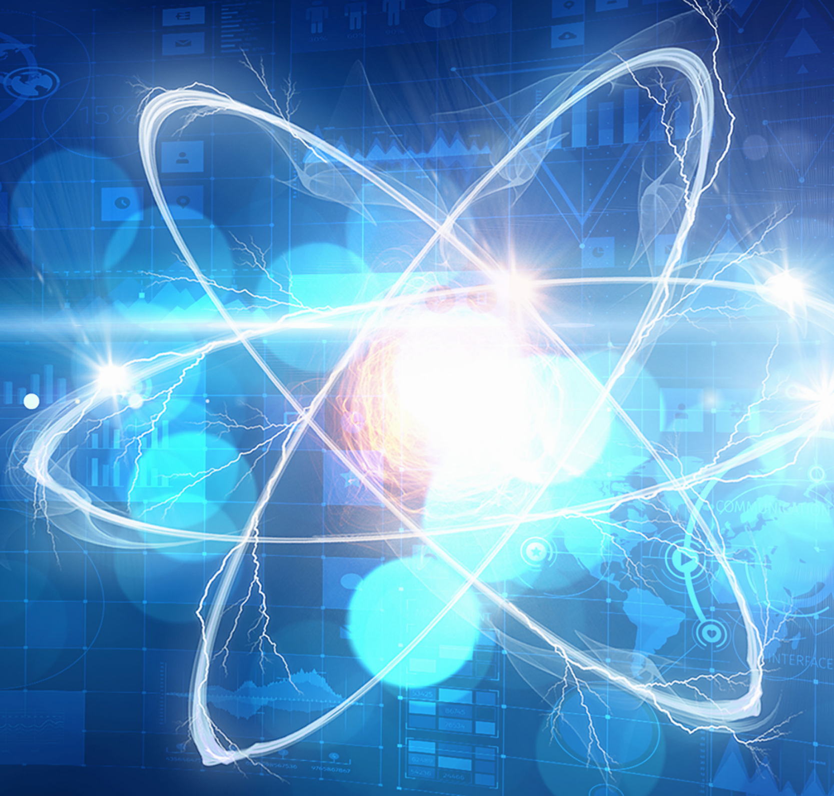
Введение в вычислительную физику экстремальных световых полей



Сергей Рыкованов
Сколковский институт науки и технологий

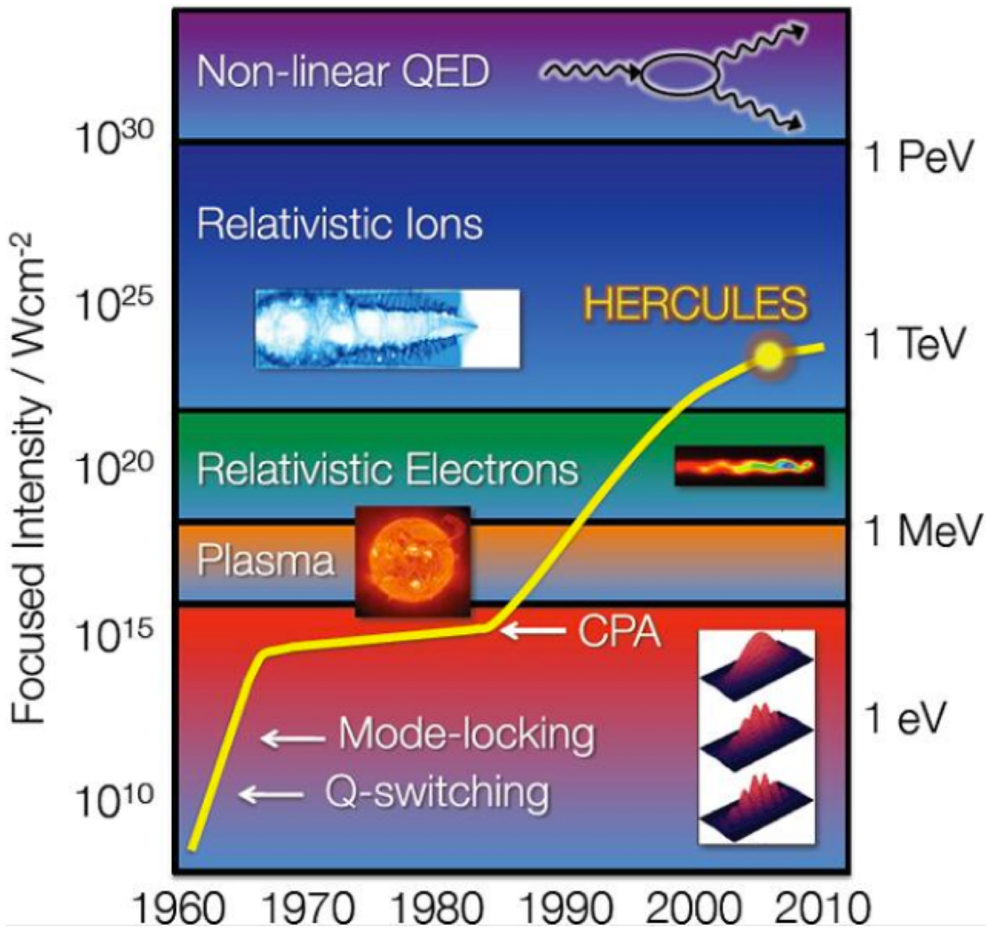
План лекции

1. Введение
2. **Электрон в плоской электромагнитной волне**
    1. Аналитическое решение
    2. Численное решение
    3. Излучение электрона
3. **Моделирование взаимодействия мощного лазерного излучения с плазмой**
    1. Метод частиц-в-ячейке (Particle-In-Cell)
    2. Примеры

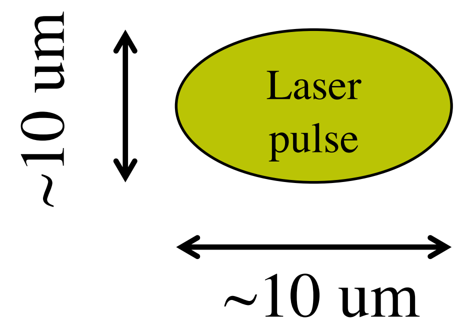
Quest for more intense laser pulses

Intensity = Energy / Unit Surface / Unit time

Sun Intensity on Earth surface ~ 0.1 W cm^{-2}



- **1985 CPA (Chirped Pulse Amplification)**
- **Petawatt laser systems (10^{15} Watt)**
- **World power production ~10^{13} Watt**
- **Ultrashort pulses (femtosecond)**
- **1 femtosecond = 10^{-15} seconds**



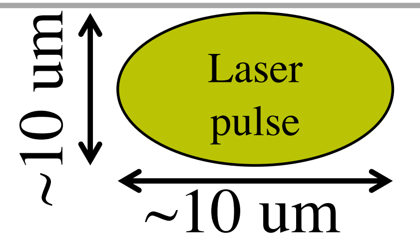
- **10 times smaller than human hair**

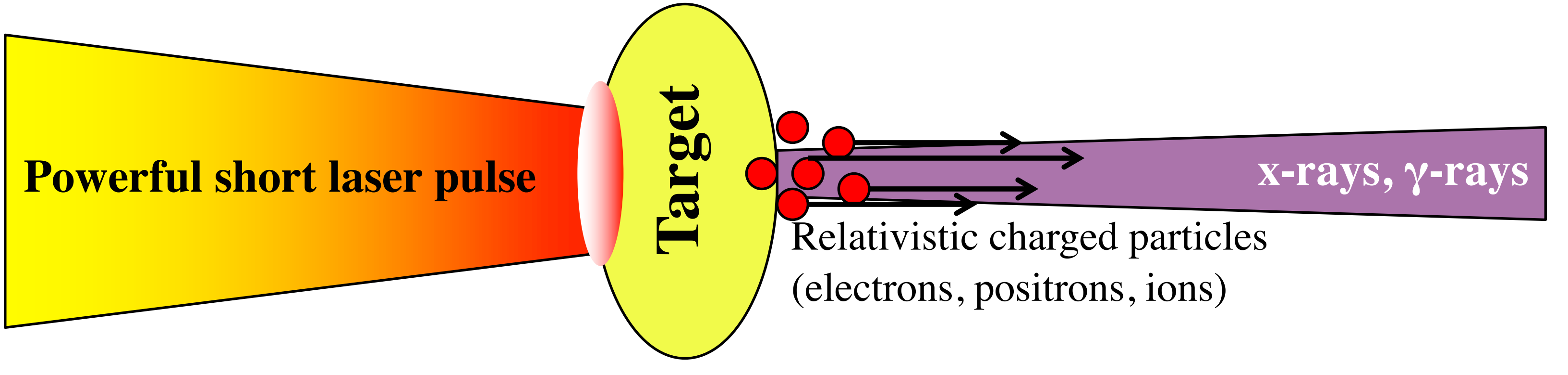
Picture credit: CUOS, University of Michigan

Skoltech
Skolkovo Institute of Science and Technology

What can one do with powerful lasers?

- **Extremely intense small light bullets**
- **So intense, they can ionize materials and create plasma**

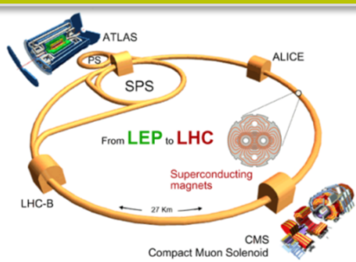




Relativistic Laser Plasma – new and extremely promising area of physics

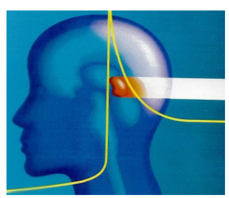
Relativistic Laser Plasma

Accelerator physics

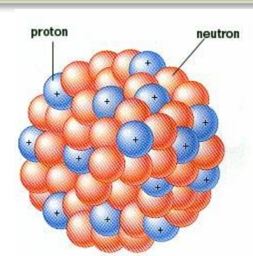


Astrophysics

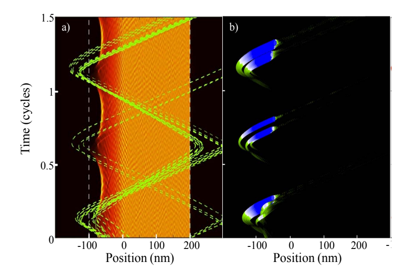
Hadron therapy



Nuclear physics



Attosecond Physics



Интенсивность лазерных импульсов: $>10^{18}$ Вт/см^2

1. Experiment

2. Theory

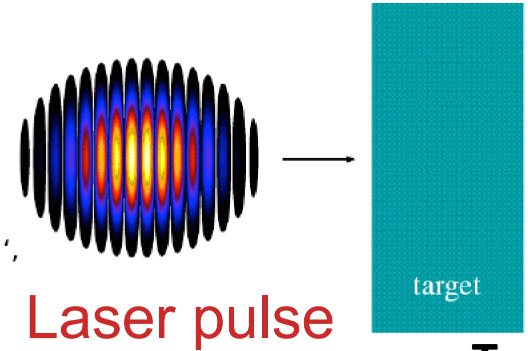
$$i\hbar\frac{\partial}{\partial t}\Psi=\hat{H}\Psi$$

3. Modeling

4. Big Data and AI

Skoltech
Skolkovo Institute of Science and Technology

Petawatt focused into small spot leads to extreme conditions



Example for a laser pulse:
E=40 Joules
tau=40 fs ($40*10^{-15}$ s)
focused to ~um spotsize

→ $I \sim 10^{22}$ W/cm^2

Typical wavelength: 1um (Nd:Glass), 0.8 um (Ti:Sa), 10um (CO2)

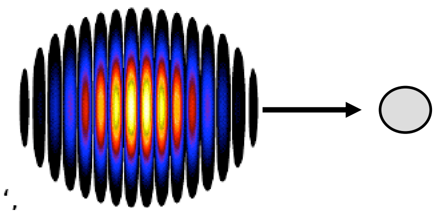
Typical targets:
- gases (Hydrogen, Hydrogen+Helium, etc);
- solid state targets (Glass, aluminum, plastic targets);
- advanced targets (nanostructured, liquid jets etc).

Targets are almost immediately **ionized** → interaction with **plasmas**

Laser pulse **electric field**: $E \sim 10^{13}$ V/m

Radiation pressure: $P_{rad} = I/c \sim 30$Gbar

Движение электрона в э-м волне
Основа основ



$$\frac{d\vec{p}}{dt} = -e\vec{E} - \frac{e}{c}\vec{v} \times \vec{B}$$

$$p = mv\gamma$$

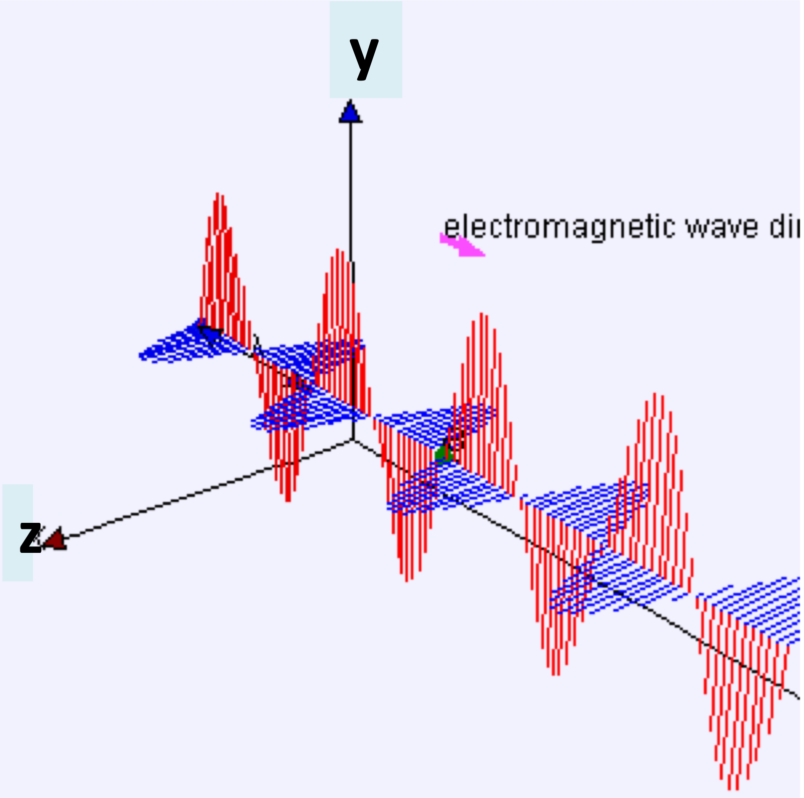
$$\gamma = \sqrt{1 + \frac{p^2}{m^2c^2}} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\frac{du^\mu}{d\tau} = -\frac{e}{c}F^{\mu\nu}u_\nu$$

Electron in the plane e-m wave: important problem!

Consider plane e-m wave (laser pulse) traveling in the +x direction:

$$\vec{A} = \vec{e}_y A(t,x) = \vec{e}_y A_0 g\left(t - \frac{x}{c}\right)\cos\left(\omega t - kx\right)$$



$$\vec{E} = -\frac{1}{c}\frac{\partial \vec{A}}{\partial t} \quad \Rightarrow \quad E_y = B_z$$

$$\vec{B} = \mathrm{rot}\vec{A}$$

$$\omega^2 = c^2 k^2$$

E-m dispersion relation in vacuum

$$\phi = t - \frac{x}{c}$$

Lightcone coordinate

Electron in the plane e-m wave: important problem!

Consider plane e-m wave (laser pulse) traveling in the +x direction:

$$\vec{A} = \vec{e}_y A(t,x) = \vec{e}_y A_0 g\left(t - \frac{x}{c}\right)\cos\left(\omega t - kx\right)$$

$$\phi = t - \frac{x}{c} \quad \Rightarrow \quad E_y = -\frac{1}{c}\frac{dA_y}{d\phi} \qquad B_z = -\frac{1}{c}\frac{dA_y}{d\phi}$$

$$E_y = B_z = \frac{\omega A_0}{c} g\left(\phi\right)\sin\left(\omega\phi\right)$$

* assuming g(phi) is slowly varying

Electron in the plane e-m wave: important problem!

Consider plane e-m wave (laser pulse) traveling in the +x direction:

$$E_y = B_z = -\frac{1}{c}\frac{dA_y}{d\phi} = \frac{\omega A_0}{c} g\left(\phi\right)\sin\left(\omega\phi\right)$$

$$\phi = t - \frac{x}{c}$$

Equations of motion:

$$\frac{d\vec{p}}{dt} = -e\vec{E} - \frac{e}{c}\vec{v}\times\vec{B}$$

$$p = mv\gamma$$

$$\gamma = \sqrt{1+\frac{p^2}{m^2c^2}} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

Electron in the plane e-m wave: important problem!

Consider plane e-m wave (laser pulse) traveling in the +x direction:

$$E_y = B_z = -\frac{1}{c}\frac{dA_y}{d\phi} = \frac{\omega A_0}{c} g\left(\phi\right)\sin\left(\omega\phi\right)$$

$$\phi = t - \frac{x}{c}$$

Equations of motion:

$$\frac{dp_x}{dt} = -\frac{ev_y}{c}B_z$$

$$\frac{dp_y}{dt} = -eE_y + \frac{ev_x}{c}B_z$$

$$p = mv\gamma$$

$$\gamma = \sqrt{1 + \frac{p^2}{m^2c^2}} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Electron in the plane e-m wave: important problem!

$$\vec{\beta} = \frac{\vec{v}}{c}, \quad \vec{u} = \frac{\vec{p}}{mc}$$

$$\tilde{t} = \omega t, \quad \tilde{x} = \frac{\omega}{c} x, \quad \tilde{\phi} = \omega \phi$$

$$\frac{dp_y}{dt} = -eE_y + \frac{ev_x}{c} B_z$$

$$\Downarrow$$

$$\frac{du_y}{d\tilde{t}} = -\frac{eE_y}{mc\omega} + \beta_x \frac{eB_z}{mc\omega}$$

Electron in the plane e-m wave: important problem!

$$\vec{\beta} = \frac{\vec{v}}{c}, \quad \vec{u} = \frac{\vec{p}}{mc}$$

$$\tilde{t} = \omega t, \quad \tilde{x} = \frac{\omega}{c} x, \quad \tilde{\phi} = \omega \phi$$

$$\frac{dp_y}{dt} = -eE_y + \frac{ev_x}{c} B_z$$

$$\frac{du_y}{d\tilde{t}} = -\frac{eE_y}{mc\omega} + \beta_x \frac{eB_z}{mc\omega}$$

$\frac{eE_y}{mc\omega} \rightarrow \tilde{E}_y$, $\quad \frac{eB_z}{mc\omega} \rightarrow \tilde{B}_z$

Normalized e-m wave strength

If we know electric or magnetic field amplitude:

$$a_0 = \frac{eE_{\mathrm{max}}}{mc\omega}$$

If we know vector potential amplitude:

$$a_0 = \frac{eA_{\mathrm{max}}}{mc^2}$$

$$\tilde{E}_y = \tilde{B}_z = -\frac{d\tilde{A}_y}{d\tilde{\phi}}$$

Additional equation: energy balance

$$mc^2 \frac{d\gamma}{dt} = -e\vec{E}\vec{v} = -eE_y v_y$$

Additional equation: energy balance

$$mc^2 \frac{d\gamma}{dt} = -e\vec{E}\vec{v} = -eE_y v_y$$

$$\frac{d\gamma}{d\tilde{t}} = -\beta_y \tilde{E}_y$$

Equations of motion in dimensionless form

(1) $$\frac{d\gamma}{d\tilde{t}} = -\beta_y \tilde{E}_y$$

(2) $$\frac{du_x}{d\tilde{t}} = -\beta_y \tilde{B}_z$$

(3) $$\frac{du_y}{d\tilde{t}} = -\tilde{E}_y + \beta_x \tilde{B}_z$$

$$E_y = B_z$$

$$\tilde{\phi} = \tilde{t} - \tilde{x}$$

Equations of motion in dimensionless form

(1) $$\frac{d\gamma}{d\tilde{t}} = -\beta_y \tilde{E}_y$$

(2) $$\frac{du_x}{d\tilde{t}} = -\beta_y \tilde{B}_z$$

(3) $$\frac{du_y}{d\tilde{t}} = -\tilde{E}_y + \beta_x \tilde{B}_z$$

$$E_y = B_z$$

$$\tilde{\phi} = \tilde{t} - \tilde{x}$$

(1) – (2):

$$\gamma - u_x = \text{const}$$

$$\gamma - u_x = 1$$

for e- initially @ rest

Equations of motion in dimensionless form

$$\gamma - u_x = 1$$

$$E_y = B_z$$

$$\frac{d\tilde{\phi}}{d\tilde{t}} = 1 - \beta_x = \frac{\gamma - u_x}{\gamma} = \frac{1}{\gamma} \quad \Leftarrow \quad \tilde{\phi} = \tilde{t} - \tilde{x}$$

$$d\tilde{\phi} = \frac{d\tilde{t}}{\gamma} = d\tau$$

τ - proper time

(3) $$\frac{du_y}{d\tilde{t}} = -\tilde{E}_y + \beta_x \tilde{B}_z$$ → replace t to phi

Equations of motion in dimensionless form

$$\gamma - u_x = 1$$

$$E_y = B_z$$

$$d\tilde{\phi} = \frac{d\tilde{t}}{\gamma} = d\tau$$

$$\tilde{\phi} = \tilde{t} - \tilde{x}$$

$$\frac{du_y}{d\tilde{t}} = -\tilde{E}_y + \beta_x \tilde{B}_z \longrightarrow \frac{du_y}{d\tilde{\phi}} = -\gamma \tilde{E}_y + u_x \tilde{B}_z$$

$$\tilde{E}_y = \tilde{B}_z = -\frac{d\tilde{A}_y}{d\tilde{\phi}} \longrightarrow \frac{du_y}{d\tilde{\phi}} = \frac{d\tilde{A}_y}{d\tilde{\phi}}$$

$$u_y = \tilde{A}_y$$

for electron initially @ rest

$$p_y = \frac{e}{c} A_y$$

HELMHOLTZ | GEMEINSCHAFT

Helmholtz-Institut Jena

We have found two integrals of motion: problem solved

$$\gamma - u_x = 1$$
$$u_y = \tilde{A}_y$$

$$u_x(\tilde{\phi}) = \frac{\tilde{A}_y^2(\tilde{\phi})}{2}$$

$$u_y(\tilde{\phi}) = \tilde{A}_y(\tilde{\phi})$$

$$\gamma(\tilde{\phi}) = 1 + \frac{\tilde{A}_y^2(\tilde{\phi})}{2}$$

Примеры на языке Python:

https://colab.research.google.com/drive/1QrVb7iY0zuaPdZYMFKif8P-fzZ3V5TUP?usp=sharing

shorturl.at/mnvN0

Plasma models: Vlasov equation

$$\frac{\partial f_e}{\partial t} + \mathbf{v}_e \cdot \nabla f_e - e\left(\mathbf{E} + \frac{\mathbf{v_e}}{c} \times \mathbf{B}\right) \cdot \frac{\partial f_e}{\partial \mathbf{p}} = 0$$

$$\frac{\partial f_i}{\partial t} + \mathbf{v}_i \cdot \nabla f_i + Z_i e\left(\mathbf{E} + \frac{\mathbf{v_i}}{c} \times \mathbf{B}\right) \cdot \frac{\partial f_i}{\partial \mathbf{p}} = 0$$

$$\nabla \times \mathbf{B} = \frac{4\pi \mathbf{j}}{c} + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t}$$

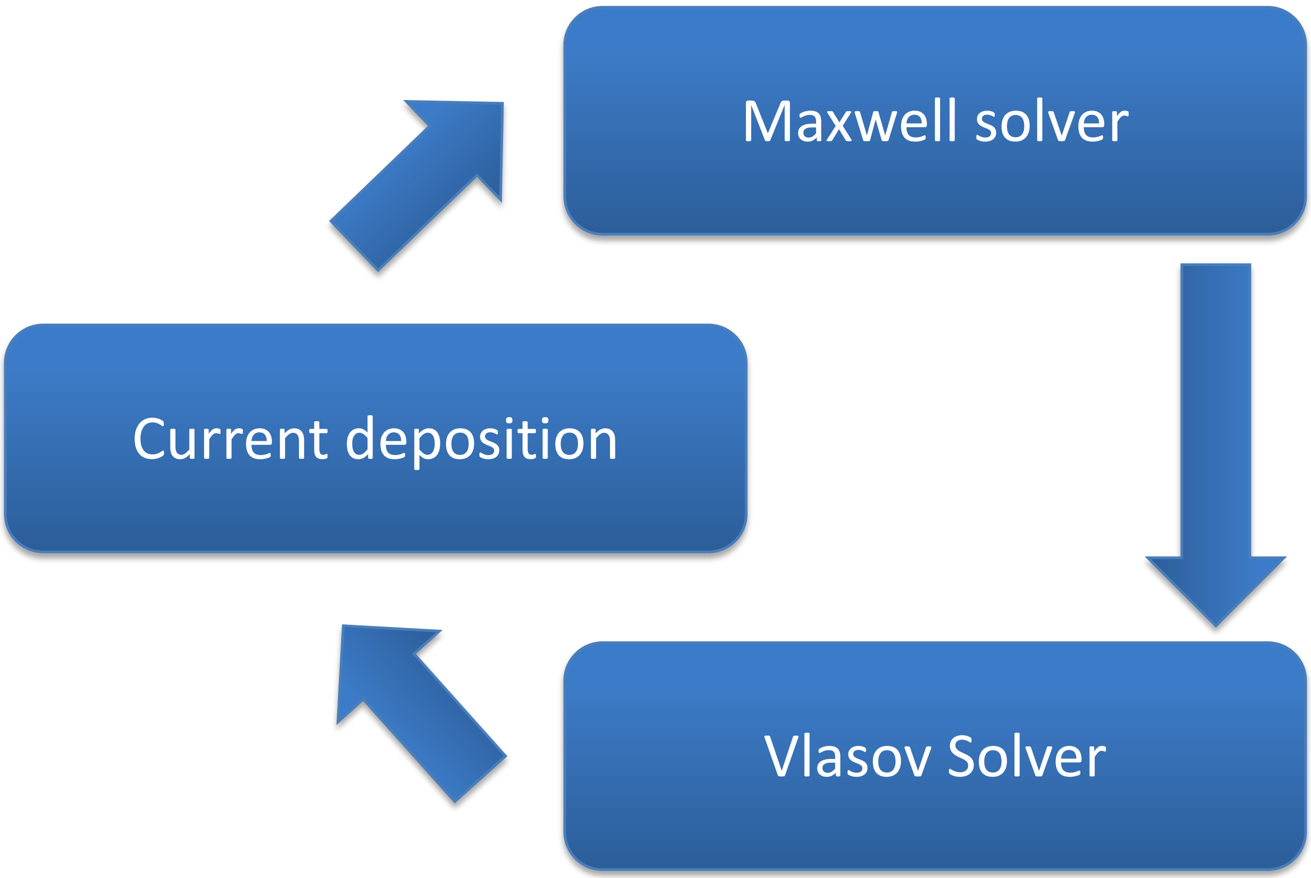
$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \cdot \mathbf{E} = 4\pi\rho$$

$$\nabla \cdot \mathbf{B} = 0$$

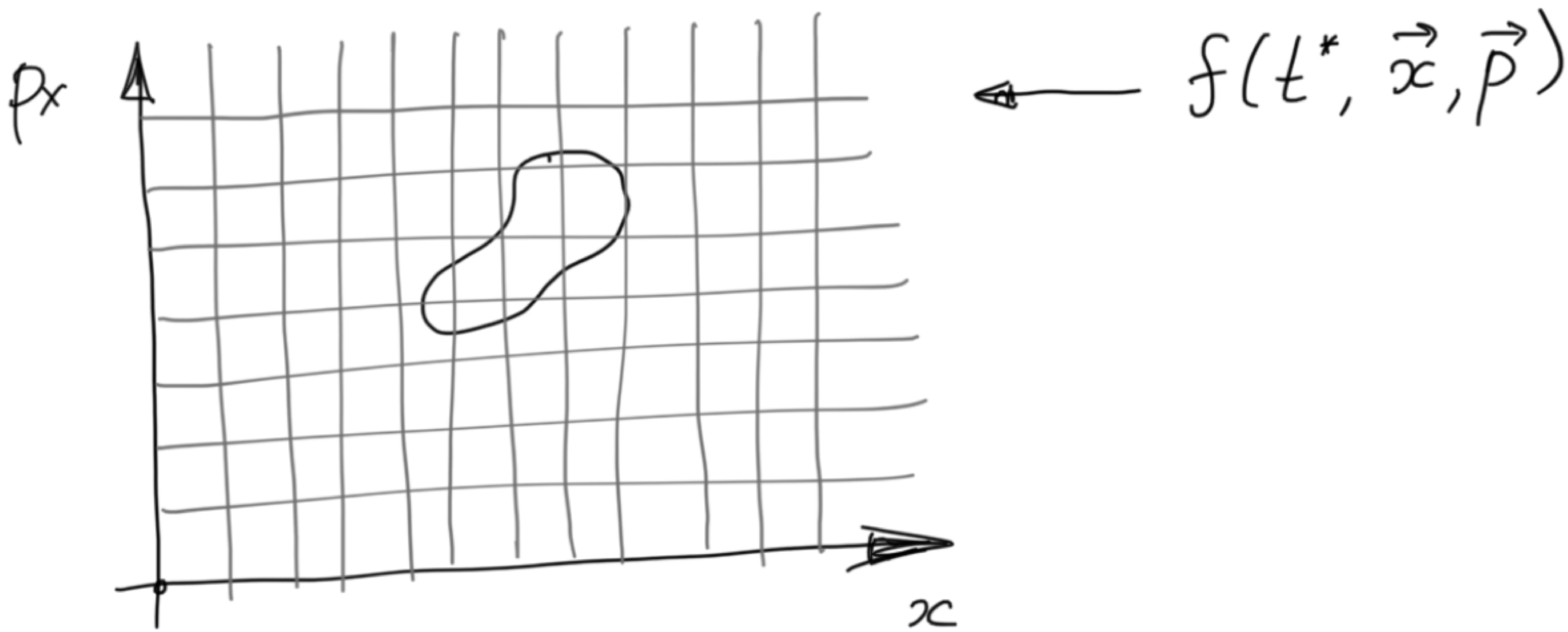
$$\rho = e\int (Z_i f_i - f_e)\mathrm{d}^3 p, \quad \mathbf{j} = e\int (Z_i f_i \mathbf{v}_i - f_e \mathbf{v}_e)\mathrm{d}^3 p, \quad \mathbf{v}_\alpha = \frac{\frac{\mathbf{p}}{m_\alpha}}{\left(1 + \frac{p^2}{(m_\alpha c)^2}\right)^{1/2}}$$

Vlasov-Maxwell equations

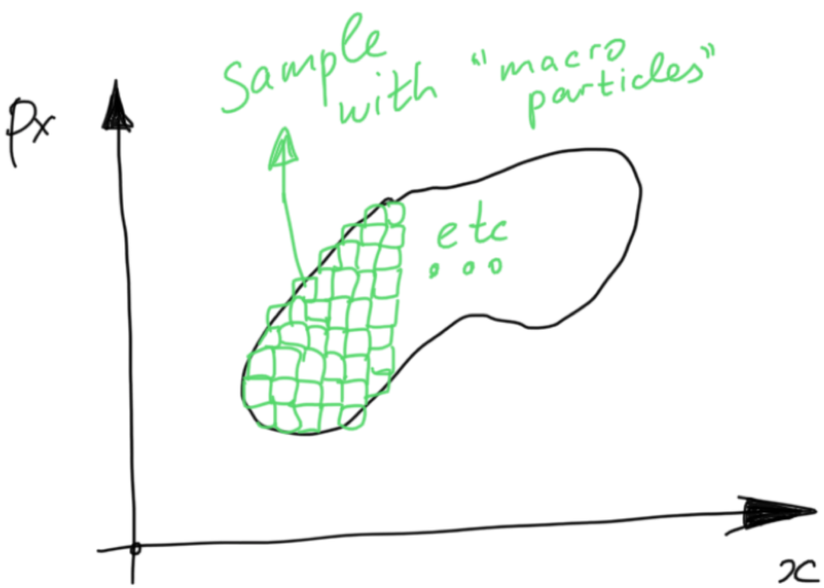


Plasma models: Vlasov equation

Cons:
- 6D space (3r 3p)
- even 1D tasks require 3-4 dimensions
- most of the simulation space is empty (i.e. cold thin plasma layer)



Plasma models: Particle-in-cell method



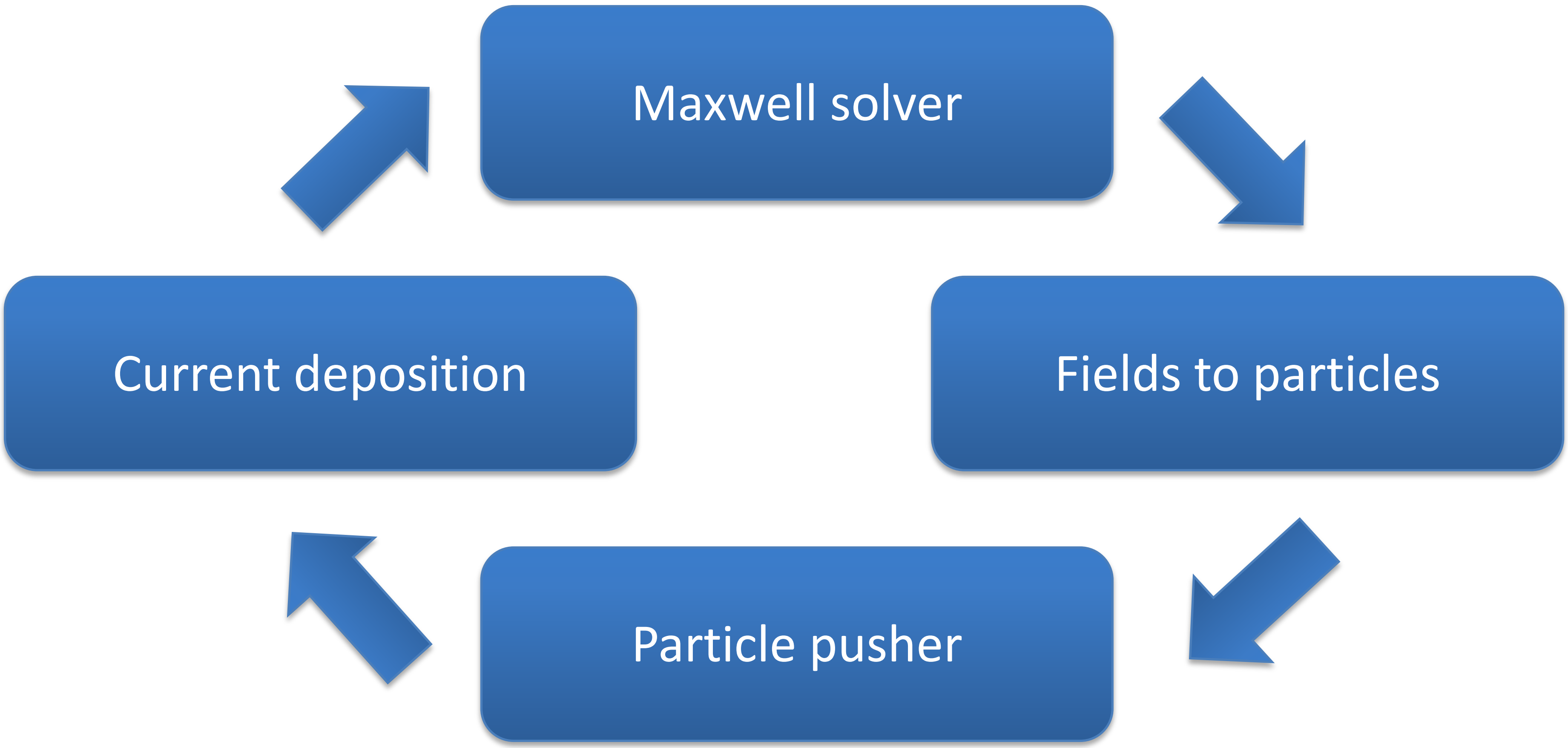
$$f = \sum_{i=1}^{N_p} S_x(\vec{x}-\vec{x}_i)\, S_p(\vec{p}-\vec{p}_i)$$

Typically:

$$S_p(\vec{p}-\vec{p}_i) = \delta(\vec{p}-\vec{p}_i)$$

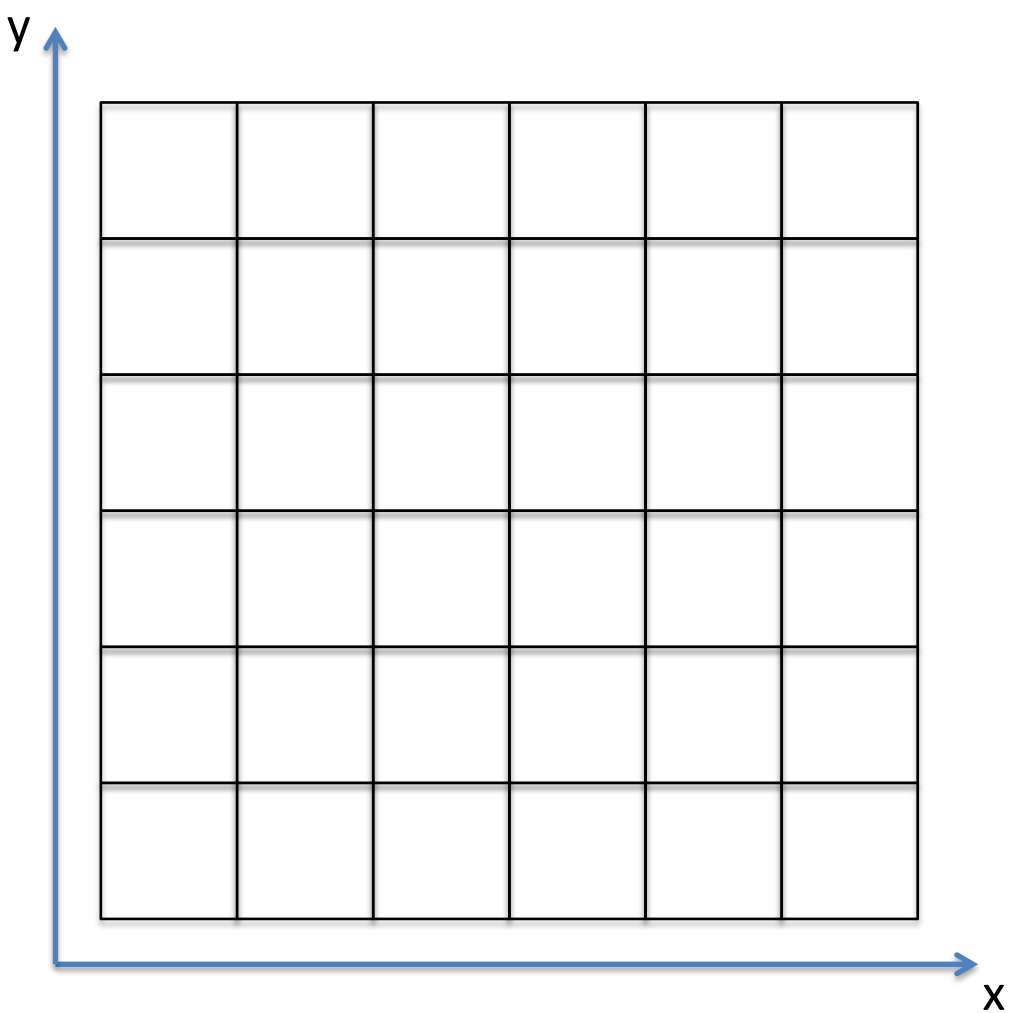
Number of "macroparticles" $\ll$ Number of real particles

Typical time step of the particle-in-cell method



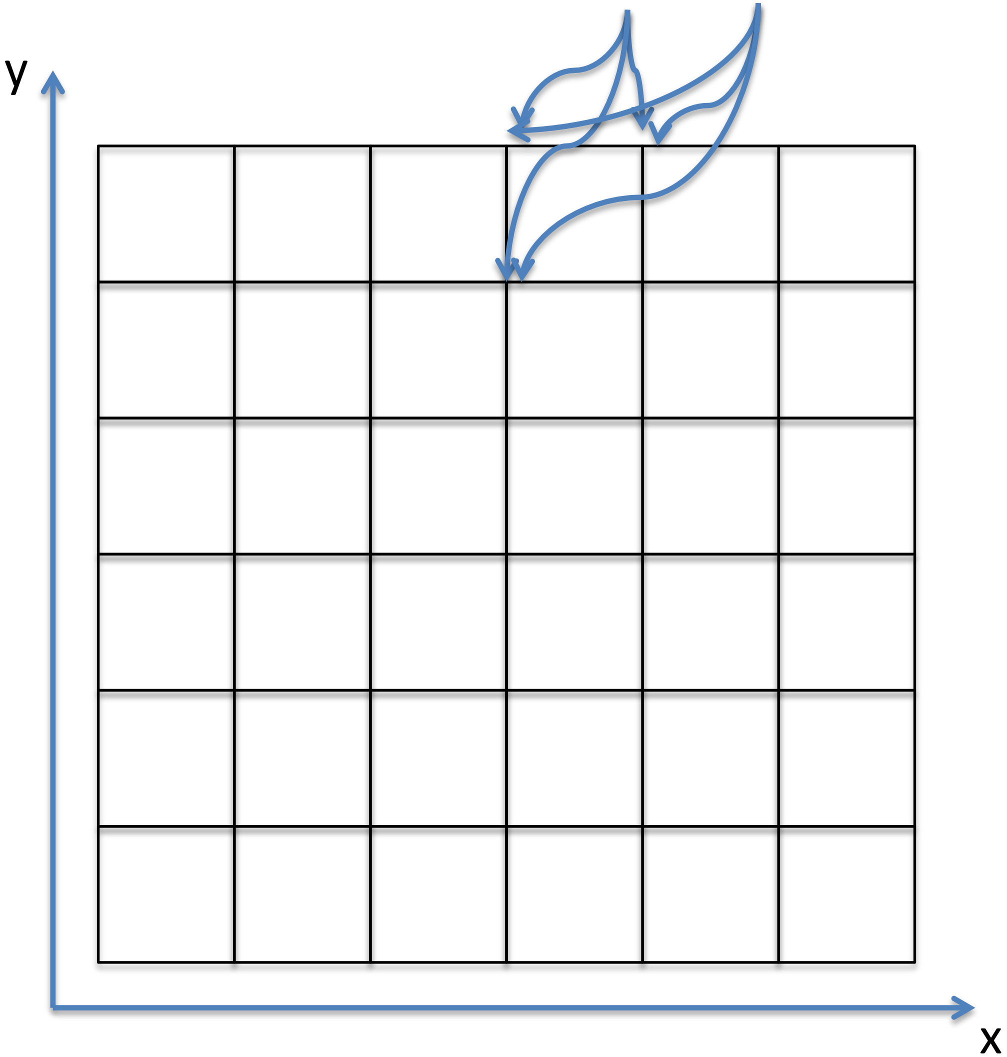
Particle-in-cell code is a workhorse in plasma simulations

Start with spatial grid



Particle-in-cell code is a workhorse in plasma simulations

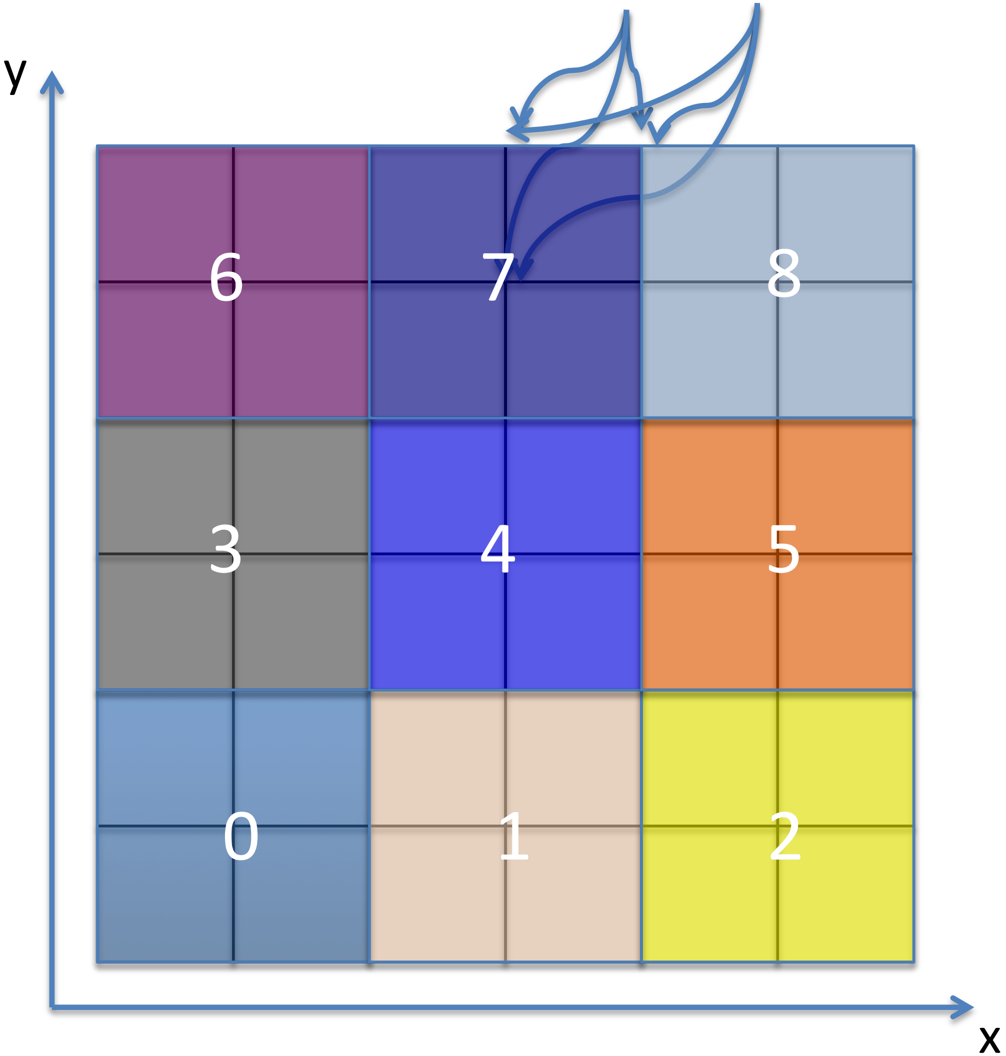
Electromagnetic fields **E** and **B** are defined on the grid nodes



- Electromagnetic fields are described by Maxwell equations
- Numerical solution is straightforward (similar to antennas modeling or computational photonics)

Particle-in-cell code is a workhorse in plasma simulations

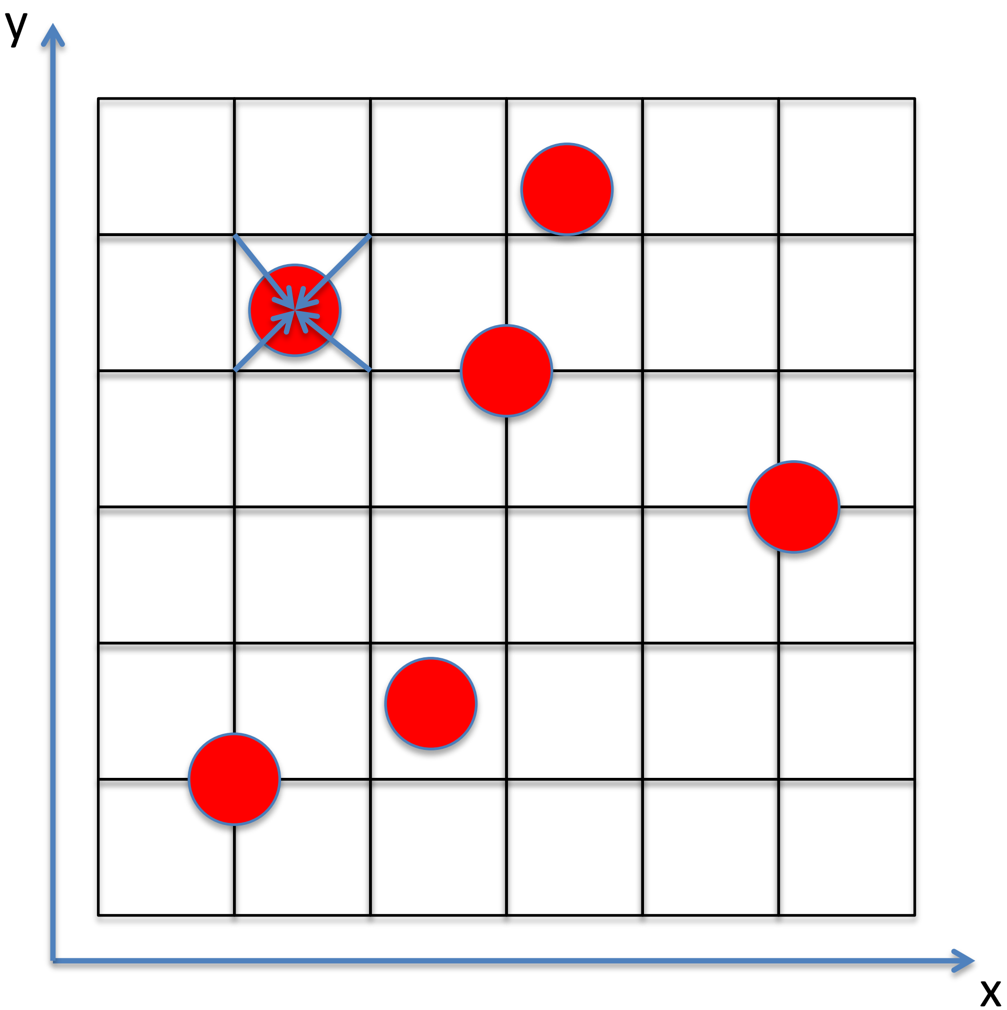
Electromagnetic fields **E** and **B** are defined on the grid nodes



- Electromagnetic fields are described by Maxwell equations
- Numerical solution is straightforward (similar to antennas modeling or computational photonics)
- parallelization is straightforward (CUDA has implementation of Maxwell solver in examples, MPI between nodes)

Particle-in-cell code is a workhorse in plasma simulations

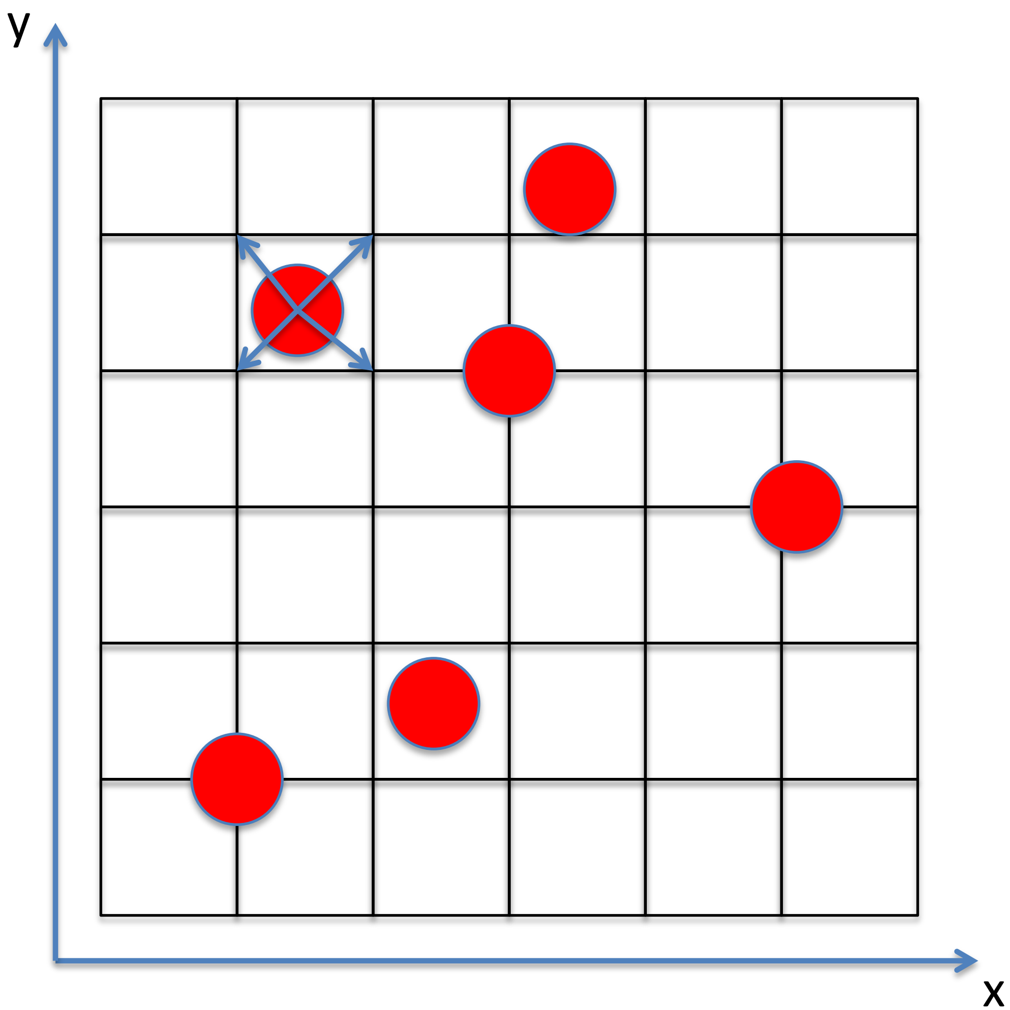
Particles (electrons and ions) can move freely



- Interpolate fields from the nodes to the particle position
- Move the particle (system of ordinary differential equations) one timestep using leapfrog, RK4, etc.

Particle-in-cell code is a workhorse in plasma simulations

Moving particles generate currents which we must plug in to Maxwell equations

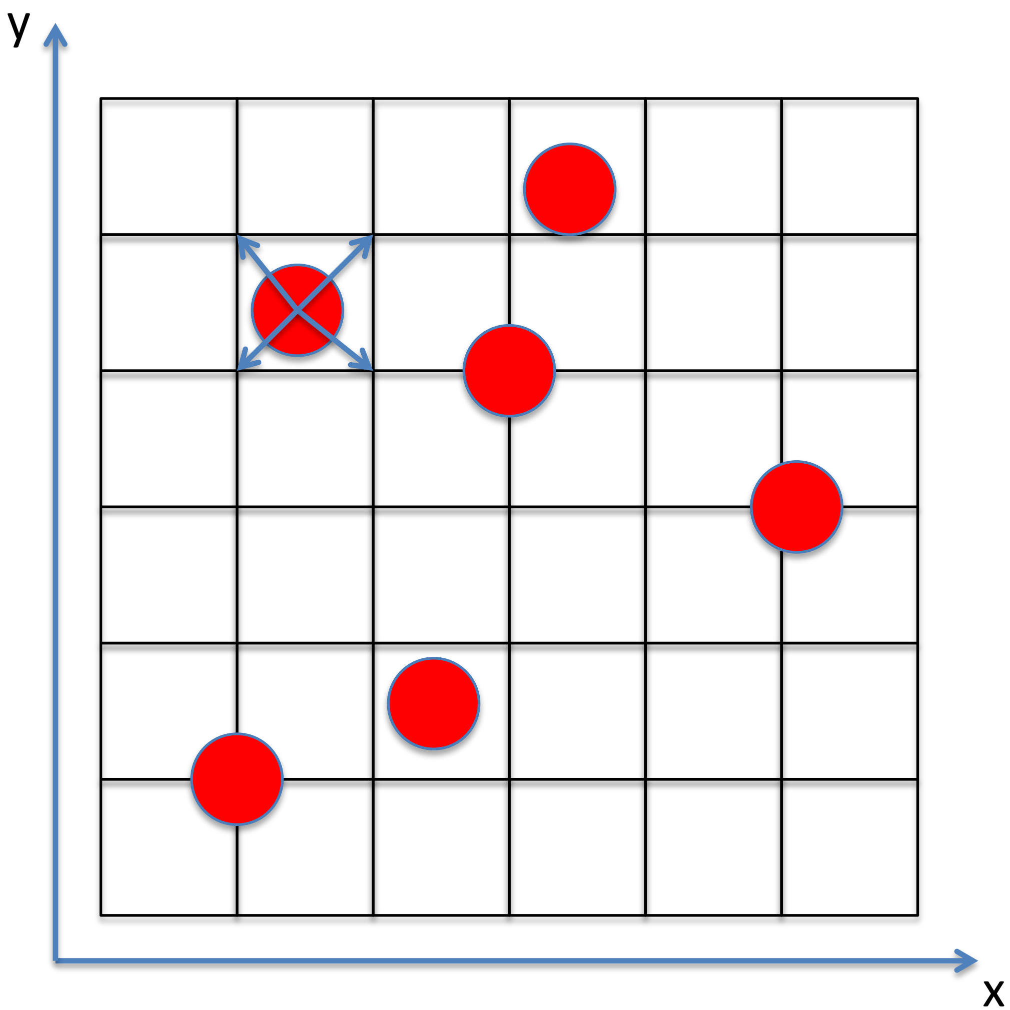


- Distribute currents based on particle motion

Simulations by S. Kuschel

Current deposition is easy for homogeneous particle distr.

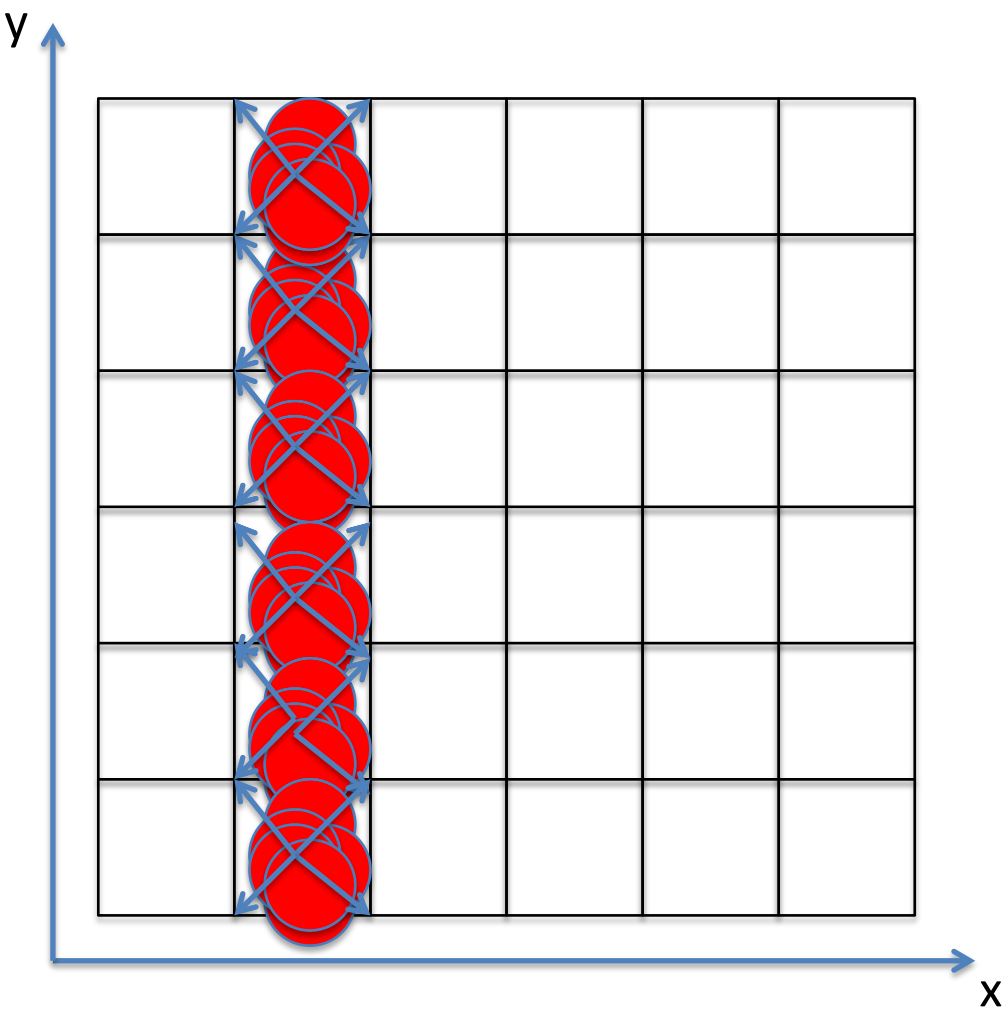
Moving particles generate currents which we must plug in to Maxwell equations



- Distribute currents based on particle motion

Serious bottleneck on SIMD architectures if not equally distr.

Moving particles generate currents which we must plug in to Maxwell equations



- Distribute currents based on particle motion

Race condition for accessing the same piece of memory

Even though the code was tested and showed good scalability for homogeneous scaling, simulations of thin plasmas are not effective on SIMD architectures.

Заключение

1. Введение - *узнали кратко о релятивистской лазерной плазме*
2. **Электрон в плоской электромагнитной волне**
    1. Аналитическое решение *(рецепт, по слайдам можно повторить вывод решения)*
    2. Численное решение (shorturl.at/mnvN0)
    3. Излучение электрона (shorturl.at/mnvN0)
3. **Моделирование взаимодействия мощного лазерного излучения с плазмой**
    1. Метод частиц-в-ячейке (Particle-In-Cell)