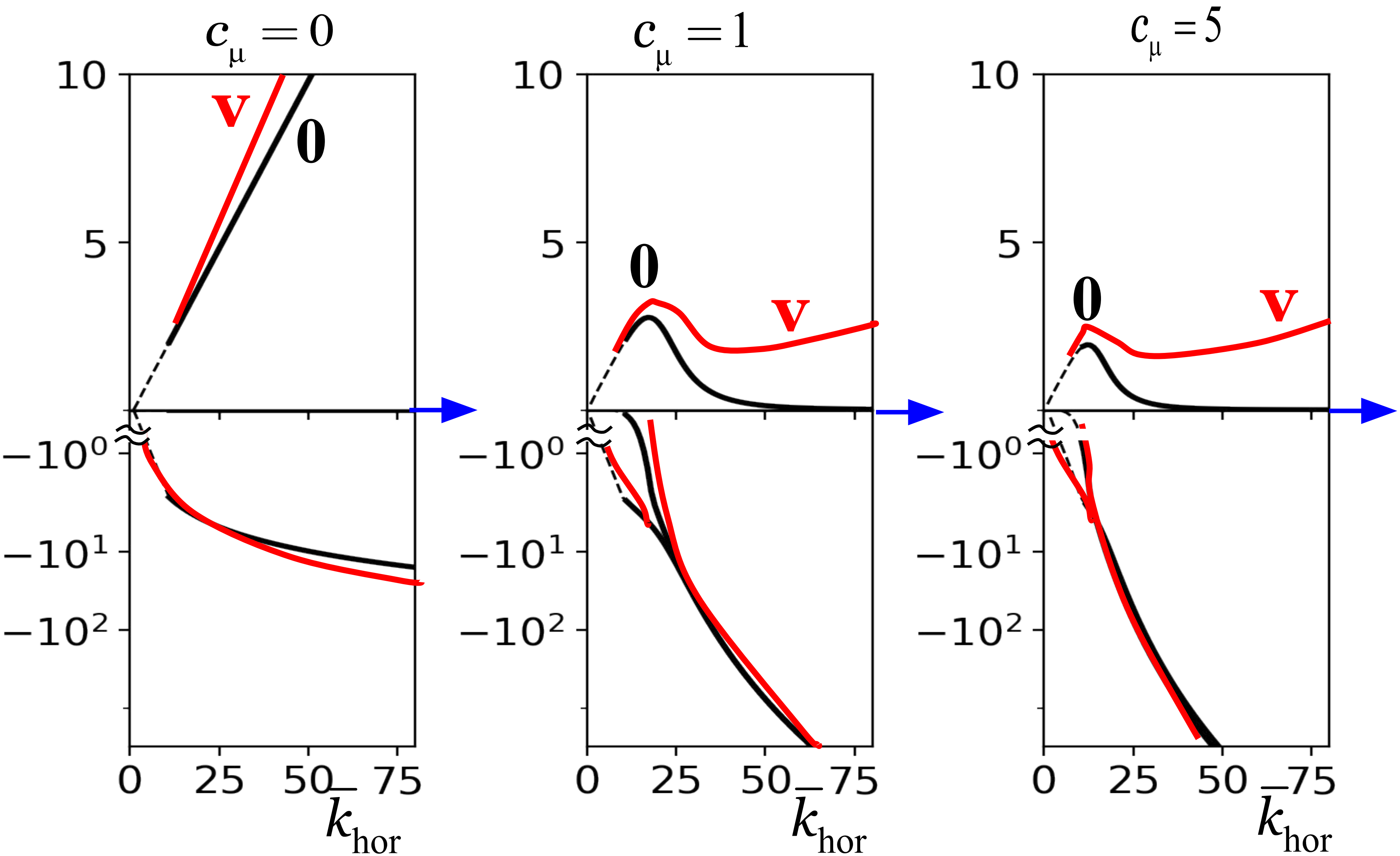
Near equatorial (θ << 1)

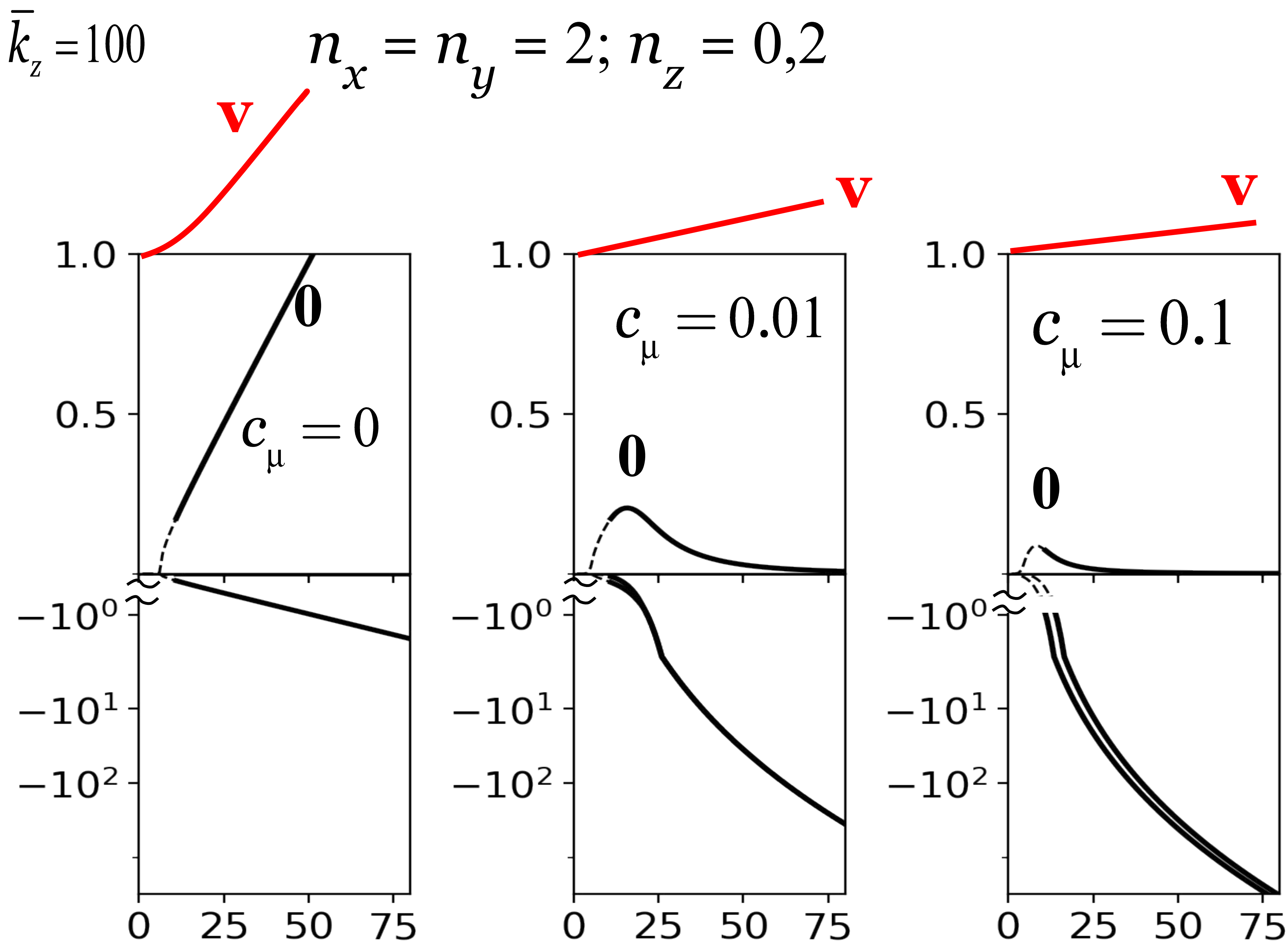
$$\bar{f}_{\text{cor}} \ll \sqrt{\left|\bar{N}^2-\bar{G}^2\right|}\frac{\bar{k}_{\text{hor}}}{\bar{k}_{\text{ver}}}, \qquad \bar{\omega}_{**}^{(2,3)}=\pm A_1\kappa_1 \underset{\kappa_1\to\infty}{\longrightarrow} \pm\infty, \qquad \left(\kappa_1=\frac{\bar{k}_{\text{hor}}}{\bar{k}_{\text{ver}}^2}\right)$$

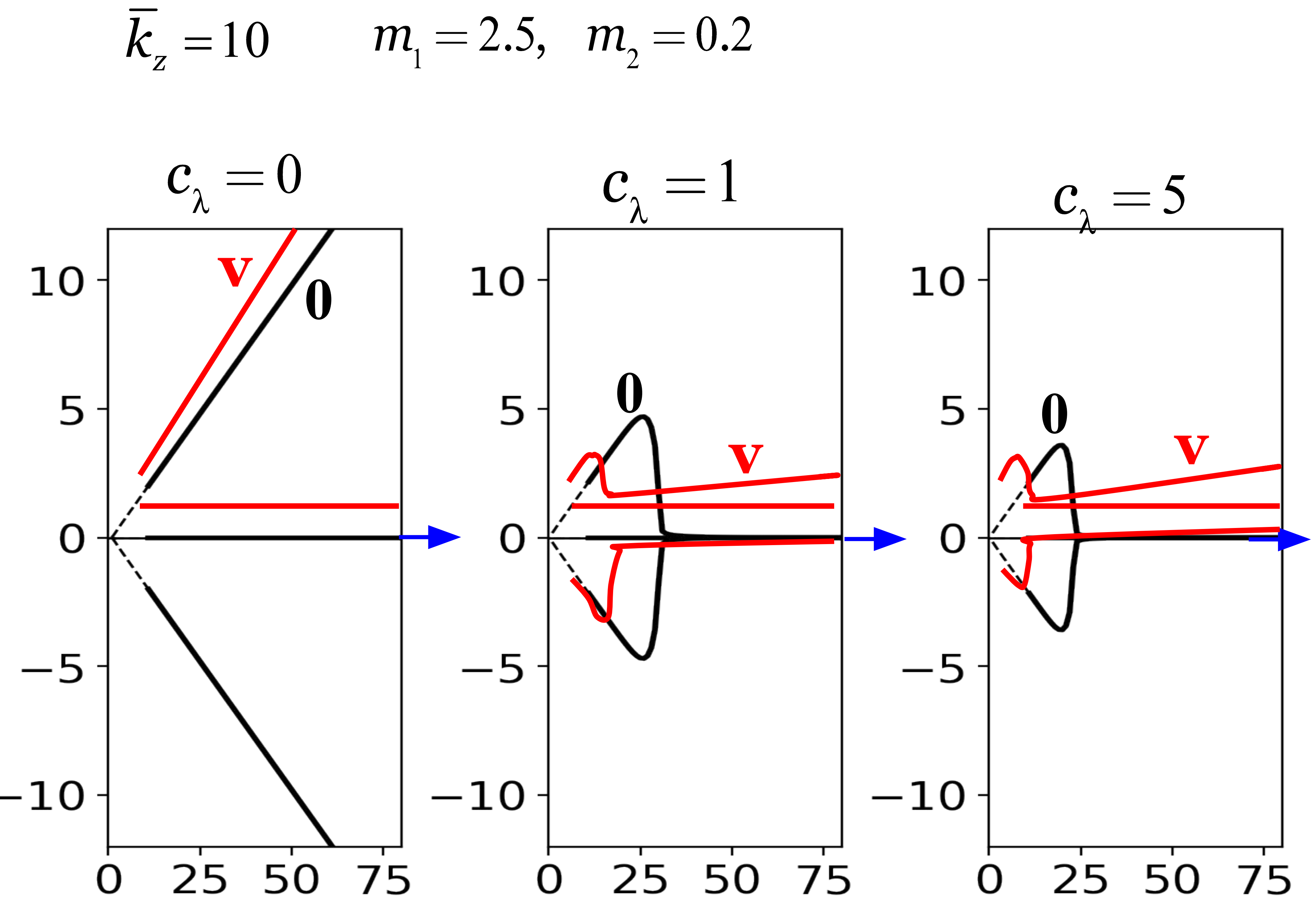
High latitudes (θ > 1) or z ≈ 4 rm

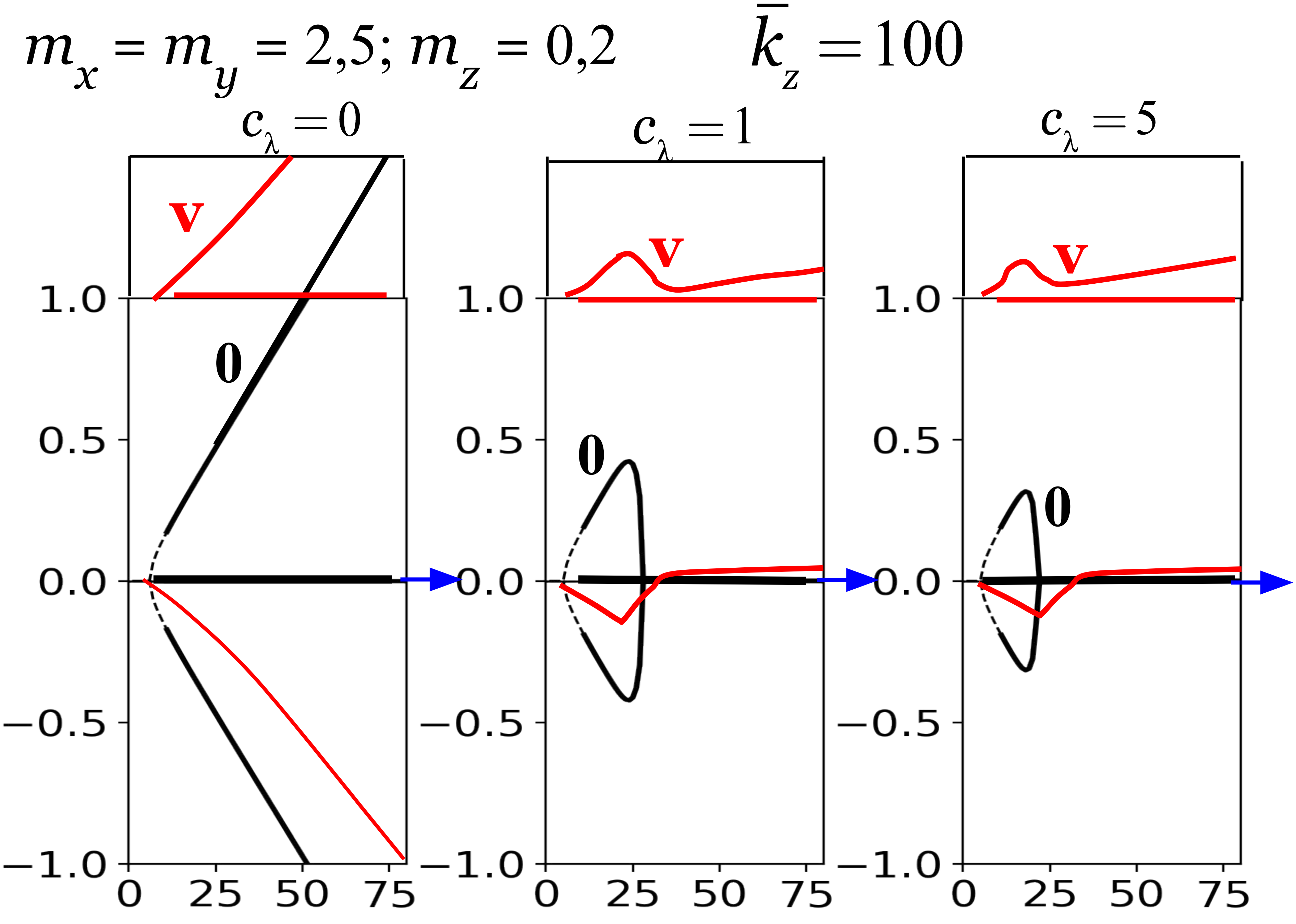
$$\bar{f}_{\text{cor}} \gg \sqrt{\left|\bar{N}^2-\bar{G}^2\right|}\frac{\bar{k}_{\text{hor}}}{\bar{k}_{\text{ver}}}, \qquad \bar{\omega}_{**}^{(2,3)}=\pm A_2\kappa_2 \underset{\kappa_2\to\infty}{\longrightarrow} \pm\infty, \qquad \left(\kappa_2=\frac{\bar{k}_{\text{hor}}^2}{\bar{k}_{\text{ver}}^3}\right)$$

$\bar{k}_z = 10 \qquad n_x = n_y = 2;\ n_z = 0,2$




$\bar{k}_z = 100$
$n_x = n_y = 2; n_z = 0,2$
v
0
$c_\mu = 0$
$c_\mu = 0.01$
$c_\mu = 0.1$
1.0
0.5
-10^0
-10^1
-10^2
0
25
50
75


$\bar{k}_z = 10 \quad m_1 = 2.5, \quad m_2 = 0.2$
$c_\lambda = 0$
$c_\lambda = 1$
$c_\lambda = 5$
v
0
10
5
0
−5
−10
0
25
50
75



$m_x = m_y = 2,5; m_z = 0,2$
$\bar{k}_z = 100$
$c_\lambda = 0$
$c_\lambda = 1$
$c_\lambda = 5$
v
v
v
0
0
0
1.0
0.5
0.0
−0.5
−1.0
0
25
50
75